

A Non-Linear Stability Analysis of Magneto convection of a couple stress Fluid in the Presence of Anisotropic and Rotational Modulation

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Abstract

The Rayleigh Bènard convection of couple stress fluid in the presence of rotational modulation and magnetic field. Nusselt number, which is obtained as function of the slow time scale, is used to quantify heat transport. The effects of Chandrasekhar number, Prandtl number, Magnetic Prandtl number, Taylor number and Couple stress parameter are studied.

Keywords-Couple stress fluid, Rayleigh Bènard convection, Magnetic field, Rotational modulation.

1. Introduction

The classical Rayleigh Bènard convection studies the convection in fluids that arise due to variation in density as a result of temperature differences in the fluid layers. Couple stress fluids are used as a continuum model in many fluid applications that involve suspended particles. Couple stress fluids have distinct features, such as polar effects. These fluids find their uses in a number of processes that occur in industry, such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubrication and colloidal and suspension solutions. The constitutive equations for couple stress fluids were given by Stokes[1]. Since then, many authors have done some extensive studies on Rayleigh Bènard convection of couple stress fluids. Some of the notable studies performed in this area are by Sharma and Thakur [3], R.C Sharma and Shivani Sharma [4], Vivek Kumar and Sudhir Kumar [5]. Various practical applications of couple stress fluid in industries led to the studies on Rayleigh Bènard convection in couple stress fluids in the presence of rotation and magnetic field. Although rotation speed modulation led to the ideas of temperature and gravity modulation, the research work in this field is scarce. Therefore in the present work, we intend to investigate the onset of Rayleigh Bènard convection in a couple stress fluid in the presence of magnetic field and rotational modulation using weak nonlinear stability analysis with the help of Ginzburg- Landau model. The objective of this paper is to study how the onset criteria for convection is affected by Prandtl number, Magnetic Prandtl number, Taylor number, Chandrasekhar number and Couple stress parameter.

II. MATHEMATICAL FORMULATION

Consider a layer of couple stress liquid confined between two infinite horizontal surfaces separated by a distance d . A Cartesian system is taken with origin in the lower boundary and z -axis vertically upward. Lower surface is maintained at higher temperature $T_0 + \Delta T$ and upper surface is maintained at temperature T_0 . A uniform magnetic field and variable rotational speed $\bar{\Omega}(t) = \Omega_0 (1 + \varepsilon^2 \delta \cos \omega t)$ is applied along the positive z axis (See Fig.1)

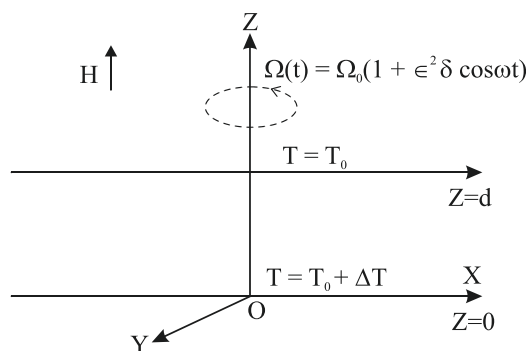


Fig.1.

The basic governing equations are:

$$\nabla \vec{q} = 0 \quad \dots (1)$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} + 2\vec{\Omega}(t) \times \vec{q} \right] = -\nabla P_{rm} + \rho \vec{g} + \mu \nabla^2 \vec{q} - \mu' \nabla^4 \vec{q} + \mu_m (\vec{H} \cdot \nabla) \vec{H} \quad \dots (2)$$

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \chi_i \nabla^2 T \quad \dots (3)$$

$$\rho = \rho_0 (1 - \alpha_i (T - T_0)) \quad \dots (4)$$

$$\frac{\partial \vec{H}}{\partial t} + (\vec{q} \cdot \nabla) \vec{H} = (\vec{H} \cdot \nabla) \vec{q} + \gamma_m \nabla^2 \vec{H} \quad \dots (5)$$

$$\nabla \cdot \vec{H} = 0 \quad \dots (6)$$

The basic state is assumed to be quiescent, i.e.,

$$\vec{q}_b = (0, 0, 0), \vec{\Omega} = \Omega_0 k, T = T_b(z), P_{rm,b} = P_{rm,b}(z), \rho = \rho_b(z), \vec{H}_b = H_0 k \quad \dots (7)$$

where the subscript b denotes the basic state.

Now we introduce infinitesimally small perturbations to the basic state to get

$$\vec{q} = \vec{q}', \rho_b + \rho', P_{rm} = P_{rm,b} + P_{rm}', T = T_b + T', \vec{H} = H_0 k + \vec{H}' \quad \dots (8)$$

Substituting (8) in (1) to (6), eliminating pressure by cross differentiation, introducing the stream functions

$$u' = \frac{\partial \Psi}{\partial z}, w' = -\frac{\partial \Psi}{\partial x}, H_x = \frac{\partial \phi}{\partial z}, H_z = -\frac{\partial \phi}{\partial x}, \text{ we get}$$

$$\rho_0 \left[\frac{\partial}{\partial t} (\nabla^2 \Psi) + J(\nabla^2 \Psi, \Psi) \right] = 2\rho_0 \vec{\Omega}(t) \frac{\partial v'}{\partial z} + \mu \nabla^4 \Psi - \mu' \nabla^6 \Psi + \mu_m H_0 \frac{\partial}{\partial z} \nabla^2 \phi + \mu_m J(\nabla^2 \phi, \phi) - \rho_0 \alpha_i \frac{\partial T'}{\partial x} g \quad \dots (9)$$

$$\rho_0 \left[\frac{\partial v'}{\partial t} + J(v', \Psi) + 2\vec{\Omega}(t) \frac{\partial \Psi}{\partial z} \right] = \mu \nabla^2 v' - u' \nabla^4 v' \quad \dots (10)$$

$$\frac{\partial T'}{\partial t} + J(T', \Psi) = \chi_i \nabla^2 T' - \frac{\nabla T}{d} \frac{\partial \Psi}{\partial x} \quad \dots$$

(11)

$$\frac{\partial \phi}{\partial t} - J(\Psi, \phi) = H_0 \frac{\partial \Psi}{\partial t} + \gamma_m \nabla^2 \phi \quad \dots (12)$$

Equations (9) to (12) are rendered dimensionless using the following transformations.

$$(x^*, z^*) = \left(\frac{x}{d}, \frac{z}{d} \right), \Psi^* = \frac{\Psi}{\chi_i}, \nabla^* = \frac{\nabla}{(1/d)}, t^* = \frac{t}{(d^2 / \chi_i)}, T^* = \frac{T}{\Delta T}, u^* = \frac{u}{(\chi_i / d)}, w^* = \frac{w}{(\chi_i / d)}, H^* = \frac{H}{H_0}$$

The dimensionless equations are given by

$$\begin{bmatrix} -\nabla^4 + C\nabla^6 & R_0 \frac{\partial}{\partial x} & -QPm \frac{\partial}{\partial z} \nabla^2 & -Ta^{1/2} (1 + \varepsilon^2 \delta \cos \omega t) \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ -\frac{\partial}{\partial z} & 0 & -Pm \nabla^2 & 0 \\ Ta^{1/2} (1 + \varepsilon^2 \delta \cos \omega t) \frac{\partial}{\partial z} & 0 & 0 & -\nabla^2 + C\nabla^4 \end{bmatrix} \begin{bmatrix} \Psi \\ T \\ \phi \\ v \end{bmatrix} =$$

$$\begin{bmatrix} -\frac{1}{Pr} \left[\frac{\partial}{\partial t} \nabla^2 + J(\nabla^2 \Psi, \Psi) \right] + QPm J(\nabla^2 \phi, \phi) \\ -\frac{\partial T}{\partial t} + J(\Psi, T) \\ -\frac{\partial \phi}{\partial t} + J(\Psi, \phi) \\ -\frac{1}{Pr} \left[\frac{\partial v}{\partial t} - J(\Psi, v) \right] \end{bmatrix} \dots (13)$$

Where $Pr = \frac{u}{\rho_0 \chi_i}$, $Ta = \left(\frac{2\rho_0 d^2 \Omega_0}{\mu} \right)^2$, $Pm = \frac{\gamma_m}{\chi_i}$, $Q = \frac{\mu_m H_0^2 d^2}{\mu \gamma_m}$, $C = \frac{\mu'}{\mu d^2}$ and $R = \frac{\alpha_l g d^3 \Delta T \rho_0}{\mu \chi_i}$

The system of equations (13) are solved for free- free isothermal boundary conditions

$$\Psi = \nabla^2 \Psi = T = \phi = D\phi = 0 \text{ at } z = 0, 1.$$

Finite Amplitude Equation, Heat and Mass Transfer

We now introduce the following asymptotic expansions in (13).

$$R = R_0 + \varepsilon^2 R_2 + \varepsilon^4 R_4 + \dots$$

$$\Psi = \varepsilon \Psi_1 + \varepsilon^2 \Psi_3 + \varepsilon^3 \Psi_3 + \dots$$

$$v = \varepsilon v_1 + \varepsilon^2 v_2 + \varepsilon^3 v_3 + \dots$$

$$T = \varepsilon T_1 + \varepsilon^2 T_2 + \varepsilon^3 T_3 + \dots$$

$$\phi = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \dots$$

At the lowest mode, we have

$$\begin{bmatrix} -\nabla^4 + C\nabla^6 & R_0 \frac{\partial}{\partial x} & -QPm \frac{\partial}{\partial z} \nabla^2 & -Ta^{1/2} \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ -\frac{\partial}{\partial z} & 0 & -Pm \nabla^2 & 0 \\ Ta^{1/2} \frac{\partial}{\partial z} & 0 & 0 & -\nabla^2 + C\nabla^4 \end{bmatrix} \begin{bmatrix} \Psi_1 \\ T_1 \\ \phi_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \dots (14)$$

The solution to the lowest order system is given as

$$\Psi_1 = A(\tau) \sin(K_c x) \sin(\pi z)$$

$$T_1 = -\frac{K_c}{\delta^2} A(\tau) \cos(K_c x) \sin(\pi z)$$

$$\phi_1 = \frac{\pi}{Pm\delta^2} A(\tau) \sin(K_c x) \cos(\pi z)$$

$$v_1 = -\frac{\pi Ta^{1/2}}{\delta^2 (1 + C\delta^2)} A(\tau) \sin(K_c x) \cos(\pi z)$$

$$\text{where } \delta^2 = K_c^2 + \pi^2$$

$$\text{The critical Rayleigh number is given by } R_0 = \frac{Ta\pi^2}{K_c^2 (1 + C\delta^2)} + \frac{Q\pi^2\delta^2}{K_c^2} + \frac{\delta^6 (1 + C\delta^2)}{K_c^2}$$

At the second order, we have,

$$\begin{bmatrix} -\nabla^4 + C\nabla^6 & R_0 \frac{\partial}{\partial x} & -Q Pm \frac{\partial}{\partial z} \nabla^2 & -Ta^{1/2} \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ -\frac{\partial}{\partial z} & 0 & -Pm \nabla^2 & 0 \\ Ta^{1/2} \frac{\partial}{\partial z} & 0 & 0 & -\nabla^2 + C\nabla^4 \end{bmatrix} \begin{bmatrix} \Psi_2 \\ T_2 \\ \phi_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \\ R_{23} \\ R_{24} \end{bmatrix}$$

where

$$R_{21} = 0$$

$$R_{22} = -\frac{A(\tau)^2 K_c^2 \pi}{2\delta^2} \sin(2\pi z)$$

$$R_{23} = -\frac{A(\tau)^2 K_c^2 \pi^2}{2Pm\delta^2} \sin(2K_c x)$$

$$R_{24} = -\frac{A(\tau)^2 K_c \pi^2 Ta^{1/2}}{2Pm\delta^2 (1 + C\delta^2)} \sin(2K_c x)$$

The second order solutions are as follows.

$$\Psi_2 = 0$$

$$T_2 = -\frac{K_c^2}{8\pi\delta^2} A(\tau)^2 \sin(2\pi z)$$

$$\phi_2 = -\frac{\pi^2}{8K_c Pm\delta^2} A(\tau)^2 \sin(2K_c x)$$

$$v_2 = \frac{\pi^2 Ta^{1/2}}{8K_c Pr\delta^2 (1 + C\delta^2) (1 + 4K_c^2)} A(\tau)^2 \sin(2K_c x)$$

$$\text{The horizontally averaged Nusselt number is given by } Nu(\tau) = 1 + \frac{K_c^2}{8\pi\delta^2} A(\tau)^2 \quad \dots(15)$$

At the third order, we have

$$\begin{bmatrix} -\nabla^4 + C\nabla^6 & R_0 \frac{\partial}{\partial x} & -Q P m \frac{\partial}{\partial z} \nabla^2 & -T a^{1/2} \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ -\frac{\partial}{\partial z} & 0 & -P m \nabla^2 & 0 \\ T a^{1/2} \frac{\partial}{\partial z} & 0 & 0 & -\nabla^2 + C\nabla^4 \end{bmatrix} \begin{bmatrix} \Psi_3 \\ T_3 \\ \phi_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \\ R_{34} \end{bmatrix}$$

where

$$\begin{aligned} R_{31} &= -\frac{1}{Pr} \frac{\partial}{\partial \tau} \nabla^2 \Psi_1 + \frac{1}{Pr} \left[J(\Psi_1, \nabla^2 \Psi_1) + J(\Psi_2, \nabla^2 \Psi_1) \right] + Q P m \left[J(\nabla^2 \phi_2, \phi_1) + J(\nabla^2 \phi_1, \phi_2) \right] - R_2 \frac{\partial T_1}{\partial x} = T a^{1/2} \delta \cos \omega t \frac{\partial v_1}{\partial z} \\ R_{32} &= J(\Psi_1, T_2) + J(\Psi_2, T_1) - \frac{\partial T_1}{\partial \tau} \\ R_{33} &= J(\Psi_1, \phi_2) + J(\Psi_2, \phi_1) - \frac{\partial \phi_1}{\partial \tau} \\ R_{34} &= -\frac{1}{Pr} \frac{\partial v_1}{\partial \tau} - T a^{1/2} \delta \cos \omega t \frac{\partial \Psi_1}{\partial z} \end{aligned}$$

Now we apply the solvability condition for the criterion of the third order solution of the system (16) to arrive at the non-autonomous Ginzburg Landau equation for stationary convection with time periodic coefficients in the form

$$A_1 A'(\tau) - A_2 A(\tau) + A_3 A(\tau)^3 = 0$$

where

$$\begin{aligned} A_1 &= \frac{\delta^2}{Pr} + \frac{R_0 K_c^2}{\delta^4} - \frac{Q \pi^2}{\delta^2 P m} - \frac{T a \pi^2}{\delta^4 (1 + C \delta^2)^2 Pr}, A_2 = \frac{K_c^2 R_2}{\delta^2} - \frac{2 T a \pi^2 \delta \cos \omega t}{\delta^2 (1 + C \delta^2)} \\ A_3 &= \frac{Q \pi^4 K_c^2}{2 P m^2 \delta^4} - \frac{Q \pi^4}{4 P m^2 \delta^2} + \frac{K_c^4 R_0}{8 \delta^4} \end{aligned}$$

The Ginzburg-Landau equation given in (17) is Bernoulli equation and hence, obtaining its analytical solution is difficult as it is autonomous in nature. In view of this, it has been solved numerically using Mathematica, subject to the initial condition $A(0) = a_0$, where a_0 is the chosen initial amplitude of convection.

III. RESULTS AND DISCUSSIONS

In this paper, a nonlinear analysis is performed to study the effect of magnetic field and rotational modulation on Rayleigh Bénard convection of a couple stress fluid. Autonomous Ginzburg-Landau equation is derived to study the effects of parameters of the problem on heat and mass transfer.

Fig (2) shows the variation of Nu with for different values of couple stress parameter C . It is observed that as C increases Nu decreases. The presence of couple stress increases the viscosity of the fluid and hence as C increases, more heating is required to make the system unstable, indicating the stabilising effect of C . Fig (3) shows the variation of Nu with for

different values of Pr . It is found that Nu increases as Pr increases, indicating the destabilising effect. Fig (4) to (6) show the variation of Nu with for different values of Ta , Pm and Q respectively. It is found that Nu decreases with increase in Ta , Pm and Q , showing the delayed onset of convection.

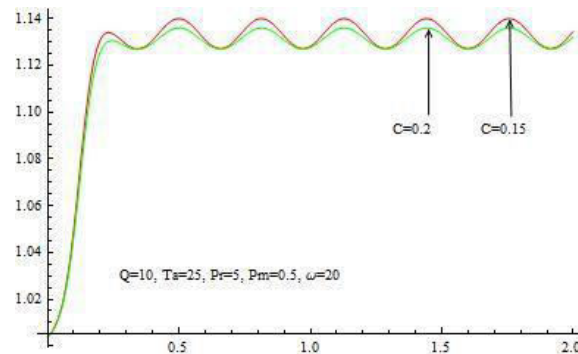


Fig. 2. Variation of Nu with τ for different values of C

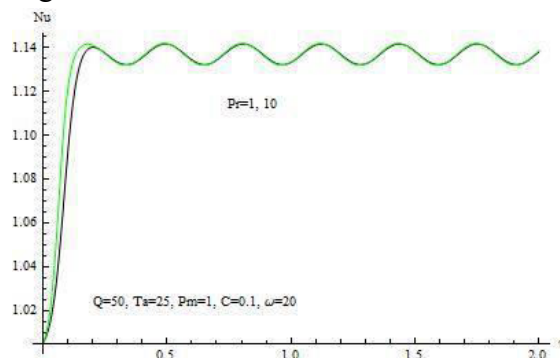


Fig. 3. Variation of Nu with τ for different values of Pr

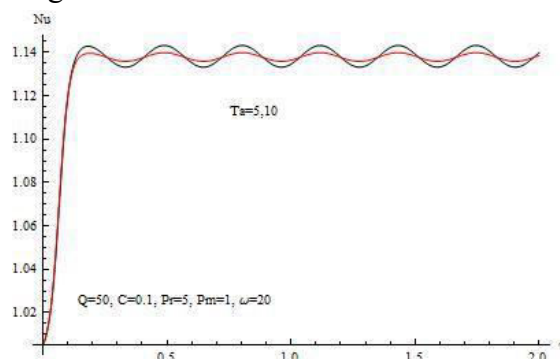


Fig. 4. Variation of Nu with τ for different values of Ta

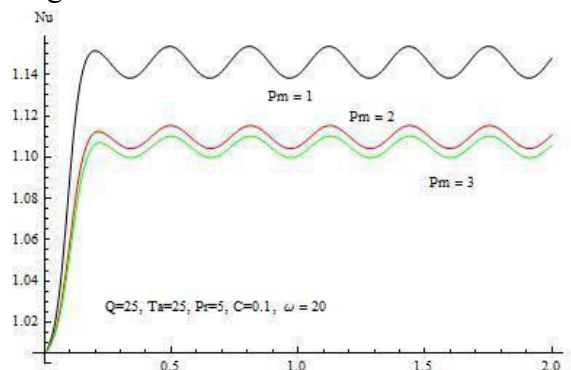


Fig. 5. Variation of Nu with τ for different values of Pm

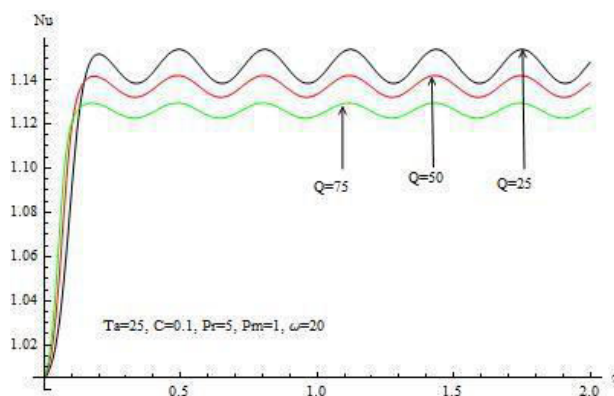


Fig. 6. Variation of Nu with τ for different values of Q

IV. CONCLUSIONS

The combined effect of magnetic field and rotational modulation on Rayleigh Bènard convection of a couple stress fluid has been studied by employing nonlinear analysis using Ginzburg- Landau model. The results have been obtained in terms of Nusselt number, and the effects of various parameters have been obtained graphically. The following are the observations.

1. The effect of increasing Couple stress parameter is to decrease the Nusselt number, thus indicating stabilising effect.
2. The effect of increasing Prandtl number is to advance the onset of convection, and thus heat and mass transfer.
3. Increasing Taylor number has a stabilising effect by delaying the onset of convection, hence reducing heat and mass transfer.
4. The effect of increasing Magnetic Prandtl number is to decrease Nusselt number, thus indicating stabilising effect.
5. The effect of increasing Chandrasekhar number is to decrease Nusselt number, hence delaying the onset of convection.

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