ISSN PRINT 2319 1775 Online 2320 7876
 UGC CARE Listed (Group -I) Journal Volume 11, Iss 13, 2022

A NEW CLASS OF CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract: This paper aims at the study of a new class of intuitionistic fuzzy topological set, namely, intuitionistic fuzzy generalized hash pre-closed (briefly, $I_f g_p^{\#}C$) set in intuitionistic fuzzy topological spaces (briefly, I_fTS). The theorems and various properties are discussed with suitable examples.

Key Words: $I_f g_p^{\#}$ Closed Set, $I_f g_p^{\#}$ Closure, $I_f g_p^{\#}$ Neighbourhood.

1. INTRODUCTION

L.A. Zadeh [22] introduced fuzzy sets to value every member of a set in mathematics in 1965. C.L. Chang [6] introduced fuzzy topological spaces in 1967. Further, K. Atanassov [2] introduced intuitionistic fuzzy sets by adding the notion of non-membership into fuzzy sets in 1983. D. Coker [8] introduced intuitionistic fuzzy topological spaces in 1997. From then on, many research works focused on the generalization of the concepts of intuitionistic fuzzy topological spaces. Extending further, we introduce intuitionistic fuzzy generalized hash pre-closed (briefly, $I_f g_p^{\#}C$) set with its characterization and properties with suitable examples.

2. PRELIMINARIES

Definition 2.1 [2] Let X be a universal set and let A be an intuitionistic fuzzy (briefly, I_f) subset in X, where $A = \{< x, \mu_A(x), \upsilon_A(x) >: x \in X\}$. Here, the functions $\mu_A: X \to [0,1]$ and $\upsilon_A: X \to [0,1]$ denote the degree of membership, namely $\mu_A(x)$ and the degree of non-membership, namely $\upsilon_A(x)$ of each element $x \in X$ to the set A respectively, and $0 \le \mu_A(x) + \upsilon_A(x) \le 1$ for each $x \in X$.

Definition 2.2 [2] Let $A = \{< x, \mu_A(x), \upsilon_A(x) >: x \in X\}$ and B= $\{< x, \mu_B(x), \upsilon_B(x) >: x \in X\}$ be two I_f subsets, then the following holds true

(a) $A \subseteq B$ iff $\mu_A \le \mu_B$ and $\mu_A(x) \ge \mu_B(x)$ for all $x \in X$
(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{< x, \upsilon_A(x), \mu_A(x) >: x \in X\}$
(d) $A \cup B = \{x | \mu_A(x) \vee \mu_B(x), \upsilon_A(x) \wedge \upsilon_B(x) : x \in X\}$

(e) $A \cap B = \{x|\mu_A(x) \wedge \mu_B(x), \upsilon_A(x) \vee \upsilon_B(x) : x \in X\}$
(f) $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$
(g) the empty set $\tilde{0} =< x, 0, 1 >$ and the whole set $\tilde{1} =< x, 1, 0 >$

Definition 2.3 [8] An I_f topology on a non-empty set X is a family τ_{if} of intuitionistic fuzzy subsets of X, satisfying the following axioms;

(1) $\tilde{0}, \tilde{1} \in \tau_{if}$
(2) $A \cap B \in \tau_{if}$ for any $A, B \in \tau_{if}$
(3) $\cup A_i \in \tau_{if}$ for any arbitrary family $\{A_i/i \in J\} \subseteq \tau_{if}$

A non-empty set X on which an I_f topology τ_{if} has been specified is called an intuitionistic fuzzy topological space, i.e., (X, τ_{if}). Any I_f set in τ_{if} is known as an intuitionistic fuzzy open (briefly, I_fO) set in X and the complement of an I_fO set is known as an intuitionistic fuzzy closed (briefly, I_fC) set in X.

Definition 2.4 [8] Let (X, τ_{if}) be an I_f topological space and $A = \{< x, \mu_A(x), \upsilon_A(x) >: x \in X\}$ be an I_f subset in X. Then the interior and closure of the above I_f subset are defined as follows.

(i) $int(A) = \cup \{G|G$ is an intuitionistic fuzzy open (briefly, I_fO) set in X and $G \subseteq A\}$
(ii) $cl(A) = \cap \{K|K$ is an intuitionistic fuzzy closed (briefly, I_fC) set in X and $A \subseteq K\}$

Definition 2.5 [8] Let X be a non-empty set and let $p \in X$ be a fixed element in X. Let $\alpha, \beta \in [0, 1]$ be two real numbers such that $\alpha + \beta \leq 1$ then

(i) an I_f point is written as $p_{(\alpha,\beta)}$ and denoted as $\tilde{p}$.
(ii) an I_f point is defined as $p_{(\alpha,\beta)}(x) = \{(\alpha, \beta),\ \text{if } x = p\ \ (0, 1)\ \ \text{otherwise}$.
(iii) an I_f point $p_{(\alpha,\beta)}$ in $X =< x, p_\alpha, p_{1-\beta}: x \in X >$ where α denotes the degree of membership of $p_{(\alpha,\beta)}$ and β denotes the degree of non-membership of $p_{(\alpha,\beta)}$.
(iv) an I_f point $p_{(\alpha,\beta)} \in A$, by the Definition 1.2.1.1 if $\alpha \leq \mu_A$ and $\beta \geq \upsilon_A$.
(v) a vanishing I_f point is written as $p_{(\beta)}$ and denoted as $\tilde{\tilde{p}}$.
(vi) a vanishing I_f point is defined as $p_{(\beta)}(x) =< x, 0, \left[p_{1-\beta}\right]^c: x \in X >$ where β denotes the degree of non-membership of $p_{(\beta)}$.

Definition 2.6 [9] Let (X, τ_{if}) be an I_fTS such that $p \in X$, then

(1) an intuitionistic fuzzy neighbourhood of $\tilde{p}$ (briefly, $I_fN(\tilde{p})$), if there exists an I_fO set N such that $\tilde{p} \in N \subseteq I_fN(\tilde{p})$.
(2) an intuitionistic fuzzy neighbourhood of $\tilde{\tilde{p}}$ (briefly, $I_fN(\tilde{\tilde{p}})$), if there exists an I_fO set N such that $\tilde{\tilde{p}} \in N \subseteq I_fN(\tilde{\tilde{p}})$.

Definition 2.7 [10] Let (X, τ_{if}) be an intuitionistic fuzzy topological space (briefly, I_fTS). Then an intuitionistic fuzzy subset $L \subseteq X$ is said to be an

(i) intuitionistic fuzzy semi-closed (briefly, I_fsC) set if $int(cl(L)) \subseteq L$.
(ii) intuitionistic fuzzy pre-closed (briefly, I_fpC) set if $cl(int(L)) \subseteq L$.
(iii) intuitionistic fuzzy alpha-closed (briefly, $I_f\alpha C$) set if $cl(int(cl(L))) \subseteq L$.
(iv) intuitionistic fuzzy semi-pre-closed (briefly, I_fspC) set [21] if $int\left(cl\left(int(L)\right)\right) \subseteq L$.

(v) intuitionistic fuzzy semi-open (briefly, $I_f sO$) set if $L \subseteq cl(int(L))$.

(vi) intuitionistic fuzzy pre-open (briefly, $I_f pO$) set if $L \subseteq int(cl(L))$.

(vii) intuitionistic fuzzy alpha-open (briefly, $I_f \alpha O$) set if $L \subseteq int\Big(cl\big(int(L)\big)\Big)$.

(viii) intuitionistic fuzzy semi-pre-open (briefly, $I_f spO$) set [21] if $L \subseteq cl\Big(int\big(cl(L)\big)\Big)$.

Definition 2.8 Let (X, τ_{if}) be an $I_f TS$. Then an intuitionistic fuzzy subset $L \subseteq X$ is said to be an

(i) intuitionistic fuzzy generalized closed (briefly, $I_f gC$) set [18] if $cl(L) \subseteq U$ whenever $L \subseteq U$ and U is an intuitionistic fuzzy open (briefly, $I_f O$) set in (X, τ_{if}).

(ii) intuitionistic fuzzy semi generalized closed (briefly, $I_f sgC$) set [20] if $scl(L) \subseteq U$ whenever $L \subseteq U$ and U is an intuitionistic fuzzy semi-open (briefly, $I_f sO$) set in (X, τ_{if}).

(iii) intuitionistic fuzzy generalized pre-closed (briefly, $I_f gpC$) set [14] if $pcl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $I_f O$ set in (X, τ_{if}).

(iv) intuitionistic fuzzy semi generalized pre-closed (briefly, $I_f sgpC$) set [4] if $pcl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $I_f sO$ set in (X, τ_{if}).

(v) intuitionistic fuzzy generalized semi-pre-closed (briefly, $I_f gspC$) set [17] if $spcl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $I_f O$ set in (X, τ_{if}).

(vi) intuitionistic fuzzy generalized alpha-closed (briefly, $I_f g\alpha C$) set [11] if $\alpha cl(L) \subseteq U$ whenever $L \subseteq U$ and U is an intuitionistic fuzzy alpha open (briefly, $I_f \alpha O$) set in (X, τ_{if}).

(vii) intuitionistic fuzzy generalized star closed (briefly, $I_f g^* C$) set [3] if $cl(L) \subseteq U$ whenever $L \subseteq U$ and U is an intuitionistic fuzzy generalized open (briefly, $I_f gO$) set in (X, τ_{if}).

(viii) intuitionistic fuzzy generalized star pre-closed (briefly, $I_f g^* pC$) set [4] if $pcl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $I_f gO$ set in (X, τ_{if}).

(ix) intuitionistic fuzzy generalized star semi-closed (briefly, $I_f g^* sC$) set [13] if $scl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $I_f gO$ set in (X, τ_{if}).

(x) intuitionistic fuzzy omega closed (briefly, $I_f \omega C$) set [19] if $cl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $I_f sO$ set in (X, τ_{if}).

(xi) intuitionistic fuzzy psi closed (briefly, $I_f \Psi C$) set [12] if $scl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $I_f sgO$ set in (X, τ_{if}).

(xii) intuitionistic fuzzy generalized hash closed (briefly, $I_f g^\# C$) set [1] if $cl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $I_f \alpha O$ set in (X, τ_{if}).

3 INTUITIONISTIC FUZZY $g_p^\#$ CLOSED SETS

Definition 3.1 An intuitionistic fuzzy (briefly I_f) subset L of an intuitionistic fuzzy topological space (X, τ_{if}) is called an intuitionistic fuzzy generalized hash pre closed (briefly, $I_f g^\# pC$) set if $pcl(L) \subseteq U$ whenever $L \subseteq U$ and U is intuitionistic fuzzy generalized alpha open (briefly, $I_f g\alpha O$) set in (X, τ_{if}).

Example 3.2 Let (X, τ_{if}) be an I_fTS where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, $A = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$, and U be an $I_f g\alpha O$ set. Then $A^C = \{< q/0.7, 0.3 >, < r/0.6, 0.4 >\}$. Now considering an I_f set $L = \{< q/0.8, 0.2 >, < r/0.7, 0.3 >\}$ in (X, τ_{if}), we observe that $pcl(L) \subseteq I_f g\alpha O = U$. Hence L is an $g_p^{\#}C$ set in (X, τ_{if}).

Theorem 3.3 Let (X, τ_{if}) be an I_fTS. Then every I_fC set is an $I_f g_p^{\#}C$ set in (X, τ_{if}).
Proof: Let L be an I_fC set and let U be any $I_f g\alpha O$ set in (X, τ_{if}) such that $L \subseteq U$. Since L is an I_fC set, we observe that $L = cl(L)$. Moreover $pcl(L) \subseteq cl(L) \subseteq U$. Hence L is an $I_f g_p^{\#}C$ set in (X, τ_{if}).
However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.4 Let (X, τ_{if}) be an I_fTS where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, and $A = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Then $A^C = \{< q/0.7, 0.3 >, < r/0.6, 0.4 >\}$. Now considering an I_f set $L = \{< q/0.2, 0.8 >, < r/0.3, 0.7 >\}$ in (X, τ_{if}), clearly, we see that L is an $I_f g_p^{\#}C$ set but not an I_fC set in (X, τ_{if}).

Theorem 3.5 Let (X, τ_{if}) be an I_fTS. Then every $I_f\alpha C$ set is an $I_f g_p^{\#}C$ set in (X, τ_{if}).
Proof: Let L be an $I_f\alpha C$ set and let U an any $I_f g\alpha O$ set in (X, τ_{if}) such that $L \subseteq U$. Since L is an $I_f\alpha C$ set, we observe that $L = \alpha cl(L)$. Moreover $pcl(L) \subseteq \alpha cl(L) \subseteq U$. Hence L is an $I_f g_p^{\#}C$ set in (X, τ_{if}).
However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.6 Let (X, τ_{if}) be an I_fTS where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, and $A = \{< q/0.7, 0.3 >, < r/0.6, 0.4 >\}$. Then $A^C = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Now considering an I_f set $L = \{< q/0.4, 0.6 >, < r/0.5, 0.5 >\}$ in (X, τ_{if}), clearly, we see that L is an $I_f g_p^{\#}C$ set but not an $I_f\alpha C$ set in (X, τ_{if}).

Theorem 3.7 Let (X, τ_{if}) be an I_fTS. Then every I_fpC set is an $I_f g_p^{\#}C$ set in (X, τ_{if}).
Proof: Let L be an I_fpC set and let U an any $I_f g\alpha O$ set in (X, τ_{if}) such that $L \subseteq U$. Since L is an I_fpC set, we observe that $L = pcl(L)$. Moreover $pcl(L) = pcl(L) \subseteq U$. Hence L is an $I_f g_p^{\#}C$ set in (X, τ_{if}).
However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.8 Let (X, τ_{if}) be an I_fTS where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, and $A = \{< q/0.7, 0.3 >, < r/0.6, 0.4 >\}$. Then $A^C = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Now considering an I_f set $L = \{< q/0.8, 0.2 >, < r/0.7, 0.3 >\}$ in (X, τ_{if}), clearly, we see that L is an $I_f g_p^{\#}C$ set but not an I_fpC set in (X, τ_{if}).

Theorem 3.9 Let (X, τ_{if}) be an I_fTS. Then every $I_f g^*C$ set is an $I_f g_p^{\#}C$ set in (X, τ_{if}).

Proof: Let L be an I_fg^*C set and let U an any $I_fg\alpha O$ set in (X, τ_{if}) such that $L \subseteq U$. Every $I_fg\alpha O$ set is an I_fgO set whenever $L \subseteq U$ and since L is an I_fg^*C set, we observe that $pcl(L) \subseteq cl(L) \subseteq U$. This shows that L is an $I_fg_p^\#C$ set in (X, τ_{if}).

However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.10 Let (X, τ_{if}) be an I_fTS where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, and $A = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Then $A^C = \{< q/0.7, 0.3 >, < r/0.6, 0.4 >\}$. Now considering an I_f set $L = \{< q/0.2, 0.6 >, < r/0.5, 0.5 >\}$ in (X, τ_{if}), clearly, we see that L is an $I_fg_p^\#C$ set but not an I_fg^*C set in (X, τ_{if}).

Theorem 3.11 Let (X, τ_{if}) be an I_fTS. Then every $I_fg^\#C$ set is an $I_fg_p^\#C$ set in (X, τ_{if}).

Proof: Let L be an $I_fg^\#C$ set and let U an any $I_fg\alpha O$ set in (X, τ_{if}) such that $L \subseteq U$. Since every $I_fg\alpha O$ set is an $I_fg\alpha O$ set and L is $I_fg^\#C$, we observe that $pcl(L) \subseteq cl(L) \subseteq U$. This shows that L is an $I_fg_p^\#C$ set in (X, τ_{if}).

However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.12 Let (X, τ_{if}) be an I_fTS where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, and $A = \{< q/0.7, 0.3 >, < r/0.6, 0.4 >\}$. Then $A^C = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Now considering an I_f set $L = \{< q/0.2, 0.8 >, < r/0.3, 0.7 >\}$ in (X, τ_{if}), clearly, we see that L is an $I_fg_p^\#C$ set but not an $I_fg^\#C$ set in (X, τ_{if}).

Theorem 3.13 Let (X, τ_{if}) be an I_fTS. Then every I_fg^*pC set is an $I_fg_p^\#C$ set in (X, τ_{if}).

Proof: Let L be an I_fg^*pC set and let U an any $I_fg\alpha O$ set in (X, τ_{if}) such that $L \subseteq U$. Every $I_fg\alpha O$ set is an I_fgO set whenever $L \subseteq U$ and since L is an I_fg^*pC set, we observe that $pcl(L) = pcl(L) \subseteq U$. This shows that L is an $I_fg_p^\#C$ set in (X, τ_{if}).

However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.14 Let (X, τ_{if}) be an I_fTS where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, and $A = \{< q/0.7, 0.3 >, < r/0.6, 0.4 >\}$. Then $A^C = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Now considering an I_f set $L = \{< q/0.8, 0.2 >, < r/0.7, 0.3 >\}$ in (X, τ_{if}), clearly, we see that L is an $I_fg_p^\#C$ set but not an I_fg^*pC set in (X, τ_{if}).

Theorem 3.15 Let (X, τ_{if}) be an I_fTS. Then every $I_fg_p^\#C$ set is an I_fgspC set in (X, τ_{if}).

Proof: Let L be an $I_fg_p^\#C$ set and let U an any open set in (X, τ_{if}) such that $L \subseteq U$. Since every open set is an $I_fg\alpha O$ set and L is an $I_fg_p^\#C$ set, we observe that $spcl(L) \subseteq pcl(L) \subseteq U$. This shows that L is an I_fgspC set in (X, τ_{if}).

However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.16 Let (X, τ_{if}) be an $I_f TS$ where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, and $A = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Then $A^C = \{< q/0.7, 0.3 >, < r/0.6, 0.4 >\}$. Now considering an I_f set $L = \{< q/0.4, 0.6 >, < r/0.5, 0.5 >\}$ in (X, τ_{if}), clearly, we see that L is an $I_f gspC$ set but not an $I_f g_p^{\#} C$ set in (X, τ_{if}).

Remark 3.17 Let (X, τ_{if}) be an $I_f TS$. Then $I_f sC$ set, $I_f g^* sC$ set, and $I_f \Psi C$ set are independent to $I_f g_p^{\#} C$ set in (X, τ_{if}). This could be verified with the following example.

Example 3.18 Let (X, τ_{if}) be an $I_f TS$ where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, and $A = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Then $A^C = \{< q/0.7, 0.3 >, < r/0.6, 0.4 >\}$. Now considering an I_f set $L = \{< q/0.2, 0.8 >, < r/0.3, 0.7 >\}$ in (X, τ_{if}), clearly, we see that L is an $I_f g_p^{\#} C$ set but not an $I_f sC$ set, an $I_f g^* sC$ set, and also an $I_f \Psi C$ set in (X, τ_{if}). Also considering an I_f set $D = \{< q/0.4, 0.6 >, < r/0.5, 0.5 >\}$ in (X, τ_{if}), clearly, we see that D is an $I_f sC$ set, an $I_f g^* sC$ set, and also an $I_f \Psi C$ set but not an $I_f g_p^{\#} C$ set in (X, τ_{if}). Hence $I_f sC$ set, $I_f g^* sC$ set, and $I_f \Psi C$ set are independent to $I_f g_p^{\#} C$ set in (X, τ_{if}).

Remark 3.19 Let (X, τ_{if}) be an $I_f TS$. Then $I_f sgC$ set and $I_f \omega C$ set are independent to $I_f g_p^{\#} C$ set in (X, τ_{if}). This could be verified with the following example.

Example 3.20 Let (X, τ_{if}) be an $I_f TS$ where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, and $A = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Then $A^C = \{< q/0.7, 0.3 >, < r/0.6, 0.4 >\}$. Now considering an I_f set $L = \{< q/0.2, 0.6 >, < r/0.5, 0.5 >\}$ in (X, τ_{if}), clearly, we see that L is an $I_f g_p^{\#} C$ set but not an $I_f sgC$ set and also an $I_f \omega C$ set in (X, τ_{if}). Also considering an I_f set $D = \{< q/0.4, 0.6 >, < r/0.5, 0.3 >\}$ in (X, τ_{if}), clearly, we see that D is an $I_f sgC$ set and also an $I_f \omega C$ set but not an $I_f g_p^{\#} C$ set in (X, τ_{if}). Hence $I_f sgC$ set and $I_f \omega C$ set are independent to $I_f g_p^{\#} C$ set in (X, τ_{if}).

Remark 3.21 Let (X, τ_{if}) be an $I_f TS$. Then $I_f gC$ set and $I_f \alpha gC$ set are independent to $I_f g_p^{\#} C$ set in (X, τ_{if}). This could be verified with the following example.

Example 3.22 Let (X, τ_{if}) be an $I_f TS$ where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, and $A = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Then $A^C = \{< q/0.5, 0.5 >, < r/0.6, 0.4 >\}$. Now considering an I_f set $L = \{< q/0.2, 0.8 >, < r/0.3, 0.7 >\}$ in (X, τ_{if}), clearly, we see that L is an $I_f g_p^{\#} C$ set but not an $I_f gC$ set and also an $I_f \alpha gC$ set in (X, τ_{if}). Also considering an I_f set $D = \{< q/0.4, 0.6 >, < r/0.5, 0.3 >\}$ in (X, τ_{if}), clearly, we see that D is an $I_f gC$ set and also an $I_f \alpha gC$ set but not an $I_f g_p^{\#} C$ set in (X, τ_{if}). Hence, $I_f gC$ set and $I_f \alpha gC$ set are independent to $I_f g_p^{\#} C$ set in (X, τ_{if}).

Remark 3.23 Let (X, τ_{if}) be an $I_f TS$. Then $I_f gpC$ set and $I_f sgpC$ set are independent to $I_f g_p^{\#} C$ set in (X, τ_{if}). This could be verified with the following example.

Example 3.24 Let (X, τ_{if}) be an $I_f TS$ where $X = \{q, r\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, and $A = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Then $A^C = \{< q/0.7, 0.3 >, < r/0.6, 0.4 >\}$. Now considering an I_f set $L = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$ in (X, τ_{if}), clearly, we see that L is an $I_f g_p^\# C$ set but not an $I_f gpC$ set and also an $I_f sgpC$ set in (X, τ_{if}). Also considering an I_f set $D = \{< q/0.4, 0.6 >, < r/0.5, 0.3 >\}$ in (X, τ_{if}), clearly, we see that D is an $I_f gpC$ set and also an $I_f sgpC$ set but not an $I_f g_p^\# C$ set in (X, τ_{if}). Hence $I_f gpC$ set and $I_f sgpC$ set are independent to $I_f g_p^\# C$ set in (X, τ_{if}).

Theorem 3.25 Let (X, τ_{if}) be an $I_f TS$. Then the arbitrary union of any two subsets of an $I_f g_p^\# C$ set is a subset of the $I_f g_p^\# C$ set in (X, τ_{if}).

Proof: Let L and D be any two subsets of an $I_f g_p^\# C$ set in (X, τ_{if}). Let U be an $I_f g\alpha O$ set in (X, τ_{if}) such that $L \cup D \subseteq U$ which implies $L \subseteq U$ and $D \subseteq U$. Since L and D are subsets of an $I_f g_p^\# C$ set, $pcl(L) \subseteq U$ and $pcl(D) \subseteq U$. Then $pcl(L) \cup pcl(D) \subseteq U$. But $pcl(L) \cup pcl(D) = pcl(L \cup D)$ which implies $pcl(L \cup D) \subseteq U$. Hence, the arbitrary union of any two subsets of an $I_f g_p^\# C$ set is a subset of the $I_f g_p^\# C$ set in (X, τ_{if}).

Theorem 3.26 Let (X, τ_{if}) be an $I_f TS$. Then the finite intersection of any two subsets of an $I_f g_p^\# C$ set is a subset of the $I_f g_p^\# C$ set in (X, τ_{if}).

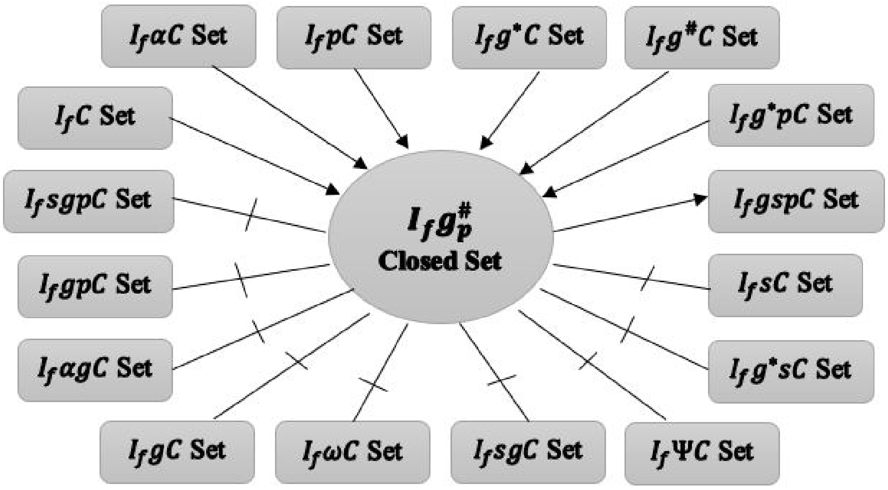
Proof: Let L and D be any two subsets of an $I_f g_p^\# C$ set in (X, τ_{if}). Let U be an $I_f g\alpha O$ set in (X, τ_{if}) such that $L, D \subseteq U$. Since L and D are subsets of an $I_f g_p^\# C$ set, $pcl(L) \subseteq U$ and $pcl(D) \subseteq U$. Then $pcl(L) \cap pcl(D) \subseteq U$. But $pcl(L) \cap pcl(D) = pcl(L \cap D)$ which implies $pcl(L \cap D) \subseteq U$. Hence, the finite interjection of any two subsets of an $I_f g_p^\# C$ set is a subset of the $I_f g_p^\# C$ set in (X, τ_{if}).

Theorem 3.27 Let (X, τ_{if}) be an $I_f TS$. If an I_f set L is an $I_f g_p^\# C$ set in (X, τ_{if}) and $L \subseteq D \subseteq g_p^\# cl(L)$ then D is also an $I_f g_p^\# C$ set in (X, τ_{if}).

Proof: Let L be an $I_f g_p^\# C$ set in (X, τ_{if}) and let U be $I_f g\alpha O$ in (X, τ_{if}) such that $D \subseteq U$. Since $L \subseteq D$, we have $L \subseteq U$. Since L is $I_f g_p^\# C$ and $pcl(D) \subseteq pcl(pcl(L)) = pcl(L) \subseteq U$. Therefore, $pcl(D) \subseteq U$. Hence D is an $I_f g_p^\# C$ set in (X, τ_{if}).

The following diagram depicts the implications and independencies of $I_f g_p^\# C$ set with other $I_f C$ sets.

ISSN PRINT 2319 1775 Online 2320 7876
 UGC CARE Listed (Group -I) Journal Volume 11, Iss 13, 2022



4. SOME FEATURES OF $I_f g_p^{\#}C$ SETS

Definition 4.1 Let (X, τ_{if}) be an I_fTS and let L be any I_f subset in I_fTS. Then $I_f g_p^{\#}$ closure of L is defined as the intersection of all $I_f g_p^{\#}C$ sets containing L, i.e., $g_p^{\#}cl(L) = \cap\{K \mid L \subseteq K, K \in I_f g_p^{\#}C$ set in $(X, \tau_{if})\}$. It is denoted as $I_f g_p^{\#}cl(L)$.

Theorem 4.2 Let (X, τ_{if}) be an I_fTS. If L is any I_f subset in (X, τ_{if}) then $L \subseteq g_p^{\#}cl(L) \subseteq cl(L)$ in (X, τ_{if}).
Proof: By the definition of $I_f g_p^{\#}$ closure, $L \subseteq g_p^{\#}cl(L)$. Since every I_fC set is $I_f g_p^{\#}C$ set, $g_p^{\#}cl(L) \subseteq cl(L)$. Hence $L \subseteq g_p^{\#}cl(L) \subseteq cl(L)$.

Theorem 4.3 Let (X, τ_{if}) be an I_fTS. If L is an $I_f g_p^{\#}C$ set in (X, τ_{if}) then $g_p^{\#}cl(L) = L$.
Proof: Let L be an $I_f g_p^{\#}C$ set in (X, τ_{if}). By the Definition 4.1, $g_p^{\#}cl(L) = \cap\{K \in (X, \tau_{if}) : L \subseteq K, K$ is $I_f g_p^{\#}C$ set in $(X, \tau_{if})\}$ and moreover L is $I_f g_p^{\#}C$ set in (X, τ_{if}). Hence $g_p^{\#}cl(L) = L$.

Theorem 4.4 Let (X, τ_{if}) be an I_fTS. If L is an I_f subset in (X, τ_{if}) then $g_p^{\#}cl\left(g_p^{\#}cl(L)\right) = g_p^{\#}cl(L)$ in (X, τ_{if}).
Proof: Let $L \in (X, \tau_{if})$. Then $g_p^{\#}cl\left(g_p^{\#}cl(L)\right) = \cap\{K \in (X, \tau_{if}) : g_p^{\#}cl(L) \subseteq K, K \in I_f g_p^{\#}C$ set in $(X, \tau_{if})\}$ which implies $g_p^{\#}cl\left(g_p^{\#}cl(L)\right) = g_p^{\#}cl(L)$.

Theorem 4.5 Let (X, τ_{if}) be an I_fTS. Then for any two I_f subsets L and D in (X, τ_{if}) the following holds true.

(i) $g_p^{\#}cl(L) \subseteq g_p^{\#}cl(D)$, if $L \subseteq D$

(ii) $g_p^{\#}cl(L) \cup g_p^{\#}cl(D) \subseteq g_p^{\#}cl(L \cup D)$

(iii) $g_p^{\#}cl(L) \cap g_p^{\#}cl(D) = g_p^{\#}cl(L \cap D)$

Proof: (i) follows from the Definition 4.1

(ii) Since $L \subseteq L \cup D$ and $N \subseteq L \cup N$ are true always, by the Theorem 4.3, $g_p^{\#}cl(L) \subseteq g_p^{\#}cl(L \cup D)$ and $g_p^{\#}cl(D) \subseteq g_p^{\#}cl(L \cup D)$. Hence $g_p^{\#}cl(L) \cup g_p^{\#}cl(D) \subseteq g_p^{\#}cl(L \cup D)$.

(iii) Since $L \cap D \subseteq L$ and $L \cap D \subseteq D$, then by Theorem 4.3 $g_p^{\#}cl(L \cap D) \subseteq g_p^{\#}cl(L)$ and $g_p^{\#}cl(L \cap D) \subseteq I_f g_p^{\#}cl(D)$. Hence $I_f g_p^{\#}cl(L \cap D) \subseteq g_p^{\#}cl(L) \cap g_p^{\#}cl(D)$. Also, $g_p^{\#}cl(L) \cap g_p^{\#}cl(D) = [\cap \{K \in (X, \tau_{if}) : L \subseteq K$ and $K \in I_f g_p^{\#}C$ in $(X, \tau_{if})\}] \cap [\cap K \in (X, \tau_{if}) : D \subseteq K$ and $K \in I_f g_p^{\#}C$ in $(X, \tau_{if})\}]$ which is $\subseteq [\cap \{K \in (X, \tau_{if}) : L \cap D \subseteq K$ and $K \in I_f g_p^{\#}C$ in $(X, \tau_{if})\}]$. Hence $g_p^{\#}cl(L) \cap g_p^{\#}cl(D) \subseteq g_p^{\#}cl(L \cap D)$. Therefore, $g_p^{\#}cl(L) \cap g_p^{\#}cl(D) = g_p^{\#}cl(L \cap D)$.

Definition 4.6 Let (X, τ_{if}) be an $I_f TS$ such that $p \in X$, then

(3) an intuitionistic fuzzy generalized hash pre neighbourhood of $\tilde{p}$ (briefly, $I_f g_p^{\#}N(\tilde{p})$), if there exists an $I_f g_p^{\#}O$ set N such that $\tilde{p} \in N \subseteq I_f g_p^{\#}N(\tilde{p})$.

(4) an intuitionistic fuzzy generalized hash pre neighbourhood of $\tilde{\tilde{p}}$ (briefly, $I_f g_p^{\#}N(\tilde{\tilde{p}})$), if there exists an $I_f g_p^{\#}O$ set N such that $\tilde{\tilde{p}} \in N \subseteq I_f g_p^{\#}N(\tilde{\tilde{p}})$.

Theorem 4.7 Let (X, τ_{if}) be an $I_f TS$, then every $I_f N(\tilde{p})$ is an $I_f g_p^{\#}N(\tilde{p})$ for every $p \in X$ in (X, τ_{if}).

Proof: Let N be an I_f neighbourhood of point $p \in X$. Then by the definition of $I_f N(\tilde{p})$, there exists an $I_f O$ set N such that $\tilde{p} \in N \subseteq I_f N(\tilde{p}) = N$. Since every $I_f O$ set is an $I_f g_p^{\#}O$ set, N is an $I_f g_p^{\#}N(\tilde{p})$.

Theorem 4.8 Let (X, τ_{if}) be an $I_f TS$, then every $I_f N(\tilde{p})$ is an $I_f g_p^{\#}N(\tilde{\tilde{p}})$ for every $p \in X$ in (X, τ_{if}).

Proof: Let N be an I_f neighbourhood of point $p \in X$. Then by the definition of $I_f N(\tilde{\tilde{p}})$, there exists an $I_f O$ set N such that $\tilde{\tilde{p}} \in N \subseteq I_f N(\tilde{\tilde{p}}) = N$. Since every $I_f O$ set is an $I_f g_p^{\#}O$ set, N is an $I_f g_p^{\#}N(\tilde{\tilde{p}})$.

Theorem 4.9 Let (X, τ_{if}) be an $I_f TS$. If an I_f set L is $I_f g_p^{\#}C$ set in (X, τ_{if}) then $g_p^{\#}cl(L) - L$ does not contain any non-empty $I_f g\alpha C$ set in (X, τ_{if}).

Proof: Let L be an $I_f g_p^{\#}C$ set and let V be a non-empty $I_f g\alpha C$ subset of $pcl(L) - L$. Then $F \subseteq pcl(L) \cap (X - L)$. Since $(X - L)$ is an $I_f g\alpha O$ set and L is $I_f g_p^{\#}C$ set, $pcl(L) \subseteq (X - L)$ which implies $F \subseteq (X - pcl(L))$. Thus $V \subseteq pcl(L) \cap (X - pcl(L)) = \phi$ which implies that $V = \phi$. Hence $pcl(L) - \mathrm{L}$ does not contain any non-empty $\mathrm{I_f g\alpha C}$ set.

Theorem 4.10 Let $(\mathrm{X}, \tau_{\mathrm{if}})$ be an $\mathrm{I_f TS}$ and let L be an $\mathrm{I_f g_p^{\#} C}$ set in $(\mathrm{X}, \tau_{\mathrm{if}})$. Then L is an $\mathrm{I_f pC}$ set if and only if $\mathrm{pcl(L) - L = \phi}$ is an $\mathrm{I_f g\alpha C}$ set in $(\mathrm{X}, \tau_{\mathrm{if}})$.

Proof: Suppose L is an $\mathrm{I_f pC}$ set then U is an $\mathrm{I_f g\alpha O}$ set such that $\mathrm{L \subseteq U}$. Now $\mathrm{pcl(L) = L}$ implies $\mathrm{pcl(L) - L = \phi}$. Hence $\mathrm{pcl(L) - L}$ is an $\mathrm{I_f g\alpha C}$ set in $(\mathrm{X}, \tau_{\mathrm{if}})$.

Conversely, let $pcl(L) - L$ be an $I_f g\alpha C$ set, then $pcl(L) - L = \phi$ which implies $pcl(L) = L$. Hence L is an $I_f pC$ set in (X, τ_{if}).

Theorem 4.11 Let (X, τ_{if}) be an $I_f TS$. If an I_f set L is both $I_f g\alpha O$ and $I_f g_p^{\#} C$ set in (X, τ_{if}) then L is an $I_f pC$ set in (X, τ_{if}).

Proof: Let L be both $I_f g\alpha O$ and $I_f g_p^{\#} C$ set in (X, τ_{if}), then $pcl(L) - L = \phi$ which implies $pcl(L) = L$. Hence L is an $I_f pC$ set in (X, τ_{if}).

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