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Topological Cordial Labeling for Some Different Types of Graphs

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Abstract

For a graph G, topological cordial labeling for G is an injective function $f: V(G) \to 2^X$, X is a non – empty set such that $|V(G)| > |X|$ that induces a function $f^*: E(G) \to \{0,1\}$ defined as $f^*(uv) = 0$ if $f(u) \cap f(v)$ is an empty set or a singleton set and 1 otherwise for all edges in G such that $|e_f(0) - e_f(1)|$ is atmost 1 where $e_f(1)$ denotes number of edges labelled with 1 and $e_f(0)$ denotes the number of edges labelled with 0. Also $\{f(V(G))\}$ forms a topology on X. The graph which admits a topological cordial labeling is called a topological cordial graph. In this paper, we discuss topological cordial labelling for some different types of graphs.

Keywords: Bistar graph, Umbrella graph, Butterfly graph, Double wheel graph, Double fan graph, Gear graph, Helm graph.

1 Introduction

In this paper we study only simple, finite and undirected graphs. The graph G has a vertex set V(G) and the edge set E(G). We refer to Bondy and Murthy[2] for notations and terminology. Acharya[1] developed another connection between graph theory and point set topology. S Selestin Lina and Asha S[3] introduced the topological cordial labeling and studied the topological cordial labeling for vertex switching of cycle C_n $(n \geq 3)$ and David's star graph. In this paper, we proved topological cordial labeling for some different types of graphs.

2 Basic Definitions

Definition 2.1

A topological cordial labeling of a graph $G = (V(G), E(G))$ with n vertices is an injective function $f: V(G) \to 2^X$ where $|X| < n$ and non-empty and $\{f(V(G))\}$ forms a topology on X, f induces a function $f^*: E(G) \to \{0,1\}$ given by

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ a\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & otherwise \end{cases}$$

for every $uv \in E(G)$ such that $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ denotes the number of edges labelled with 0 and $e_f(1)$ denotes the number of edges labelled with 1. The graph G which admits topological cordial labeling is called a topological cordial graph.

Definition 2.2

Bistar graph $B_{n,n}$ is obtained by joining an edge to the centre vertices of two copies of $K_{1,n}$.

Definition 2.3

An Umbrella graph $U_{n,m}$ is obtained by joining the central vertex of a fan graph F_m to a path P_n.

Definition 2.4

Butterfly graph $BF_{m,n}$is obtained by joining two cycles C_m and n pendent edges with the common vertex.

Definition 2.5

A double wheel graph DW_n of size n can be composed of $2C_n + K_1$. It consists of two cycles of size n where vertices of two cycles are all connected to a central vertex.

Definition 2.6

Let G be a graph with centre vertex. Then $< G, K_{1,m} >, m \geq 1$ is the graph obtained by attaching the centre vertex of $K_{1,m}$ to centre vertex of the graph G.

Definition 2.7

A Gear graph G_n is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices of the n cycle.

Definition 2.8

The Helm graph H_n is the graph obtained from a wheel by attaching a pendent edge at each vertex of the n cycle.

Definition 2.9

The double fan graph DF_n consists of two fan graph that have a common path.

3 Topological cordial labeling for some different types of graphs

Theorem 3.1 The bistar graph $B_{n,n}$ is a topological cordial graph for all $n \in \mathbb{N}$.

Proof:

Let $G = B_{n,n}$ be a bistar graph.

Let $V(G) = \{u_1, u_2, \dots, u_n, v_1, \dots, v_n\}$.

Then $|V(G)| = 2n + 2$

Let $E(G) = \{u_1 u_{i+1} / \ 1 \leq i \leq n\} \cup \{v_1 v_i / \ 1 \leq i \leq n\}$

$|E(G)| = 2n + 1$

Take $X = \{1,2, \dots, |V(G)| - 1\}$

A function $f: V(G) \rightarrow 2^X$ defined by

$f(u_1) = \phi$, $\quad f(u_i) = \{1,2, \dots, i-1\}$, $i \in \{1,2, \dots, n\}$,

$f(v_i) = \{1, \dots, n-1+i\}$, $i \in \{1,2, \dots, n\}$

Here all the vertex labels are distinct and form a topology on X.

Then the induced function $f^*: E(G) \rightarrow \{0,1\}$ given by

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & otherwise \end{cases}$$

for all $uv \in E(G)$.

Here $f^*(u_1u_i) = 0$, $for\ 1 \leq i \leq n$ and $f^*(v_1v_i) = 1\ for\ 1 \leq i \leq n$.

Here $e_f(1) = n\ and\ e_f(0) = n+1$, $|e_f(0) - e_f(1)| \leq 1$.

Hence f is a topological cordial labeling and G is a topological cordial graph.

Example 3.2

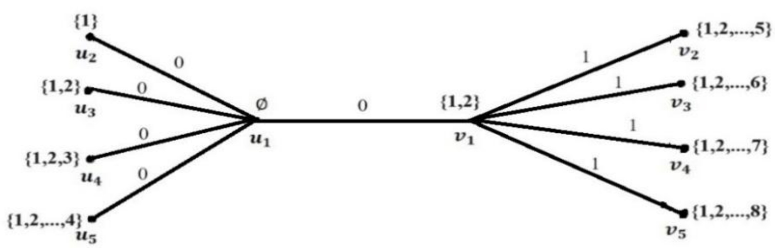

Fig 3.1 Topological cordial labeling of $B_{4,4}$

Theorem 3.3 The graph $< U_{m,m}, K_{1,m-3} >, m > 3$ is a topological cordial graph.

Proof:

Let the vertices of $U_{m,m}$ be $u_1, u_2, \dots, u_m, v_1, \dots, v_m$ and the new vertices be w,$w_1, w_2, \dots, w_{m-3}$.

Let $v_1$be the centre vertex of $U_{m,m}$.

Let $G = < U_{m,m}, K_{1,m-3} >$

Let $V(G) = \{u_1, u_2, \dots, u_m, v_1, \dots, v_m, w_1, w_2, \dots, w_{m-3}\}$ and $|V(G)| = 3m - 3$,

$$E(G) = \{u_1u_{i+1}/\ 2 \leq i \leq m-1\} \cup \{u_iv_1/1 \leq i \leq m\} \cup \{v_iv_{i+1}/1 \leq i \leq m-1\} \cup \{v_1w_i/1 \leq i \leq m-3\}$$

Then $|E(G)| = 4m - 5$

Let $X = \{1,2, \dots, |V(G)| - 1\}$

Define a function $f: V(G) \to 2^X$ as follows:

`$f(v_1) = \phi, f(v_i) = \{1,2,\dots,i\}$ $for\ 2 \le \mathrm{i} \le \mathrm{m}$, $f(w_{\mathrm{m}-3}) = \{1\}$,

$f(u_i) = \{1,2,\dots,\mathrm{m}+\mathrm{i}\}$ for $1 \le \mathrm{i} \le \mathrm{m}$, $f(w_{\mathrm{i}}) = \{1,2,\dots,2m+i\}$ $for\ 1 \le i < m-3$

Here all the vertex labels are distinct and form a topology on X.

Then the induced function $f^*: E(G) \to \{0,1\}$ given by

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & otherwise \end{cases}$$

for all $uv \in E(G)$.

Here $f^*(v_1v_2) = 0$, $f^*(v_{\mathrm{i}}v_{\mathrm{i}+1}) = 1$ for i $\in \{2,\dots,m\}$, ,$f^*(u_{\mathrm{i}}u_{\mathrm{i}+1}) = 1$ $for\ 1 \le i \le m-1$, $f^*(v_1w_{m-3}) = 0$, $f^*(v_1u_i) = 0$ $for\ 1 \le i \le \mathrm{m}$, $f^*(v_1w_i) = 0$ $for\ 1 \le i < \mathrm{m}-3$.

Then $e_f(0)\ is\ 2m-2$ and $e_f(1)$ is $2m-3$

Therefore $|e_f(0) - e_f(1)| \le 1$.

Hence f is a topological cordial labeling and G is a topological cordial graph.

Example 3.4

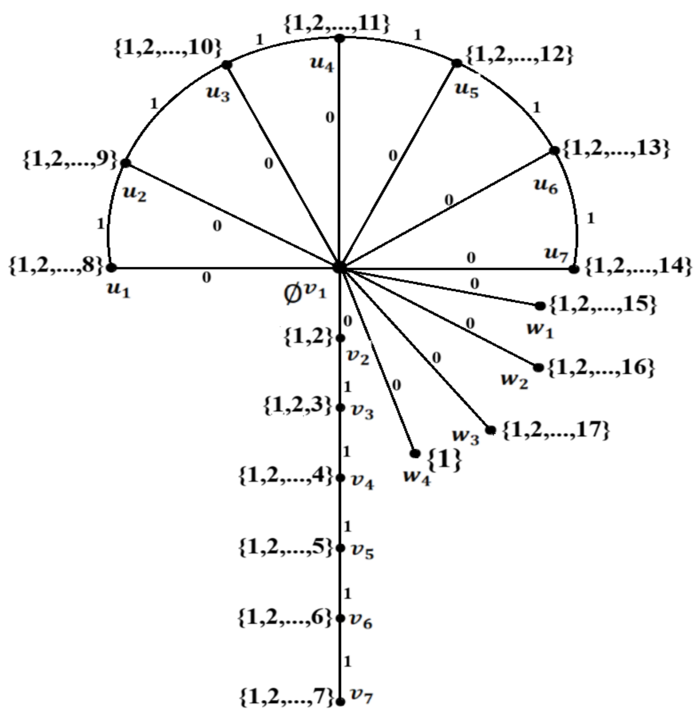

Fig 3.2 Topological cordial labeling of $< U_{7,7}, K_{1,4} >$

Theorem 3.5 The Butterfly graph $BF_{m,n}$ is a topological cordial graph if $n = 2m-8,\ m \ge 4$.

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Proof:

Let $G = BF_{m,2m-8}$, $m \geq 4$

Let v_1 be the common centre vertex of G

Let $V(G) = \{v_1, \ldots, v_m, v_{m+1}, \ldots, v_{2m-1}, u_1, u_2, \ldots, u_{2m-8}\}$ be the vertices of G

Then $|V(G)| = 4m - 9$,

$$E(G) = \{v_i v_{i+1} / 1 \leq i \leq m-1, m+1 \leq i \leq 2m-2\} \cup \{v_1 v_m\} \cup \{v_1 v_{2m-1}\} \cup \{v_1 v_{m+1}\} \cup \{v_1 u_i / 1 \leq i \leq 2m-8\}$$

Then $|E(G)| = 4m - 8$

Let $X = \{1,2, \ldots, |V(G)| - 1\}$

Define a function $f: V(G) \to 2^X$ as follows:

$f(v_1) = \phi, f(u_1) = \{1\}, f(v_i) = \{1, \ldots, i\}, i \in \{2, \ldots, 2m-1\}$,

$f(u_i) = \{1, \ldots, 2(m-1) + i\}, 2 \leq i \leq 2m - 8$

Here all the vertex labels are distinct and form a topology on X.

Then the induced function $f^*: E(G) \to \{0,1\}$ given by

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & otherwise \end{cases}$$

for all $uv \in E(G)$.

Here $f^*(v_1 v_i) = 0$ for $i = 2, m, m+1, 2m-1$, $f^*(v_1 u_i) = 0$ for $1 \leq i \leq 2m-8$, $f^*(v_i v_{i+1}) = 1$ for $2 \leq i \leq m, m+1 \leq i \leq 2m-2$

Then $e_f(0) = 2m - 4$ and $e_f(1) = 2m - 4$

Therefore $|e_f(0) - e_f(1)| \leq 1$.

Hence f is a topological cordial labeling and G is a topological cordial graph.

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Example 3.6

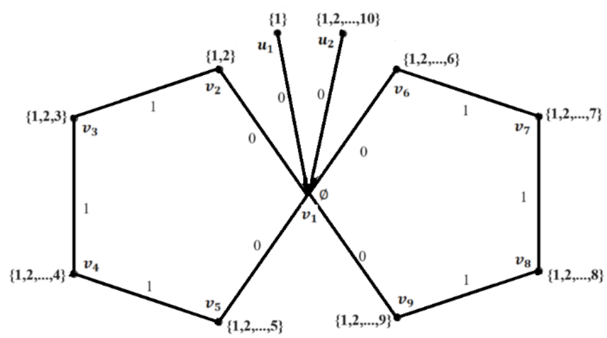

Fig 3.3 Topological cordial labeling of $BF_{5,2}$

Theorem 3.7 The graph $< DW_n, K_{1,1} >$, $n \geq 3$ is a topological cordial graph.

Proof:

Let $G =< DW_n, K_{1,1} >$

Then $V(G) = \{v_0, v_1, \dots, v_{n+1}, u_1, u_2, \dots, u_n\}$ be the vertices of G, where v_0 is the centre vertex of DW_n.

Then $|V(G)| = 2n + 2$

$E(G) = \{v_1v_i / 2 \leq i \leq n+1\} \cup \{v_1u_i/1 \leq i \leq n\} \cup \{u_iu_{i+1}/1 \leq i \leq n-1\} \cup \{v_iv_{i+1}/2 \leq i \leq n\}$
$\cup \{v_1v_0\} \cup \{u_nu_1\} \cup \{v_2v_{n+1}\}$.

Then $|E(G)| = 4n + 1$

Let $X = \{1,2, \dots, |V(G)| - 1\}$

Define a function $f: V(G) \to 2^X$ as follows:

$$f(v_1) = \phi, \quad f(v_0) = \{1\}, \quad f(v_i) = \{1,2, \dots, i\}, i \in \{2, \dots, n+1\}.$$

$$f(u_i) = \{1,2, \dots, n+1+i\}, i \in \{1,2, \dots, n\},$$

Here all the vertex labels are distinct and form a topology on X.

Then the induced function $f^*: E(G) \to \{0,1\}$ given by

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & otherwise \end{cases}$$

for all $uv \in E(G)$.

Here $f^*(v_1v_i) = 0\ for\ 2 \leq i \leq n+1, f^*(v_1u_i) = 0\ for\ 1 \leq i \leq n, f^*(v_1v_0) = 0$

$f^*(u_iu_{i+1}) = 1\ for\ 1 \leq i \leq n-1, f^*(u_nu_1) = 1, f^*(v_iv_{i+1}) = 1\ for\ 2 \leq i \leq n,$

$f^*(v_2v_{n+1}) = 1.$

Then $e_f(0) = 2n + 1$ and $e_f(1) = 2n$

Therefore $|e_f(0) - e_f(1)| \leq 1$.

Hence f is a topological cordial labeling and G is a topological cordial graph.

Example 3.8

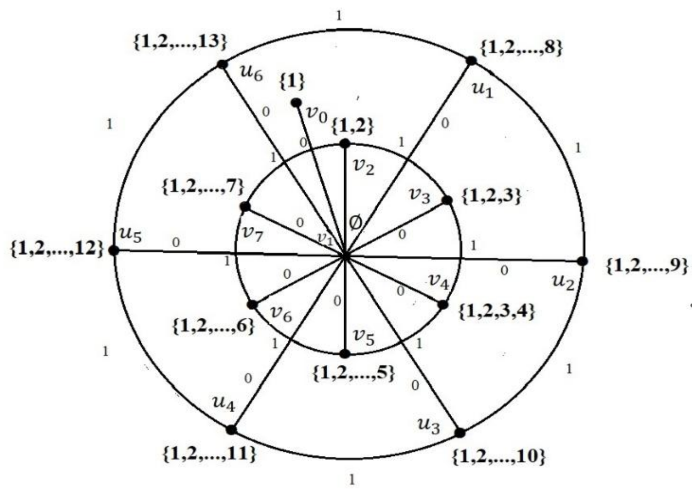

Fig 3.4 Topological cordial labeling of $< DW_6, K_{1,1} >$

Theorem 3.9 The graph $< DF_n, K_{1,n} >, n \geq 2$ is a topological cordial graph.

Proof:

Let $\{v, u, u_1, u_2, \dots, u_n\}$ be the vertices of DF_n

Let $\{v_0, v_1, v_2, \dots, v_n\}$ be the vertices of $K_{1,n}$

Let $E(DF_n) = \{vu_i / 1 \leq i \leq n\} \cup \{uu_i / 1 \leq i \leq n\} \cup \{u_i u_{i+1} / 1 \leq i \leq n - 1\}$

Let G be the graph obtained by attaching the centre vertex of DF_n say v to centre vertex of $K_{1,n}$

$G =< DF_n, K_{1,n} >$

Then $V(G) = \{v, u, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ be the vertex set of G

Then $|V(G)| = 2n + 2$,

$E(G) = \{vv_i / 1 \leq i \leq n\} \cup \{vu_i / 1 \leq i \leq n\} \cup \{uu_i / 1 \leq i \leq n\} \cup \{u_i u_{i+1} / 1 \leq i \leq n\}$.

Then $|E(G)| = 4n - 1$

Let $X = \{1,2, \dots, |V(G)| - 1\}$

Define a function $f: V(G) \to 2^X$ as follows:

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$f(v) = \phi, \quad f(v_i) = \{1,2, \dots, i\}\ for\ 1 \le i \le n.$

$f(u) = \{1,2, \dots, 2n+1\},\ f(u_i) = \{1,2, \dots, n+i\}\ \ for\ 1 \le i \le n,$

Here all the vertex labels are distinct and form a topology on X.

Then the induced function $f^*: E(G) \to \{0,1\}$ given by

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & \qquad otherwise \end{cases}$$

for all $uv \in E(G)$.

Here $f^*(vv_i) = 0\ for\ 1 \le i \le n, f^*(vu_i) = 0\ for\ 1 \le i \le n,$

$f^*(u_i u_{i+1}) = 1\ for\ 1 \le i \le n-1\ , f^*(uu_i) = 1\ for\ 1 \le i \le n.$

Then $e_f(0) = 2n$ and $e_f(1) = 2n - 1$

Therefore $|e_f(0) - e_f(1)| \le 1.$

Hence f is a topological cordial labeling and G is a topological cordial graph.

Example 3.10

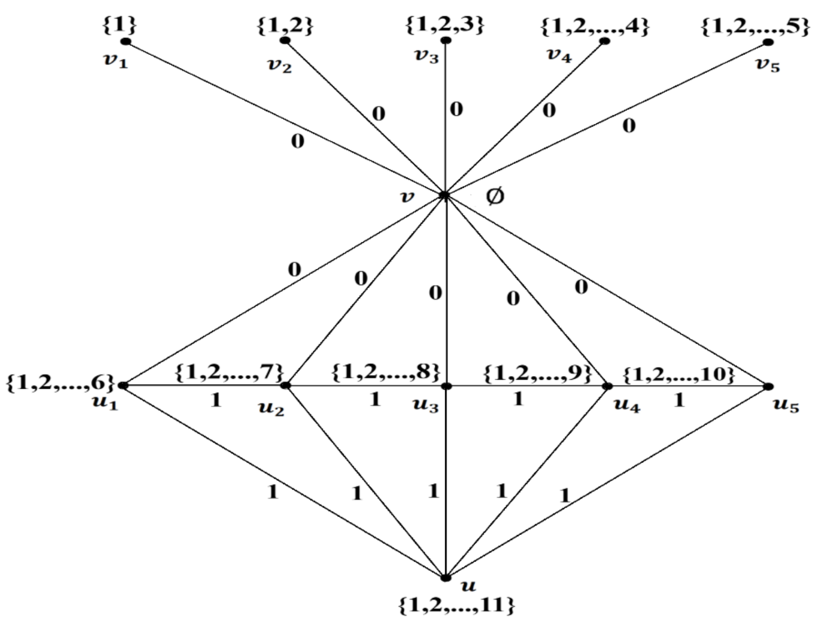

Fig 3.5 Topological cordial labeling of $< DF_5, K_{1,n} >$

Theorem 3.11 A graph obtained from Gear graph G_n by attaching the centre vertex of $K_{1,n-3}$ to any vertex of G_n of degree 2 or 3 is a topological cordial graph.

Proof:

Let $\{v, v_1, v_2, \dots, v_{2n}\}$ be the vertex set of G_n

Let $\{u, u_1, u_2, \dots, u_{n-3}\}$ be the vertex set of $K_{1,n-3}$

Let $E(G_n) = \{vv_i / i = 2m+1 \; for \; m = 1,2,\dots,n-1\} \cup \{v_i v_{i+1} / 1 \le i \le 2n-1\} \cup \{v_{2n} v_1\}$

Let G be the graph obtained by attaching $K_{1,n-3}$ to any vertex of G_n for some v_k where $1 \le k \le n$.

Then $V(G) = \{v, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_{n-3}\}$be the vertex set of G

Then $|V(G)| = 2n+1$,

$E(G) = E(G_n) \cup \{v_k u_i / \; 1 \le i \le n-3\}$.

Then $|E(G)| = 3n$

Let $X = \{1,2,\dots,|V(G)|-1\}$

Define a function $f: V(G) \to 2^X$ as follows:

$f(v) = \phi, f(v_k) = \{1\}, \quad f(u_i) = \{1,2,\dots,i+1\} \; for \; 1 \le i \le n-3$.

For $1 \le i \le n, f(v_i) = \begin{cases} \{1,2,\dots,n+i-3\} & for \; i < k \\ \{1,2,\dots,n+i-4\} & for \; i > k \end{cases}$

Here all the vertex labels are distinct and form a topology on X.

Then the induced function $f^*: E(G) \to \{0,1\}$ given by

$$f^*(uv) = \begin{cases} 1 \;\; if \; f(u) \cap f(v) \; is \; not \; an \; empty \; set \; and \; not \; a \; singleton \; set \\ 0 \qquad\qquad\qquad otherwise \end{cases}$$

for all $uv \, \epsilon \, E(G)$.

Here $f^*(v_k u_i) = 0 \; for \; 1 \le i \le n-3$,

$f^*(v_i v_{i+1}) = 1 \; for \; 1 \le i \le 2n-1 \; and \; i \ne k \;, f^*(v_1 v_{2n}) = 1$.

$f^*(v_k v_{k+1}) = 0 \; and \; f^*(v_{k-1} v_k) = 0$, If $k = 1$, then $v_{k-1} = v_{2n}$ and if $k = 2n$, then $v_{k+1} = v_1$

Then $e_f(0) = 2n-1$ and $e_f(1) = 2n-2$

Therefore $|e_f(0) - e_f(1)| \le 1$

Hence f is a topological cordial labeling and G is a topological cordial graph.

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Example 3.12

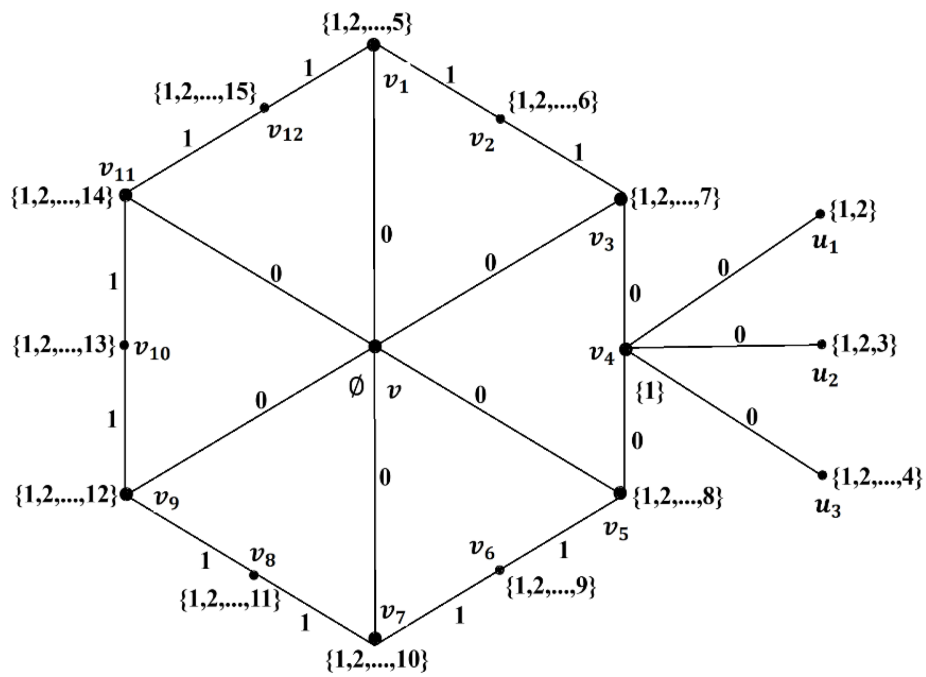

Fig 3.6 Topological cordial labeling of vertex of degree 2 of G_6 attached to centre vertex of $K_{1,n-3}$

Theorem 3.13 A graph obtained from Helm graph H_n by attaching centre vertex of $K_{1,n-1}$ to any pendent vertex of H_n is a topological cordial graph.

Proof:

Let $\{v, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$ be the vertex set of H_n, where $u_1, u_2, \ldots, u_n$ be the pendent

Let $\{w, w_1, w_2, \ldots, w_{n-1}\}$ be the vertex set of $K_{1,n-1}$

Let $E(H_n) = \{vv_i/1 \leq i \leq n\} \cup \{v_i u_i/\ 1 \leq i \leq n\} \cup \{v_n v_1\}$

Let G be the graph obtained by attaching $K_{1,n}$ to any pendent vertex of H_n for some v_k ,where $1 \leq k \leq n$.

Then $V(G) = \{v, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n, w_1, w_2, \ldots, w_{n-1}\}$ be the vertex set of G

Then $|V(G)| = 3n$

$E(G) = E(H_n) \cup \{u_k w_i/\ 1 \leq i \leq n-1\}$.

Then $|E(G)| = 4n - 1$

Let $X = \{1,2, \ldots, |V(G)| - 1\}$

Define a function $f: V(G) \rightarrow 2^X$ as follows:

$f(v) = \phi,, \quad f(v_i) = \{1,2,\dots,i+1\}\ for\ 1 \le i \le n,\ \ f(u_k) = \{1\}$

For $1 \le i \le n, f(u_i) = \begin{cases} \{1,2,\dots,n+1+i\} & for\ i < k \\ \{1,2,\dots,n+i\} & for\ i > k \end{cases}$

$f(w_i) = \{1,2,\dots,2n+i\}$ for $1 \le i \le n-1$

Here all the vertex labels are distinct and form a topology on X.

Then the induced function $f^*: E(G) \to \{0,1\}$ given by

$$f^*(uv) = \begin{cases} 1 & if\ f(u) \cap f(v)\ is\ not\ an\ empty\ set\ and\ not\ a\ singleton\ set \\ 0 & otherwise \end{cases}$$

for all $uv\, \epsilon\, E(G)$.

Here $f^*(vv_i) = 0\ for\ 1 \le i \le n,$

$f^*(u_k w_i) = 0\ for\ 1 \le i \le n-1,\ f^*(v_i v_{i+1}) = 1\ for\ 1 \le i \le n-1.$

$f^*(v_i u_i) = 1\ for\ 1 \le i \le n, \qquad f^*(v_k u_k) = 0$

Then $e_f(0) = 2n$ and $e_f(1) = 2n-1$

Therefore $|e_f(0) - e_f(1)| \le 1$

Hence f is a topological cordial labeling and G is a topological cordial graph.

Example 3.14

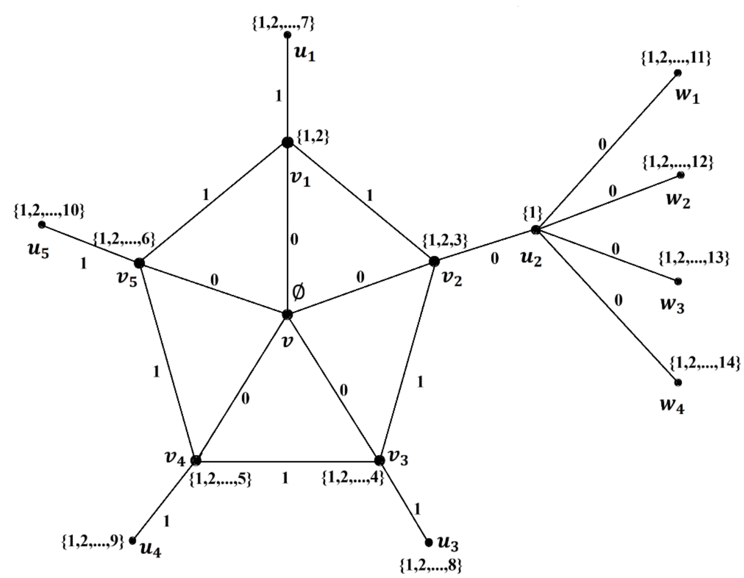

Fig 3.7 Topological cordial labeling of pendent vertex of H_5 attached to centre vertex of $K_{1,n-1}$

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4 Conclusion

Topological cordial labeling for few graphs were studied. In this study, the conditions for a bistar graph, Umbrella graph, butterfly graph, wheel graph, double fan graph, Helm graph and Gear graph to be a topological cordial graphs were found. Further we extend to study the topological cordial labeling of path related graphs.

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