DOUBLE TRIANGULAR DECOMPOSITION OF GRAPHS

S.Chitra Devi[1*], M.Subbulakshmi[2], S.Chandrakala[3]

[1]Research Scholar, Reg. No. 19222052092002, G.Venkataswamy Naidu College, Kovilpatti.
[2]Associate Professor, PG and Research Department of Mathematics, G.Venkataswamy Naidu College, Kovilpatti.
[3]Associate Professor, Department of Mathematics, T.D.M.N.S College, T.Kallikulam.
Affiliated to Manonmaniam Sundaranar University, Tirunelveli.
E – mail: schitradevimaths2016@gmail.com

ABSTRACT: Let G = (V, E) be a simple connected graph of order p and size q. If $\{G_1, G_2, \ldots, G_n\}$ are edge disjoint subgraphs of G such that $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) \cup \ldots \cup E(G_n)$ then $\{G_1, G_2, \ldots, G_n\}$ is said to be a Decomposition of a graph G. A graph G is said to have Double Triangular Decomposition (DTD) if G can be decomposed into two copies of $\{G_1, G_2, \ldots, G_n\}$ such that each subgraph G_i is connected and $|E(G_i)| = \frac{i(i+1)}{2}$ for $1 \leq i \leq n$. In this paper we investigate Double Triangular Decomposition of graphs.

Keywords – Triangular decomposition, Double Triangular Decomposition.

1 Introduction

A graph G, referred to here is an undirected connected graph without loops or multiple edges. A path of length n is denoted by P_n. A cycle on n vertices is denoted by C_n. A wheel on n vertices is denoted by W_n. [4] The concept of Continuous Monotonic Decomposition of graphs was introduced by N.Gnana Dhas and J.Paulraj Joseph. [5] S.Asha, and R.kala discussed on Continuous Monotonic Decomposition of some special class of Graphs. [3] Joseph Varghese and A.Antonysamy discussed on Double Continuous Monotonic Decomposition of Graphs. Terms not defined here are used in the sense of Harary [1]. In this paper we investigate Double Triangular Decomposition of graphs.

Definition 1.1 A decomposition of a graph G is a collection of edge disjoint subgraphs $\{G_1, G_2, G_3, \ldots, G_n\}$ of G such that every edge of G belongs to exactly one of the subgraph G_i.

Definition 1.2 A graph G of size $q = \binom{n+2}{3}$ is said to have a Triangular decomposition (TD) if G can be decomposed into n - subgraphs $\{G_1, G_2, G_3, \ldots, G_n\}$ such that each G_i is connected and $|E(G_i)| = \binom{i+1}{2}$ for $1 \leq i \leq n$.

Definition 1.3 A graph G of size $q = 2\binom{n+2}{3}$ is said to have Double Triangular Decomposition (DTD) if G can be decomposed into two copies of $\{G_1, G_2, \ldots, G_n\}$ such that each subgraph G_i is connected and $|E(G_i)| = \binom{i+1}{2}$ for $1 \leq i \leq n$.

2. Double Triangular Decomposition of Graphs

Lemma 2.1. If $k \equiv 0 (\mathrm{mod}\,3)$, then every graph admits a Double Triangular Decomposition $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k\}$ where k is the number of decomposition.
Proof. We have k = 3r, r∈N. We prove this theorem by using induction method. When r = 1, k = 3. Now $\frac{k(k+1)(k+2)}{3} = \frac{3x4x5}{3} = 20$ can be decomposed into $\{2G_1, 2G_2, 2G_3\}$. Hence the result is true for r = 1.

Assume that the result is true for r-1. Then k = 3(r-1) = 3r-3 and q' $= \frac{(3r-3)(3r-2)(3r-1)}{3}$ can be decomposed into $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{3(r-1)}\}$. Now to prove the result is true for r. Then k = 3r and $q = \frac{(3r)(3r+1)(3r+2)}{3}$. We have to prove that $q = \frac{(3r)(3r+1)(3r+2)}{3}$ can be decomposed into $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{3r}\}$.

Now $q = \frac{(3r)(3r+1)(3r+2)}{3}$

$= \frac{3(9r^3 + 9r^2 + 2r)}{3}$

$= \frac{3(r-1)(9r^2-9r+2)}{3} + 2\{\frac{(18r^2-12r+2)}{2} + \frac{(9r^2+3r)}{2}\}$

$= \frac{(3r-3)(9r^2-3r-6r+2)}{3} + 2\{\frac{(3r-1)(6r-2)}{2} + \frac{(3r)(3r+1)}{2}\}$

$= q' + 2\{\frac{(3r-2)(3r-1)}{2} + \frac{(3r-1)(3r)}{2} + \frac{(3r)(3r+1)}{2}\}$.

Therefore $q = \frac{(3r)(3r+1)(3r+2)}{3}$ can be decomposed into $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{3r}\}$. Hence by induction hypothesis if $k \equiv 0 (\mathrm{mod}\,3)$, then every graph admits a Double Triangular Decomposition $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k\}$.

This complete the proof.

Lemma 2.2. If $k+1 \equiv 0 (\mathrm{mod}\,3)$, then every graph admits a Double Triangular Decomposition $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k\}$ where k is the number of decomposition.
Proof. We have k = 3r-1, r∈N. We prove this theorem by using induction method. When r = 1, k = 2. Now $\frac{k(k+1)(k+2)}{3} = \frac{2x3x4}{3} = 8$ can be decomposed into $\{2G_1, 2G_2\}$. Hence the result is true for r = 1.

Assume that the result is true for r-1. Then k = 3(r-1)-1 = 3r-4 and q' $= \frac{(3r-4)(3r-3)(3r-2)}{3}$ can be decomposed into $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{3(r-1)-1}\}$. Now to prove the result is true for r. Then k = 3r-1 and $q = \frac{(3r-1)(3r)(3r+1)}{3}$. We have to prove that $q = \frac{(3r-1)(3r)(3r+1)}{3}$ can be decomposed into $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{3r-1}\}$.

Now $q = \frac{(3r-1)(3r)(3r+1)}{3}$

$= \frac{27r^3-3r}{3}$

$= \frac{(3r-4)(9r^2-15r+6)}{3} + 2\{\frac{(3r-2)(6r-4)}{2} + \frac{(3r-1)(3r)}{2}\}$

$= q' + 2\{\frac{(3r-3)(3r-2)}{2} + \frac{(3r-2)(3r-1)}{2} + \frac{(3r-1)(3r)}{2}\}$.

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Therefore q = $\frac{(3r-1)(3r)(3r+1)}{3}$ can be decomposed into {$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{3r-1}$}.
Hence by induction hypothesis if k+1≡ 0(mod3), then every graph admits a Double Triangular Decomposition {$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k$}.

This complete the proof.

Lemma 2.3. If k+2 ≡ 0(mod3), then every graph admits a Double Triangular Decomposition {$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k$} where k is the number of decomposition.

Proof. We have k = 3r-2, r∈N. We prove this theorem by using induction method. When r = 1, k = 1. Now $\frac{k(k+1)(k+2)}{3} = \frac{1x2x3}{3} = 2$ can be decomposed into {$2G_1$}. Hence the result is true for r =1.

Assume that the result is true for r-1. Then k=3(r-1)-2 = 3r-5 and q' = $\frac{(3r-5)(3r-4)(3r-3)}{3}$ can be decomposed into {$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{3(r-1)-2}$}. Now to prove the result is true for r. Then k = 3r-2 and q = $\frac{(3r-2)(3r-1)(3r)}{3}$. We have to prove that q = $\frac{(3r-2)(3r-1)(3r)}{3}$ can be decomposed into {$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{3r-2}$}.

Now q = $\frac{(3r-2)(3r-1)(3r)}{3}$

$= \frac{3(9r^3-9r^2+2r)}{3}$

$= \frac{3(3r-5)(3r^2-7r+4)}{3} + 2\{\frac{(9r^2-21r+12)}{2} + \frac{(18r^2-24r+8)}{2}\}$

$= \frac{(3r-5)(9r^2-9r-12r+12)}{3} + 2\{\frac{(3r-4)(3r-3)}{2} + \frac{(3r-2)(6r-4)}{2}\}$

$= q' + 2\{\frac{(3r-4)(3r-3)}{2} + \frac{(3r-3)(3r-2)}{2} + \frac{(3r-2)(3r-1)}{2}\}$.

Therefore q = $\frac{(3r-2)(3r-1)(3r)}{3}$ can be decomposed into {$2G_1, 2G_2, \ldots\ldots\ldots, 2G_{3r-2}$}. Hence by induction hypothesis if k+2≡ 0(mod3), then every graph admits a Double Triangular Decomposition {$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k$}.

This complete the proof.

Theorem 2.4. For any integer m, P_m has a Double Triangular Decomposition {$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k$} if and only if there exists an integer k satisfying the following properties:

(i) $m = \frac{k^3+3k^2+2k+3}{3}$

(ii) $k = \begin{cases} 3r-2 & r \in N \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

where k denotes the number of decompositions.

Proof. Let G = P_m. Then q(G) = m - 1. Assume G has a Double Triangular Decomposition. By the definition of Double Triangular Decomposition, q(G) = $\frac{k(k+1)(k+2)}{3}$.

Hence m-1 = $\frac{k(k+1)(k+2)}{3}$.

⇨ $m = \frac{k^3+3k^2+2k+3}{3}$.

Since m is an integer, k = $\begin{cases} 3r-2 & r \in N \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

Conversely assume (i) m = $\frac{k^3+3k^2+2k+3}{3}$ (ii) k = $\begin{cases} 3r-2 & r \in N \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

Consider G = P_m. Then q(G) = m-1. Let V = $\{x_j \,/\, 1 \le j \le m\}$ be the vertex set and E = $\{x_j x_{j+1} / 1 \le j \le m-1\}$ be an edge set. By lemma 2.1, 2.2 and 2.3, G can be decomposed into $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k\}$. Thus G admits Double Triangular Decomposition.

Table 2.5. List of first 10 DTD of P_m

m	q(G)	Double Triangular Decomposition
3	2	$2G_1$
9	8	$2G_1, 2G_2$
21	20	$2G_1, 2G_2, 2G_3$
41	40	$2G_1, 2G_2, 2G_3, 2G_4$
71	70	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_5$
113	112	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_6$
169	168	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_7$
241	240	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_8$
331	330	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_9$
441	440	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{10}$

Illustration 2.6.

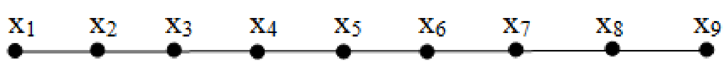


Figure 1: Path Graph P_9

Double Triangular Decomposition of P_9 as follows

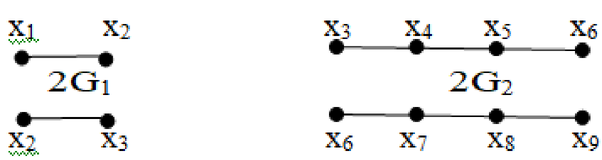


Figure 2: Double Triangular Decomposition $\{2G_1, 2G_2\}$ **of P_9**

Theorem 2.7. For any integer m, C_m has a Double Triangular Decomposition $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k\}$ if and only if there exists an integer k satisfying the following properties:

(i) $m = \frac{k(k+1)(k+2)}{3}$

(ii) $k = \begin{cases} 3r-2 & r \in N-\{1\} \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

where k denotes the number of decompositions.

Proof. Let $G = C_m$. Then $q(G) = m$. Assume G has a Double Triangular Decomposition. By the definition of Double Triangular Decomposition, $q(G) = \frac{k(k+1)(k+2)}{3}$. Hence $m = \frac{k(k+1)(k+2)}{3}$.

Since m is an integer, $k = \begin{cases} 3r-2 & r \in N-\{1\} \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

Conversely assume (i) $m = \frac{k(k+1)(k+2)}{3}$ (ii) $k = \begin{cases} 3r-2 & r \in N-\{1\} \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

Consider $G = C_m$. Then $q(G) = m$. Let $V(G) = \{x_1, x_2, x_3, \ldots, x_m\}$ be the vertex set and $E = \{x_j x_{j+1} / 1 \leq j \leq m-1\} \cup \{x_1 x_m\}$ be the edge set. By lemma 2.1, 2.2 and 2.3, G can be decomposed into $\{2G_1, 2G_2, 2G_3, \ldots, 2G_k\}$. Thus G admits Double Triangular Decomposition.

Table 2.8: List of first ten DTD of C_m

m	q(G)	Double Triangular Decomposition
8	8	$2G_1, 2G_2$
20	20	$2G_1, 2G_2, 2G_3$
40	40	$2G_1, 2G_2, 2G_3, 2G_4$
70	70	$2G_1, 2G_2, 2G_3, \ldots, 2G_5$
112	112	$2G_1, 2G_2, 2G_3, \ldots, 2G_6$
168	168	$2G_1, 2G_2, 2G_3, \ldots, 2G_7$
240	240	$2G_1, 2G_2, 2G_3, \ldots, 2G_8$
330	330	$2G_1, 2G_2, 2G_3, \ldots, 2G_9$
440	440	$2G_1, 2G_2, 2G_3, \ldots, 2G_{10}$
572	572	$2G_1, 2G_2, 2G_3, \ldots, 2G_{11}$

Illustration 2.9.

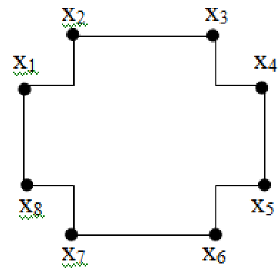
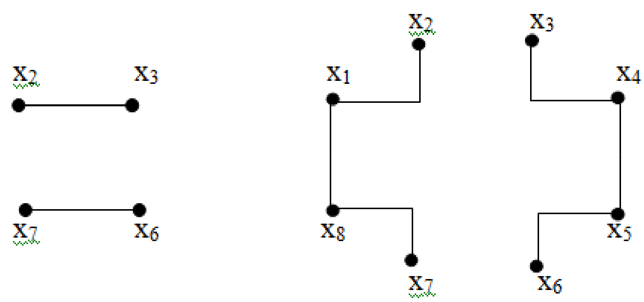


Figure 3: Cycle Graph C_8

Double Triangular Decomposition of C_8 as follows



$2G_1$ **$2G_2$**

Figure 4: Double Triangular Decomposition $\{2G_1, 2G_2\}$ of C_8

Theorem 2.10. For any integer m, W_m has a Double Triangular Decomposition $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k\}$ if and only if there exists an integer k satisfying the following properties:

(i) $m = \frac{k^3+3k^2+2k}{6}$

(ii) $k = \begin{cases} 3r-2 & r \in N-\{1\} \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

where k denotes the number of decompositions.

Proof. Let $G = W_m$. Then $q(G) = 2m$. Assume G has a Double Triangular Decomposition. By definition, $q(G) = \frac{k(k+1)(k+2)}{3}$.

$$\text{Hence } 2m = \frac{k(k+1)(k+2)}{3}.$$

$$\Rightarrow \quad m = \frac{k^3+3k^2+2k}{6}.$$

Since m is an integer, $k = \begin{cases} 3r-2 & r \in N-\{1\} \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

Conversely assume (i) $m = \frac{k^3+3k^2+2k}{6}$ (ii) $k = \begin{cases} 3r-2 & r \in N-\{1\} \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

Consider $G = W_m$. Let $V(W_m) = \{z, x_1, x_2, x_3, \ldots\ldots, x_m\}$ be the vertex set and $E(W_m) = \{zx_j/1 \leq j \leq m\} \cup \{x_j x_{j+1}/1 \leq j \leq m-1\} \cup \{x_m x_1\}$ be the edge set. By lemma 2.1, 2.2 and 2.3, G can be decomposed into $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k\}$. Thus G admits Double Triangular Decomposition.

Table 2.11: List of first ten DTD of W_m

m	q(G)	Double Triangular Decomposition
4	8	$2G_1, 2G_2$
10	20	$2G_1, 2G_2, 2G_3$
20	40	$2G_1, 2G_2, 2G_3, 2G_4$
35	70	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_5$
56	112	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_6$
84	168	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_7$
120	240	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_8$
165	330	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_9$
220	440	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{10}$
286	572	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{11}$

Illustration 2.12.

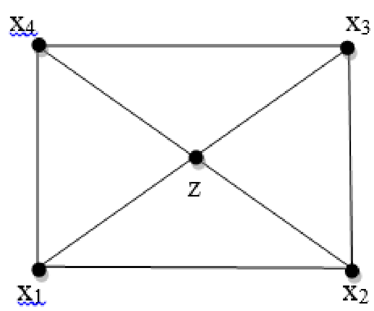


Figure 5: Wheel Graph W_4

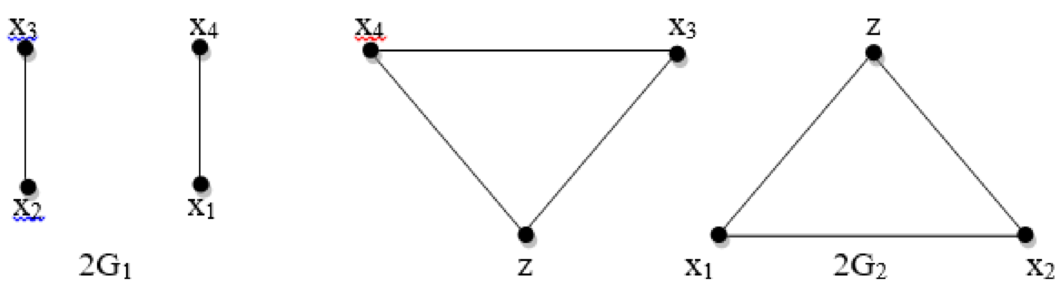


Figure 6: Double Triangular Decomposition $\{2G_1, 2G_2\}$ of W_4.

Theorem. 2.13. For any integer m, $K_{1,m}$ has a Double Triangular Decomposition $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k\}$ if and only if there exists an integer k satisfying the following properties:

(i) $m = \frac{k(k+1)(k+2)}{3}$

(ii) $k = \begin{cases} 3r-2 & r \in N \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

where k denotes the number of decompositions.

Proof. Let $G = K_{1,m}$. Then $q(G) = m$. Assume G has a Double Triangular Decomposition. By the definition of Double Triangular Decomposition, $q(G) = \frac{k(k+1)(k+2)}{3}$.

Hence $m = \frac{k(k+1)(k+2)}{3}$.

Since m is an integer, $k = \begin{cases} 3r-2 & r \in N \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

Conversely assume (i) $m = \frac{k(k+1)(k+2)}{3}$ (ii) $k = \begin{cases} 3r-2 & r \in N \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

Let $V(G) = \{x_1\} \cup \{y_j / 1 \leq j \leq m\}$ be the vertex set and $E(G) = \{x_1 y_j / 1 \leq j \leq m\}$ be the edge set. By lemma 2.1, 2.2 and 2.3, G can be decomposed into $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k\}$. Thus G admits Double Triangular Decomposition.

Table 2.14: List of first 10, DTD of $K_{1,m}$

m	q(G)	Double Triangular Decomposition
2	2	$2G_1$
8	8	$2G_1, 2G_2$
20	20	$2G_1, 2G_2, 2G_3$
40	40	$2G_1, 2G_2, 2G_3, 2G_4$
70	70	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_5$
112	112	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_6$
168	168	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_7$
240	240	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_8$
330	330	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_9$
440	440	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{10}$

Illustration. 2.15.

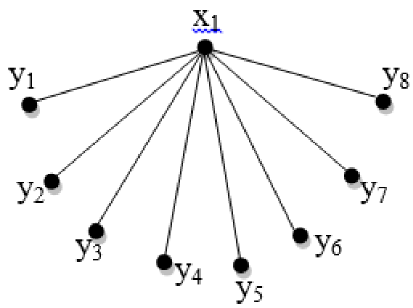


Figure 7. Bipartite Graph $K_{1,8}$.

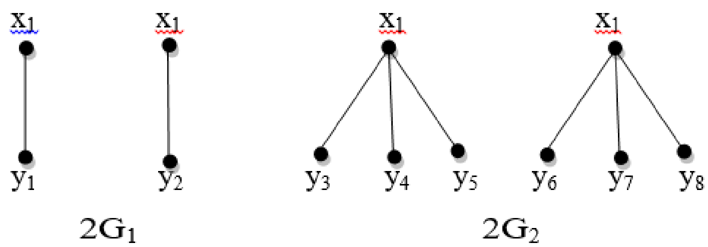


Figure 8. Double Triangular Decomposition $\{2G_1, 2G_2\}$ **of** $K_{1,8}$..

Theorem 2.16. For any odd integer m, the Firecracker Graph F_m has a Double Triangular Decomposition $\{2G_1, 2G_2, 2G_3, \ldots\ldots, 2G_k\}$ if and only if there exists an integer k satisfying the following properties:

(i) $m = \frac{k^3+3k^2+2k+3}{3}$.

(ii) $k = \begin{cases} 3r-2 & r \in N \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

where k denotes the number of decompositions.

Proof. Let $G = F_m$. Then $q(G) = m - 1$. Assume G has a Double Triangular Decomposition. By the definition of Double Triangular Decomposition, $q(G) = \frac{k(k+1)(k+2)}{3}$.

Hence $m\text{-}1 = \frac{k(k+1)(k+2)}{3}$.

⇨ $m = \frac{k^3+3k^2+2k+3}{3}$.

Since m is an integer, $k = \begin{cases} 3r-2 & r \in N \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

Conversely assume (i) $m\text{-}1 = \frac{k(k+1)(k+2)}{3}$ (ii) $k = \begin{cases} 3r-2 & r \in N \\ 3r-1 & r \in N \\ 3r & r \in N \end{cases}$

Consider $G = F_m$. Then $q(G) = m\text{-}1$. By lemma 2.1, 2.2 and 2.3, G can be decomposed into $\{2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_k\}$. Thus G admits Double Triangular Decomposition.

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Table 2.17: List of first 10, DTD of Firecracker Graph F_n

m	q(G)	Double Triangular Decomposition
3	2	$2G_1$
9	8	$2G_1, 2G_2$
21	20	$2G_1, 2G_2, 2G_3$
41	40	$2G_1, 2G_2, 2G_3, 2G_4$
71	70	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_5$
113	112	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_6$
169	168	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_7$
241	240	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_8$
331	330	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_9$
441	440	$2G_1, 2G_2, 2G_3, \ldots\ldots\ldots, 2G_{10}$

Illustration 2.18.

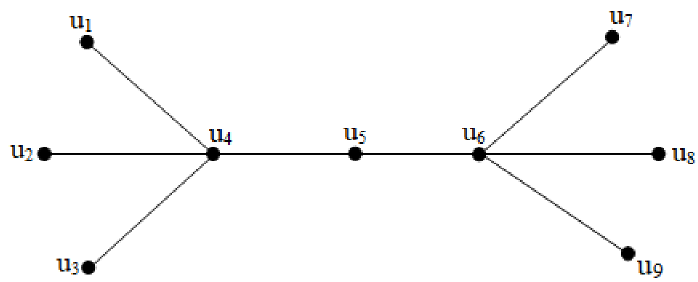

Figure 9: Firecracker Graph F_9

Double Triangular Decomposition of F_9 as follows

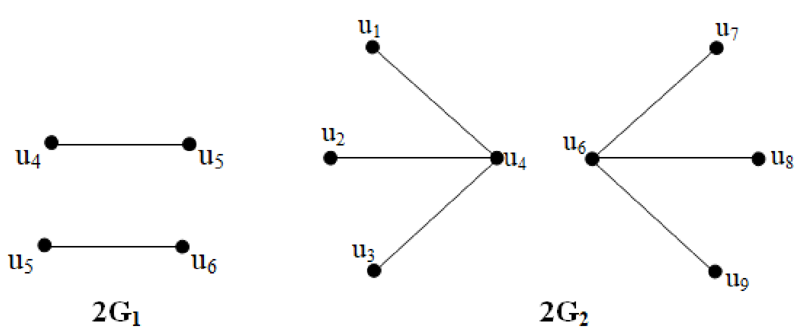

Figure 10: Double Triangular Decomposition $\{2G_1, 2G_2\}$ of F_9.

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