ANALYZING POPULATION DYNAMICS THROUGH DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORM METHODS

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ABSTRACT

Laplace transformation is an important chapter of Mathematical Analysis. It has wide application in different fields of techniques and engineering beside , basic sciences and mathematics. In this paper we will become acquainted with the basic concepts of operational mathematics, some properties of Laplace Transform and its application in demography. The purpose of this paper is to utilize the Laplace transformation to extract the logistic equation and to use the logistic equation in population growth.

Keywords: Differential equation, Laplace transform , Inverse Laplace transform , Population growth model projection.

INTRODUCTION

The operational methods are original and quite widespread method in solving different problems. They are applied in various fields of science. In this paper, we used Laplace transform for solving population growth. In everyday life, mathematical application and models are commonly used. The ability to forecast the population in the past some interesting questions about the population growth mathematical models. A population is "a group of the same species of crops, animals or other organisms which live together and reproduces ". The logistic equation is an equation with separable variables, but we will use the transformation of Laplace and its own properties to find the exponential equation.

Definition of Laplace transform:

Let it be f(t) a complex function of the real " t " argument given in the segment [a , b]. As it is known, such a function may appear in the form:

f (t) = u(t) + iv(t)

We will be limited to the integral of the functions f (t) that meets these conditions:

a) The function f(t) is continuous in $(0, +\infty)$ combined with its derivatives up to any necessary order with the exception of possibly a finite number of first type of critical point in each finite segment.

b) f (t) = 0 for t < 0

c)The function f(t) increases in absolute value no sooner than an exponential function. So,

there are numbers M > 0 and S_0> 0 such that for each term t, we have: | f(t) | < $Me^{S_0 t}$

[2],[5],[6].

Definition : The Laplace transform of the function f(t) called integral

$F(s)=\int_0^{+\infty} f(t)e^{-st} \, dt$ ………………..(1)

Where s is a complex number s = p + iw provided that $Re_s = p > S_0$.

The function F(s) is called the Laplace function picture. The function is referred to as the initial function. As mentioned earlier, the completion of the three requirements above is necessary to be the initial function f(t). The following features only fulfill those three requirements are called original. From the practical perspective, this tightening original class is not very reasonable.

If F(s) is a reflection of the function f(t), then it says:

$$L[f(t)] \rightarrow F(s) \quad \text{or} \quad F(s) \rightarrow f(t)$$

Theorem: If the function F(s) is an image, then it says: F(s) → 0 when R_{es}= p →+ ∞.

Based on this theorem, it can be said that the necessary condition for a given function F(s) to be an image of a function f (t) is to complete the reconciliation: $\lim_{Res \to \infty} F(s) = 0$

Examples of Laplace Transforms :
1. The Laplace transform of impulse function:

$$\sigma(t) = \begin{cases} 0 \text{ for } t < 0 \\ 1 \text{ for } t \geq 0 \end{cases}$$

Laplace transform application in population growth.

The function σ (t) is original because it meets the three conditions for being such,

Lets find the reflection of σ (t)

$$L\{ \sigma(t) \} = \int_0^{+\infty} e^{-st} .1 \, dt = \left[\frac{e^{-st}}{-s} \right]$$

$$= \frac{1}{s}$$

2. The Laplace transform of function f (t) = t is $F(s) = \int_0^{+\infty} e^{-st} . t \, dt$

$$= \left[\frac{t\,e^{-st}}{-s} + \frac{1}{s} \int_0^{+\infty} e^{-st} \, dt \right]$$

$$= \frac{1}{s^2}$$

3.The Laplace transform of function $f(t)=e^{at}$ where a ∈ c is given by

$$F(s) = \int_0^{+\infty} e^{-st} . e^{at} \, dt$$

$$= \int_0^{+\infty} e^{-(s-a)t} \, dt$$

$$= \frac{1}{s-a}$$

provided that Re (s-a) >0 so

$$L\{ e^{at} \} = - \frac{1}{s-a}$$

Properties of Laplace transform:

The use of operational methods simplifies the solution of many problems and in particular, the solution fo differential equations. To find out the inverse when the original is provided, or vice versa, read made table that represent the connections between the original and reflection can be used. However, the problem cannot be solved only with a table because they cannot long include all the cases of reflections that may appear. For this reason, different methods of finding the original have been elaborated but for the presentation of these methods, first of all, the properties of the reflection of Laplace should be known. We are now going to present some of the basic properties of the Laplace transformation.

Linearity property of Laplace transform: If $L\{f_1(t)\} = F_1(s)$ and $L\{f_2(t)\} = F_2(s)$ then

$$L\{c_1 f_1(t) + c_2 f_2(t)\} = c_1 F_1(s) + c_2 F_2(s)$$

This is clear since the integral of the sum is equal to the sum ofthe integral. However, we will prove it:

$$L\{c_1 f_1(t) + c_2 f_2(t)\} = \int_0^{+\infty} e^{-st} [c_1 f_1(t) + c_2 f_2(t)]\, dt$$

$$= \int_0^{+\infty} e^{-st} c_1 f_1(t)\, dt + \int_0^{+\infty} e^{-st} c_2 f_2(t)\, dt$$

$$= c_1\int_0^{+\infty} e^{-st} f_1(t)\, dt + c_2\int_0^{+\infty} e^{-st} f_2(t)\, dt$$

$$= c_1 F_1(s) + c_2 F_2(s)$$

Shifting Property of Laplace transform:

If $L\{f(t)\} = F(s) \Rightarrow L\{e^{\lambda t} f(t)\} = F(s - \lambda)$

$$L\{e^{\lambda t} f(t)\} = \int_0^{+\infty} e^{-st} . e^{\lambda t} f(t)dt = \int_0^{+\infty} e^{-(s-\lambda)t} f(t)\, dt = F(s-\lambda)$$

Where $Re(s-\lambda) > s_0$

Original derivation property of Laplace transform:

If $L\{f(t)\} = F(s) \Rightarrow L\{f^I(t)\} = s F(s) - f(0)$

Proof

$$L\{f^I(t)\} = \int_0^{+\infty} e^{-st} f^I(t)\, dt$$

and integration by parts gives $u = e^{-st}$ $V = f(t)$

$du = -s\, e^{-st} dt$ $dv = f^I(t)\, dt$ $L\{f^I(t)\} = [e^{-st} f(t) + s\int_0^{+\infty} e^{-st} f^I(t)\, dt = -f(0) + s F(s)$

But $|f(t)| < M\, e^{s_0 t}$ and $Re\, s = p > s_0$ on the order hard $F(s) = \int_0^{+\infty} e^{-st} f(t)dt$, hence

$$L\{f^I(t)\} = -f(0) + s F(s)$$

We are not going to consider the other properties as far as these properties are sufficient for our study.

Laplace transform for population growth:

How can people alter their size? The population is influenced by four variables. In a population, the birth rate is increasing and the population death rate is decreasing. New entries also exist in population (migration) or population (immigration).

ISSN PRINT 2319 1775 Online 2320 7876
, UGC CARE Listed (Group -I) Journal Volume 11, Iss 05, 2022

There are also fresh entries. The above-mentioned statements have an easy equation [1]

$\Delta P = B - D + I - E$

Where ΔP = the change in the size of population in time t

B = birth rate

D = death rate

I = migration rate

E = immigration rate

If the population is supposed to be "closed", then we don't have a mobility of the population, so that

$\Delta P = B - D$

The simplest model is if density independent population growth is regarded. Independence in density implies that the population size does not affect the birth and death rate. Therefore, birth and death are directly proportional to

$P'(t)= rP(t)$ ....(4)

(The English mathematician Thomas R Malthus brought this model to population growth in 1798). The speed of change per capita $r = P'/p$ in this model is continuous, positive when it increases and negative when p decreases [1] . Applying the Laplace transform on both sides of (4), we have [3]

$L\{P'(t)\}=L\{rP(t)$ ....(5)

Now applying the original derivation property on (5), we have

$SP(s) - P(0) = r P(s)$ where P(0) is the population at time t = 0

$(s-r)P(s) = P(0)$

$P(s)=\frac{P(0)}{(s-r)}$ ....(6)

Operating inverse Laplace transform on both sides of (6), we have

$L^{-1}\{ P(s) \} =L^{-1} \{\frac{P(0)}{(s-r)}\} = P(0)\, e^{\pi}$ ..(7)

So, $P(t) = P(0)\, e^{\pi}$

Now using the equation (7), we can find r and calculate the population for different years. We will apply the equation (7) for the Albanian population.

Using the data with t = 0 corresponding to the year 1990 , we have P(0) = 3188,380 we can solve r using the fact that P = 3,080,124 when t = 10 which is the year 2000 [1] [4] By using (7)

$3,08,124 = 3188,380e^{10r}$ => $r = -\ 0.00345$

The general solution is given to us

$P(t) = 3,188.380e^{-0.00345}$

We are now going to compute the population at later years and compare it to the actual data. The following chart represents it [1][7][8]

Year	Actual	Predicted
2001	3063320	3069648
2002	3057018	3059076
2003	3044993	3048540
2004	3034231	3038041
2005	3019634	3027577
2006	3003329	3017150

2007	2981755	3006759
2008	2958266	2996404
2009	2936355	2986084
2010	2918674	2975800
2011	2907368	2965551
2012	2903008	2955337
2013	2897770	2945159
2014	2892394	2935016
2015	2885796	2924907
2016	2875592	2914834
2017	2876591	2904795
2018	2870324	2894791
2019	2862427	2884821

As shown in 2001 and 2019, we have a good agreement between actual and predicted data. It provides us some hope to predict the future. So, we can plug in our data to estimate the population in million that year:
(P) 2025 = $3{,}188{,}380e^{-0.00345435}$

= 2825719

Conclusions:

Laplace transformation is the easiest and most expensive way to solve many of the different technical issues which occur when solving them directly. The transformation of Laplace allows us to readily achieve a definitive analytical solution to solve population growth. The Malthus model allows us to predict the population size. Clearly, the forecast information is almost the same as the actual information. An instance of how accurate this model is the forecast about the predicted and current information on the population for 2001 – 2019 are very much in common with each other. This gives us an incredible tool to predict future population growth.

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