 

# HEAT TRANSFER IN MAGNETO HYDRO DYNAMIC FLOW OVER A STRETCHING SHEET WITH VISCOUS AND OHMIC DISSIPATIONS

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**Abstract**

Here momentum boundary layer is due to linearly stretching of boundary sheet. In addition transverse magnetic field and normal electric field are considered in this paper, momentum equation are solved numerically by employing fifth order Runge-Kutta-Fehlberg method with shooting. The effects of various physical parameters like viscoelastic parameter, Prandtl number, local Reynolds number, Eckert number Hartmann number and applied electric parameter on various momentum and heat transfers characteristics are analysed.

**Keywords:** Viscoelastic fluid, boundary layer flow, skin-friction, heat transfer, stretching surface, viscous dissipation ,Ohmic dissipation, uniform transverse magnetic field and normal electric field .

## 1. Introduction

industrial applications such as polymer sheet or filament extrusion from a dye or long thread between feed roll or wind-up roll, glass fiber and paper production, drawing of plastic films, liquid films in condensation process etc are the few examples of stretching sheet. Initiating works of SAKIADIS [1] there was an extensive theoretical study on this problem over last five decades. However, some experimental work revealed that most fluids associated in these applications are more viscoelastic in nature than viscous.

Like momentum transfer heat transfer phenomenon too plays an equally important role in industrial and technological applications as the rate of cooling influences a lot to the quality of the final product with desired characteristics.

## 2.Formulation of the Problem

the works of BEARD and WALTERS [2] the unsteady state two-dimensional boundary layer equation for viscoelastic fluid in Cartesian co-ordinate system my be written as ( DANDAPAT et al. [9])

$$\rho\left[\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial x_k}\right]=\frac{\partial p}{\partial x}-\mu\frac{\partial^2 u_i}{\partial x_k\partial x_k}-k_0\left\{\begin{array}{l}\frac{\partial}{\partial t}\left(\frac{\partial^3 u_i}{\partial x_k\ \partial x_k}\right)+u_m\frac{\partial^3 u_i}{\partial x_k\partial x_k\partial x_k}-\frac{\partial u_i}{\partial x_m}\frac{\partial^2 u_m}{\partial x_k\partial x_k}\\ -2\frac{\partial^2 u_i}{\partial x_m\partial x_k}\frac{\partial u_m}{\partial x_k}\end{array}\right\} \quad (2.1)$$

## Governing Equations

Steady state two-dimensional incompressible electrically conducting viscoelastic fluid flow over a continues stretching sheet has been considered for investigation. flow is considered to be generated by stretching of an elastic boundary sheet issued from a slit with the application of two equal and opposite forces in such way that velocity of the boundary sheet (Fig.1). The flow region is exposed under uniform transverse magnetic fields $\vec{B}$ and uniform normal electric field $\vec{E}$,. Such application of electric field and magnetic field stabilizes the boundary layer flow (DANDAPAT and MUKHOPADHYAY [3]).

The governing boundary layer equations for momentum and heat transfers are the modified version of the equation (2.1) and also of the governing equations of ANDERSSON [4], KHAN and SANJAYANAND [5] and CORTELL [6] and those are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right\}-\frac{\sigma}{\rho}\left(E_0B_0+B_0^2u\right) \tag{2.3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\left(u\,B_0+E_0\right)^2\sigma}{\rho c_p} \tag{2.4}$$

### 3. Boundary conditions on velocity

$$\begin{aligned} &u=U_w(x)=bx,\ v=0 \quad &&at\quad y=0\\ &u=0 &&as\quad y\to\infty. \end{aligned} \tag{3.1}$$

### Solution of the momentum boundary layer equation

To solve the governing boundary layer equation (2.3) we use the following transformations.

$$u=bxf_\eta,\ v=-\sqrt{b\gamma}\,f(\eta)\quad \eta=\sqrt{\frac{b}{\gamma}}y. \tag{3.2}$$

Here $f(\eta)$ is the dimensionless stream function and $\eta$ is the pseudo-similarity variable. Substitution of the transformations of equation (3.2) in the equation (2.3.

$$f_\eta^2-ff_{\eta\eta}=f_{\eta\eta\eta}-k_1^*\left[2f_\eta f_{\eta\eta\eta}-ff_{\eta\eta\eta\eta}-f_{\eta\eta}^2\right]-Mn^2\left(E_1+f_\eta\right) \tag{3.3}$$

where $k_1^*=\frac{k_0 b}{\gamma}$ is the dimensionless viscoelastic parameter, $Mn=\sqrt{\frac{\sigma}{\rho b}}B_0$ is Hartmann number, $E_1=\frac{E_0}{B_0U_w}$ is the local normal electric parameter and the subscript $\eta$ stands for differentiation with respect to $\eta$.

the boundary conditions on stream function $f$ non-dimensional form.

$$f(0)=0,\qquad f_\eta(0)=1,\qquad f_\eta(\infty)=0 \tag{3.4}$$

Thus terms of order $k^2, k^3$ and higher orders may be neglected and therefore we may seek the solution of the equation (3.3) in the form

$$f = f_0(\eta) + k_1^* f_1(\eta) + 0\left(k_1^{*2}\right) \tag{3.5}$$

Substituting the series expansion of the equation for $f_0(\eta)$ and $f_1(\eta)$.

$$f_{0\eta}^2 - f_0 f_{0\eta\eta} = f_{0\eta\eta\eta} - Mn^2\left(E_1 + f_{0\eta}\right) \tag{3.6}$$

$$f_{1\eta\eta\eta} - Mn^2 f_{1\eta} - 2f_{0\eta}f_{1\eta} + f_0 f_{1\eta\eta} + f_1 f_{0\eta\eta} = 2f_{0\eta}f_{0\eta\eta\eta} - f_0 f_{0\eta\eta\eta\eta} - f_{0\eta\eta}^2 \tag{3.7}$$

Making use of the series expansion of the equation (3.5) in the boundary conditions (3.4) we obtain boundary conditions for $f_0(\eta)$ and $f_1(\eta)$ in the following form.

$$f_0(0) = 0, \quad f_{0\eta}(0) = 1, \; f_{0\eta}(\infty) = 0 \tag{3.8}$$

$$f_1(0) = 0, \quad f_{1\eta}(0) = 0, \; f_{1\eta}(\infty) = 0 \tag{3.9}$$

## 4.Similarity Solution of the heat transfer equation

**Case A: prescribed surface temperature (PST)**

In order to solve the thermal boundary layer equation (2.4) in PST case we consider non-isothermal temperature boundary condition as follows.

$$\begin{aligned} T &= T_w = T_\infty + A_0\left(\frac{x}{l}\right)^2 && at \quad y = 0 \\ T &\to T_\infty && as \quad y \to \infty \end{aligned} \tag{4.1}$$

Keeping in mind to obtain similarity equation from the thermal boundary layer equation (2.7) we define dimensionless temperature variable $\theta(\eta)$ of the form:

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{4.2}$$

where expression for $T_w - T_\infty$ is given by equation (4.1). Making use of the equation (4.2) in the dimensional energy equation (2.4) we arrive at the following form of non-dimensional thermal boundary layer equation.

$$\theta_{\eta\eta} + \Pr\left(f\theta_\eta - 2f_\eta\theta\right) = -\Pr E\left\{f_{\eta\eta}^2 + Mn^2\left(f_\eta^2 + E_1^2 + 2E_1 f_\eta\right)\right\} \tag{4.3}$$

where $\Pr = \dfrac{\gamma}{\alpha}$ is the Prandtl number and $E = \dfrac{b^2 l^2}{A_0 c_p}$ is the Eckert number.

Temperature boundary conditions of the equation (4.1) take the following non-dimensional form.

$$\theta(0) = 1, \qquad \theta(\infty) = 0 \tag{4.4}$$

**Case B: prescribed boundary heat flux (PHF)**

In order to solve the thermal boundary layer equation (2.4) in PHF case we consider variable heat flux boundary condition of the following form:

$$k\left(\frac{\partial T}{\partial y}\right)_w = A_1\left(\frac{x}{l}\right)^2$$

(4.5)

$T \to T_\infty$

we define dimensionless temperature variable $g(\eta)$ of the form:

$$g(\eta) = \frac{T - T_\infty}{A_1\left(\frac{x}{l}\right)^2 \frac{1}{k}\sqrt{\frac{\gamma}{b}}} \tag{4.6}$$

Making use of the equation (4.6) in the dimensional energy equation (2.4) we arrive at the following form of non-dimensional thermal boundary layer equation in PHF case.

$$g_{\eta\eta} + \Pr\left(fg_\eta - 2f_\eta g\right) = -\Pr E\left\{f_{\eta\eta}^2 + Mn^2\left(f_\eta^2 + E_1^2 + 2E_1 f_\eta\right)\right\} \tag{4.7}$$

where $\Pr = \frac{\gamma}{\alpha}$ is the Prandtl number and $E = \frac{b^2 l^2 k}{A_1 c_p \sqrt{\frac{\gamma}{b}}}$ is the Eckert number in PHF case.

In view of the transformation equation (4.6) the temperature boundary conditions of the equation (4.5) take the following non-dimensional form.

$$g_\eta(0) = -1, \qquad g(\infty) = 0. \tag{4.8}$$

The solution of the equation (4.7) subject to the boundary conditions of the equation (4.8) is obtained numerically by applying the fifth order Runge-Kutta-Fehlberg integration scheme with shooting and outline of the numerical solution procedure is described in the next section.

## 6. Results and Discussion

The equations (3.6)-(3.7) are highly nonlinear ordinary differential equations and (4.3) is a non-homogeneous quasi-ordinary differential equation with variable coefficient. In order to solve these equations numerically we follow most efficient numerical shooting method and series solntion method.

Here in Fig. 2. is plotted graphically the velocity profile $f_\eta(\eta)$ and stream function $f(\eta)$ for various values of viscoelastic parameter $k_1^*$ and local normal electric parameter $E_1$. It reveals that the combined effect of local normal electric parameter $E_1$ and viscoelastic parameter $k_1^*$ is to reduce velocity significantly little away from the boundary layer sheet.

Temperature profiles for various values of viscoelastic parameter $k_1^*$, Hartmann number $Mn$, Prandtl number $\Pr$, local normal electric parameter $E_1$ and Eckert number $E$ are plotted in the Figs. 6and9.

Graphs are plotted for temperature profile in the Figures 11, in PHF case. The analysis of the graphs reveal the similar qualitative effects of Hartmann number $Mn$, Prandtl number $\Pr$, viscoelastic parameter $k_1^*$, Eckert number $E$ and local normal electric parameter $E_1$ on temperature profile, but with different magnitudes. However, interestingly we notice that boundary sheet would always attain higher temperature in presence of magnetic and electric fields.

Table 1. Analysis of the tabular data shows that magnetic field and viscous dissipation reduce the numerical value of heat transfer gradient $-\theta_\eta(0)$ in absence of normal electric field. This means the rate of heat transfer across the stretching sheet would be reduced in such a situation. However, in absence of local normal electric field the effect of increasing values of Prandtl number Pr is to increase the numerical value of $-\theta_\eta(0)$. This means there would be enhancement of rate of heat transfer across the stretching sheet. In presence of local normal electric parameter $E_1$, the effect of increasing values of Prandtl number Pr is to decrease the rate of heat transfer for smaller values of local normal electric parameter, but for larger values of $E_1$ the direction of heat transfer would be changed in case of viscoelastic fluid. When the strength of local normal electric parameter $E_1$ is further increased then the effect of increasing values of Prandtl number Pr is to increase the rate of heat transfer in the reverse direction.

## 7. Conclusion

A numerical local similar solution has been presented on momentum and heat transfer characteristics in an incompressible electrically conducting viscoelastic boundary layer fluid flow over a linear stretching sheet where the fluid is influenced by a transverse magnetic field and normal electric field. Highly non-linear fourth order stream function equation has been solved using three boundary conditions only following the method employed by Rajagopal et al. [2] and Mahapatra and Gupta [6] which uses series expansion of stream function $f(\eta)$ up to first order term of viscoelastic parameter $k_1^*$. Fifth order Runge-Kutta –Fehlberg method with shooting is used to solve stream function and heat transfer equations numerically. The results are analysed for the situation when stretching boundary is prescribed by non-isothermal prescribed surface temperature (PST) and variable boundary heat flux which vary quadratically with the flow directional coordinate x. The effects of various physical parameters like viscoelastic parameter, Prandtl number, local Reynolds number, Eckert number, Hartmann number and local normal electric parameter on various momentum and heat transfers characteristics are obtained. Some of the important conclusions of this paper are

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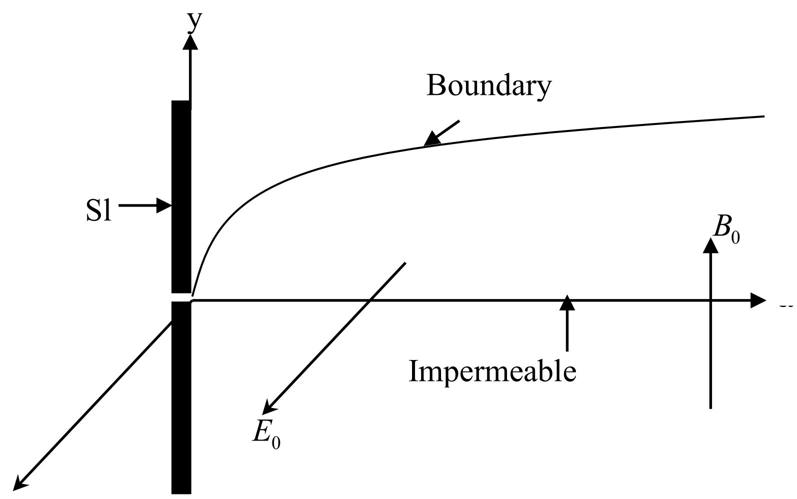


Fig.1. Boundary layer over an impermeable linear stretching sheet with electro- magnetic field

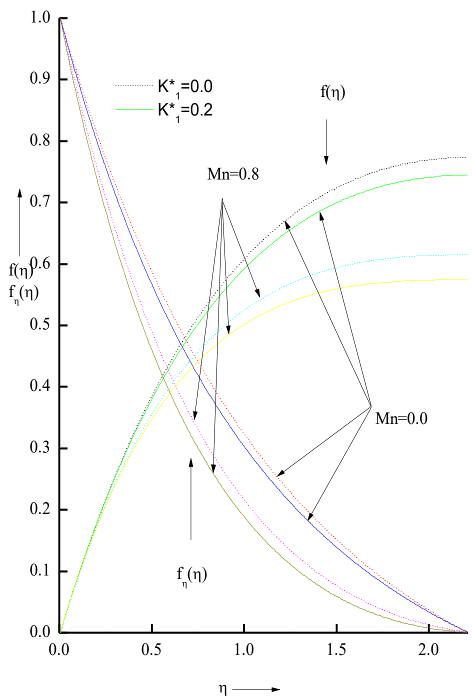


Fig.2. Graph of stream function f(η) and velocity function $f_\eta(\eta)$ vs. η for di of viscoelastic parameter $k^*_1$ and Hartmann number Mn when $E_1$=0.2.

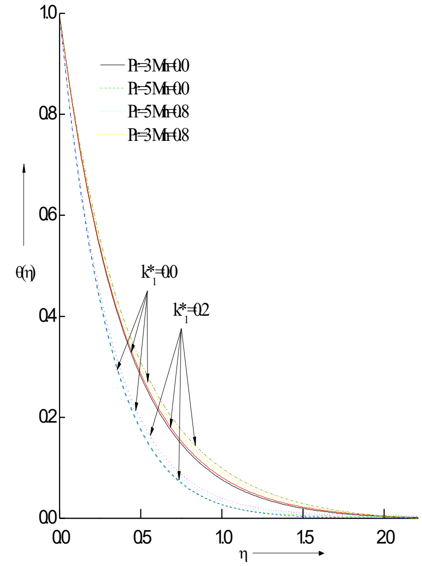


Fig.6. Dimensionless temperature profile θ(η) vs. η for various values of Prandtl number Pr and Hartmann number Mn when E=0.0 and $E_1$=0.0 in PST case.

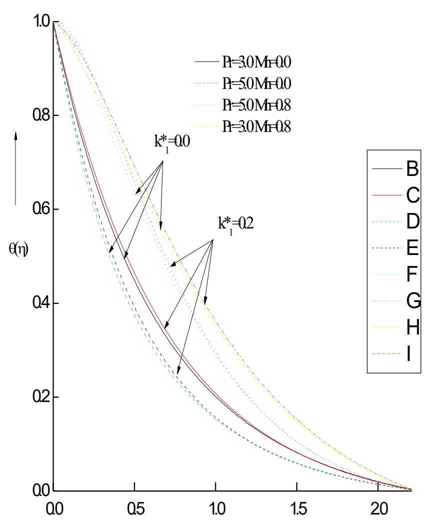


Fig.8. Dimensionless temperature profile θ(η) vs. η for various values of Prandtl number Pr and Hartmann number Mn when E=1.0 and $E_1$=0.2 in PST case.

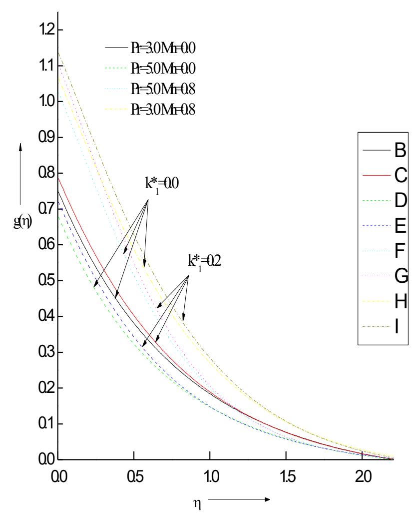


Fig.11. Dimensionless temperature profile g(η) vs. η for various values of Prandtl number Pr and Hartmann number Mn When E=1.0 ans $E_1$=0.0 in PHF case.

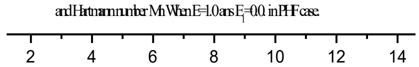

  

Table. 1

Wall temperature gradient $-\theta_\eta(0)$ in PST case for different values of Hartmann numbers $Mn$, number $E$, local electric parameter $E_1$, Prandtl number $\Pr$ and viscoelastic parameter $k_1^*$

| $k_1^*$ | $Mn$ | $E$ | $E_1$ | $\Pr$ | $-\theta_\eta(0)$ |
|---|---|---|---|---|---|
| 0.0<br>0.2 | 0.0 | 0.0 | 0.0 | 3.0 | 2.469<br>2.459 |
| 0.0<br>0.2 | | | | 5.0 | 3.260<br>3.244 |
| 0.0<br>0.2 | 0.8 | 1 | 0.0 | 3.0 | 0.836<br>0.672 |
| 0.0<br>0.2 | | | | 5.0 | 0.911<br>0.673 |
| 0.0<br>0.2 | | | 0.2 | 3.0 | 0.360<br>0.189 |
| 0.0<br>0.2 | | | | 5.0 | 0.242<br>-0.006 |
| 0.0<br>0.2 | | | 0.8 | 3.0 | -1.806<br>-1.962 |
| 0.0<br>0.2 | | | | 5.0 | -2.823<br>-3.087 |