

A New Class of Star Generalized Open Sets Weaker than $\text{Semi}^* \alpha$ - Open Sets

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Abstract:

In this paper we intend to explore the possibility of introducing a novel class, namely $\widehat{S}_p^*$ - Open Sets using pre-open set and generalized closure. An undaunted effort has been made to study its characterization to relate this newly constructed set with the well-established sets that are available by earlier studies. An analysis of the properties of $\widehat{S}_p^*$ open sets have borne interesting results. The results are substantiated with appropriate examples.

Key Words: *Pre-Open, Generalized Pre Closure, $\widehat{S}_p^*$ - Open Sets, $\widehat{S}_p^*$ interior.*

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1. INTRODUCTION

A.S. Mashhour et al, [6] introduced pre-open sets in topological spaces in 1982. Since the introduction of pre-open sets, many topologists generalized various concepts of topology using pre-open sets instead of open sets. In 1970, Levine [4] defined and studied generalized closed sets. Using Levine's generalized closed sets, Dunham [3] introduced the concept of generalized closure and characterized some of its properties. In this paper, analogous to H. Maki, et al's pre-generalized closed sets, we define a new class of sets, namely $\widehat{S}_p^*$ -open sets, using the pre-generalized closure operator instead of closure to due to W. Dunham in the definition of pre-open set. We further show that the concept of $\widehat{S}_p^*$ -open set is weaker than $\text{semi}^* \alpha$ -open sets but stronger than the concept of semi-pre-open sets (β open set) and is independent of g -open sets, pre-open sets, and semi^* -pre-open sets. We study the characterization of $\widehat{S}_p^*$ -open sets. We investigate the basic properties of $\widehat{S}_p^*$ -open sets. We also define $\widehat{S}_p^*$ -interior of a subset and study some of its basic properties. We also establish that the class of $\widehat{S}_p^*$ -closed sets are placed between the class of α -closed sets and the class of semi-pre-closed sets.

2. PRELIMINARIES

Definition 2.1 Let A be a subset of a topological space (X, τ) . Then

- (i) A is semi-open set [4] if there is an open set U in X such that $U \subseteq A \subseteq Cl(U)$ or equivalently if $A \subseteq Cl(Int(A))$.
- (ii) A is pre-open set [6] if $A \subseteq Int(Cl(A))$ and pre-closed if $Cl(Int(A)) \subseteq A$. [35]
- (iii) A is α -open set [7] if $A \subseteq Int(Cl(Int(A)))$ and α -closed if $Cl(Int(Cl(A))) \subseteq A$. [36]

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- (iv) A is semi pre-open set [11] if $A \subseteq Cl(Int(Cl(A)))$ and semi pre-closed if $Int(Cl(Int(A))) \subseteq A$.

The pre-closure of A , α -closure of A and semi pre closure of A are analogously defined and they are respectively denoted by $PCL(A)$, $\alpha Cl(A)$ and $SPCL(A)$.

Definition 2.2 Let A be a subset of a topological space (X, τ) . Then

- A is called a generalized closed set [5] (briefly g -closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- A is called a semi generalized closed set [1] (briefly sg -closed) if $SCL(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ) .
- A is called a pre generalized closed set [6] (briefly pg -closed) if $PCL(A) \subseteq U$ whenever $A \subseteq U$ and U is pre-open in (X, τ) .
- A is called a semi star generalized closed set [2] (briefly S^*g -closed) if $SCL(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ) .
- A is called a semi generalized star open set [11] (briefly S_g^* -open) if there is an open set U in X such that $U \subseteq A \subseteq Scl^*(U)$.
- A is called a semi star open set [9] (briefly S^* -open) if there is an open set U in X such that $U \subseteq A \subseteq Cl^*(U)$ or equivalently if $A \subseteq Cl^*(Int(A))$.
- A is called an alpha star open set [10] (briefly α^* -open) if there is an open set U in X such that $U \subseteq A \subseteq Cl^*(U)$ or equivalently if $A \subseteq Int^*(Cl(Int(A)))$.
- A is called a semi star alpha open set [8] (briefly $S^*\alpha$ -open) if there is an open set U in X such that $U \subseteq A \subseteq Cl^*(U)$ or equivalently if $A \subseteq Int^*(Cl(Int(A)))$.

3. $\widehat{S}_p^*$ OPEN SETS

Definition 3.1. A subset A of a topological space (X, τ) is called a $\widehat{S}_p^*$ -open set, if there is an open set U such that $U \subseteq A \subseteq PCL^*(U)$. The collection of all $\widehat{S}_p^*$ -open sets in (X, τ) is denoted by $\widehat{S}_p^*O(X, \tau)$ or $\widehat{S}_p^*O(X)$.

Theorem 3.2. If A is a subset of a topological space (X, τ) , then the following statements are equivalent:

- A is $\widehat{S}_p^*$ -open
- $A \subseteq PCL^*(Int(A))$
- $PCL^*(Int(A)) = PCL^*(A)$
- $PCL^*(A \cap Cl(Int(A))) = PCL^*(A)$

Proof (i) $\Rightarrow$ (ii): If A is $\widehat{S}_p^*$ -open, then there is an open set U in X such that $U \subseteq A \subseteq PCL^*(U)$. Now $U \subseteq A$ and U is an open implies $U = Int(U) \subseteq Int(A)$ which implies that $A \subseteq PCL^*(U) \subseteq PCL^*(Int(A))$

(ii) $\Rightarrow$ (iii): Since PCL^* is a kuratowski closure operator, $A \subseteq PCL^*(Int(A))$ implies $PCL^*(A) \subseteq PCL^*(PCL^*(Int(A))) = PCL^*(Int(A))$. Since $Int(A) \subseteq A$, $PCL^*(Int(A)) \subseteq PCL^*(A)$. Thus $PCL^*(Int(A)) = PCL^*(A)$

(iii) $\Rightarrow$ (i): Take $U = Int(A)$. Then U is an open set such that $U \subseteq A \subseteq PCL^*(A) = PCL^*(Int(A)) = PCL^*(U)$. Therefore, by definition 2.1.1, A is $\widehat{S}_p^*$ -open.

(iii) $\Leftrightarrow$ (iv): Follows from the fact that for any subset A , $PInt(A) = A \cap Cl(Int(A))$

Theorem 3.3. Every open set is a $\widehat{S}_p^*$ -open set.

Proof: Let A be an open set in X , $\text{Int}(A) = A$ and $A \subseteq \text{PCL}^*(A)$, then we have $A \subseteq \text{PCL}^*(\text{Int}(A))$. Therefore, A is a $\widehat{S}_p^*$ -open set.

Remark 3.4. The Converse of the above Theorem 3.3 need not be true as can be seen from the following example.

Example 3.5 Consider a topological space (X, τ) where $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then $\widehat{S}_p^* O(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{c, d, a\}, \{d, a, b\}\}$. Here the subsets $\{a, c\}$ and $\{a, d\}$ are $\widehat{S}_p^*$ -open but not open.

Theorem 3.6. Arbitrary union of $\widehat{S}_p^*$ -open sets is $\widehat{S}_p^*$ -open.

Proof: Let $\{A_\beta : \beta \in \Delta\}$ be a collection of $\widehat{S}_p^*$ -open sets in X . Since each A_β is $\widehat{S}_p^*$ -open, there is an open set U_β in X such that $U_\beta \subseteq A_\beta \subseteq \text{PCL}^*(U_\beta)$. Hence, $\cup U_\beta \subseteq \cup A_\beta \subseteq \cup \text{PCL}^*(U_\beta) \subseteq \text{PCL}^*(\cup U_\beta)$. Since each U_β is open, $\cup U_\beta$ is open and hence by Definition 3.1, $\cup A_\beta$ is $\widehat{S}_p^*$ -open.

Remark 3.7. The intersection of any two $\widehat{S}_p^*$ -open sets need not be $\widehat{S}_p^*$ -open as can be seen from the following example.

Example 3.8. Consider a topological space (X, τ) where $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}\}$. Here $\{a, c\}$ and $\{b, c\}$ are $\widehat{S}_p^*$ -open but their intersection $\{c\}$ which is not $\widehat{S}_p^*$ -open.

Theorem 3.9. If A is a $\widehat{S}_p^*$ -open set and B is an open set in a topological space (X, τ) , then $(A \cup B)$ is $\widehat{S}_p^*$ -open set.

Proof: The proof follows from Theorem 3.6.

Theorem 3.10. Let $A \subseteq B \subseteq \text{PCL}^*(A)$. If A is $\widehat{S}_p^*$ -open in the topological space (X, τ) , then B is also $\widehat{S}_p^*$ -open.

Proof: Suppose A is $\widehat{S}_p^*$ -open in X , then there is an open set U in X such that $U \subseteq A \subseteq \text{PCL}^*(U)$. Since $A \subseteq B$, $U \subseteq A$ and $U \subseteq B$ which implies $\text{PCL}^*(U) \subseteq \text{PCL}^*(B)$. Therefore, $U \subseteq B \subseteq \text{PCL}^*(U)$ and hence B is $\widehat{S}_p^*$ -open.

Theorem 3.11. If A is a $\widehat{S}_p^*$ -open set in X and B is an open set in X , then $A \cap B$ is a $\widehat{S}_p^*$ -open set in X .

Proof: Since A is $\widehat{S}_p^*$ -open in X , there is an open set U in X such that $U \subseteq A \subseteq \text{PCL}^*(U)$. Since B is open in X , $U \cap B$ is also open. Therefore, $(U \cap B) \subseteq (A \cap B) \subseteq \text{PCL}^*(U) \cap B \subseteq \text{PCL}^*(U \cap B)$. Thus, by Definition 3.1 $A \cap B$ is $\widehat{S}_p^*$ -open.

Theorem 3.12. Every α -open set in X is a $\widehat{S}_p^*$ -open set.

Proof: Let A be an α -open set in X , then $\alpha \text{Int}(A) = A$ and hence $A \subseteq \text{PCL}^*(A) = \text{PCL}^*(\alpha \text{Int}(A)) = \text{PCL}^*(\text{Int}(A))$. By involving Theorem 3.2, A is $\widehat{S}_p^*$ -open.

Remark 3.13. The Converse of the above theorem is not true as can be seen from the following example.

Example 3.14. Consider a topological space (X, τ) where $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}\}$. Then the subsets $\{a\}$, $\{b\}$, and $\{a, b\}$ are α -open sets and they are $\widehat{S}_p^*$ -open sets but the $\widehat{S}_p^*$ -open sets $\{a, c\}$ and $\{b, c\}$ are not α -open sets.

Theorem 3.15. Every semi^* -open set in topological space (X, τ) , is $\widehat{S}_p^*$ -open.

Proof: Let A be a semi^* -open set. Then $A \subseteq Cl^*(Int(A)) \subseteq PCl^*(Int(A))$ which implies $A \subseteq PCl^*(Int(A))$. Hence A is $\widehat{S}_p^*$ -open set.

Remark 3.16. The converse of the above theorem need not be true from the following example.

Example 3.17. Consider a topological space (X, τ) where $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}\}$ the subset $\{a\}$ is a semi^* -open set which is also a $\widehat{S}_p^*$ -open set but not a semi^* -open set.

Theorem 3.18. Every $S^*\alpha$ -open in a topological space (X, τ) is $\widehat{S}_p^*$ -open set.

Proof: Let A be a $S^*\alpha$ -open set in a topological space (X, τ) . Then $A \subseteq Cl^*(\alpha(Int(A)))$ which implies that $A \subseteq PCl^*(Int(A))$. Hence A is $\widehat{S}_p^*$ -open set.

Remark 3.19. The converse of the above theorem is not true as seen from the following example.

Example 3.20. Consider a topological space (X, τ) where $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then the subsets $\{a, c\}$ and $\{b, c\}$ are $\widehat{S}_p^*$ -open sets but are not $S^*\alpha$ -open sets.

Theorem 3.21. Every S_g^* -open set is a $\widehat{S}_p^*$ -open set.

Proof: Let A be a S_g^* -open set in a topological space (X, τ) . Then $A \subseteq SCl^*(Int(A)) \subseteq PCl^*(Int(A))$ which implies $A \subseteq PCl^*(Int(A))$. Hence A is a $\widehat{S}_p^*$ -open set.

Remark 3.22. The converse of the above theorem is not true as shown in the following example.

Example 3.23. Consider a topological space (X, τ) where $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then the subset $\{a, b\}$ is a S_g^* -open set and is also $\widehat{S}_p^*$ -open set but the subsets $\{a, c\}, \{d\}$ are $\widehat{S}_p^*$ -open but not S_g^* -open.

Theorem 3.24. Every $\widehat{S}_p^*$ -open set in a topological space (X, τ) is α^* -open set.

Proof: Let A be a $\widehat{S}_p^*$ -open set. Then it follows from Theorem 2.1.35 every $\widehat{S}_p^*$ -open set is a pre^* -open set. And we know that every pre^* -open is α -open. Therefore, $\widehat{S}_p^*$ -open is α^* -open.

Remark 3.25. Converse of the above theorem is not true as seen from the following example.

Example 3.26. Consider a topological space (X, τ) where $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. Then the subset $\{a\}$ is an α^* -open set but not a $\widehat{S}_p^*$ -open set.

Theorem 3.27. Every $\widehat{S}_p^*$ -open set in a topological space (X, τ) is a sg -open set.

Proof: It follows that every $\widehat{S}_p^*$ -open set is semi-open and every semi-open is semi-generalized open. Therefore every $\widehat{S}_p^*$ -open set is semi-generalized open.

Remark 3.28. Converse of the above theorem is not true as seen from the following example.

Example 3.29. Consider a topological space (X, τ) where $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$. Then the subsets $\{a\}$ and $\{b\}$ are semi-generalized open sets but not $\widehat{S}_p^*$ -open sets.

Remark 3.30. The concept of $\widehat{S}_p^*$ -open set is independent of the concepts of pre-open sets as seen from the following example.

Example 3.31. Consider a topological space (X, τ) where $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$, the subset $\{b, c\}$ is a pre-open set but not a $\widehat{S}_p^*$ -open set. However, in the topological space (X, τ) , where $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}\}$, the subset $\{b, c\}$ is a $\widehat{S}_p^*$ -open set but not a pre-open set. Hence pre-open set and $\widehat{S}_p^*$ -open set are independent.

Remark 3.32. The concept of $\widehat{S}_p^*$ -open set is independent of the concepts of g-open sets as seen from the following example.

Example 3.33. Consider a topological space (X, τ) where $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$, the subset $\{b, c, d\}$ is not a g-open set but $\widehat{S}_p^*$ -open set and subset $\{c\}$ is a g-open set but not a $\widehat{S}_p^*$ -open set.

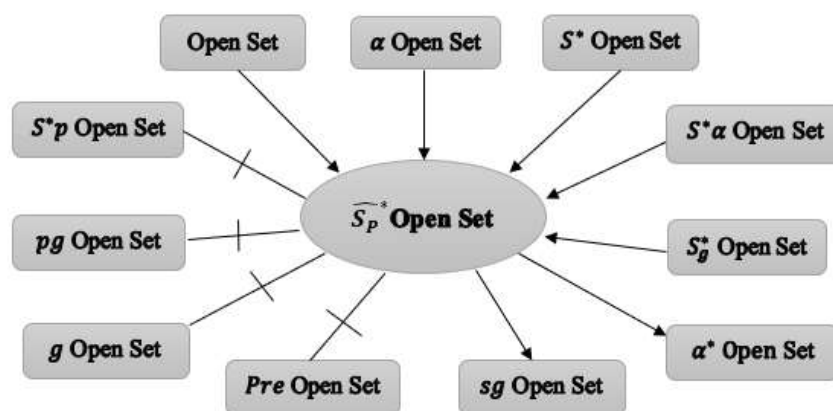
Remark 3.34. The concept of $\widehat{S}_p^*$ -open set is independent of the concepts of pg -open sets as seen from the following example.

Example 3.35. Consider a topological space (X, τ) where $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}\}$. Then the subsets $\{a, c\}$ and $\{b, c\}$ are $\widehat{S}_p^*$ -open sets and not pre-generalized-open sets. However, in the topological space (X, τ) where $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$, the subsets $\{a\}$, $\{b\}$, $\{a, c\}$ and $\{b, c\}$ are pre-generalized-open sets but not $\widehat{S}_p^*$ -open sets. Hence pg open set and $\widehat{S}_p^*$ -open set are independent.

Remark 3.36. The concept of $\widehat{S}_p^*$ -open set is independent of the concepts of $semi^*$ -pre-open sets as seen from the following example.

Example 3.37. Consider a topological space (X, τ) where $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$, the subset $\{a, c\}$ is a $semi^*$ -pre-open set but not a $\widehat{S}_p^*$ -open set. However, in the topological space (X, τ) , where $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}\}$, the subset $\{a, c\}$ is a $\widehat{S}_p^*$ -open set but not a $semi^*$ -pre-open set. Hence $semi^*$ -pre-open set and $\widehat{S}_p^*$ -open set are independent.

Remark 2.1.38. From the above discussions we have the following diagram.



4. $\widehat{S}_p^*$ - INTERIOR

In this section we introduce the concept of $\widehat{S}_p^*$ -interior of a subset and investigate its characterization. We also define $\widehat{S}_p^*$ -interior point of a subset. We also give the characterizations for a subset to be $\widehat{S}_p^*$ -open.

Definition 4.1. The $\widehat{S}_p^*$ -interior of A is defined as the union of all $\widehat{S}_p^*$ -open sets of X that are contained in A and is denoted by $\widehat{S}_p^*Int(A)$.

Definition 4.2. Let $A \subseteq X$. A point x in X is called a $\widehat{S}_p^*$ -interior point of A if A contains a $\widehat{S}_p^*$ -open set containing x .

Theorem 4.3. Let A be a subset of a topological space X then

- (i) $\widehat{S}_p^*Int(A)$ is the largest $\widehat{S}_p^*$ -open set contained in A .
- (ii) A is $\widehat{S}_p^*$ -open if and only if $\widehat{S}_p^*(Int(A)) \subseteq A$
- (iii) $\widehat{S}_p^*(Int(A))$ is the set of all $\widehat{S}_p^*$ -interior points of A
- (iv) A is $\widehat{S}_p^*$ -open if and only if every point of A is $\widehat{S}_p^*$ -interior point of A

Proof: (i) Since $\widehat{S}_p^*(Int(A))$ is the union of $\widehat{S}_p^*$ -open subsets of A , by Theorem 3.6, $\widehat{S}_p^*(Int(A))$ is a $\widehat{S}_p^*$ -open set that is contained in A . Also, it contains every $\widehat{S}_p^*$ -open subset of A . Hence $\widehat{S}_p^*(Int(A))$ is the largest $\widehat{S}_p^*$ -open set contained in A .

(ii) Suppose A is $\widehat{S}_p^*$ -open and since $A \subseteq A$ implies $\widehat{S}_p^*Int(A)$ follows from definition. Conversely suppose that $\widehat{S}_p^*(Int(A)) = A$. By (i) above, $\widehat{S}_p^*(Int(A))$ is $\widehat{S}_p^*$ -open and hence A is $\widehat{S}_p^*$ -open.

(iii) By the definition of $\widehat{S}_p^*$ -interior, a point $x \in \widehat{S}_p^*(Int(A))$ if and only if x belongs to some $\widehat{S}_p^*$ -open subset of A . That is, if and only if x is a $\widehat{S}_p^*$ -interior point of A .

(iv) Follows from (ii) and (iii)

Theorem 4.4. In any topological space X , the following statements hold

- (i) $\widehat{S}_p^*Int(\phi) = \phi$
- (ii) $\widehat{S}_p^*Int(X) = X$
- (iii) $\widehat{S}_p^*Int(A) \subseteq A$
- (iv) For any subsets A and B of a topological space X , $A \subseteq B$ implies $\widehat{S}_p^*Int(A) \subseteq \widehat{S}_p^*Int(B)$

Proof: Clearly (i), (ii), (iii) and (iv) follow from the Definition 4.1.

Theorem 4.5 In any topological spaces (X, τ) , the following statements hold:

- (i) $\widehat{S}_p^*Int(A) \cup \widehat{S}_p^*Int(B) \subseteq \widehat{S}_p^*Int(A \cup B)$
- (ii) $\widehat{S}_p^*Int(A \cap B) \subseteq \widehat{S}_p^*Int(A) \cap \widehat{S}_p^*Int(B)$
- (iii) $Int(\widehat{S}_p^*Int(A)) = Int(A)$
- (iv) $\widehat{S}_p^*Int(Int(A)) = Int(A)$

Proof: (i) Since $A \subseteq (A \cup B)$, using theorem 4.4 (iv), we have $\widehat{S}_p^*Int(A) \subseteq \widehat{S}_p^*Int(A \cup B)$. Similarly, $\widehat{S}_p^*Int(B) \subseteq \widehat{S}_p^*Int(A \cup B)$. Thus (i) is proved.

(ii) Since $A \cap B \subseteq A$, from Theorem 4.4 (iv), we have $\widehat{S}_p^*Int(A \cap B) \subseteq \widehat{S}_p^*Int(A)$. Similarly, $\widehat{S}_p^*(Int(A \cap B)) \subseteq \widehat{S}_p^*(Int(B))$ and proves (ii)

(iii) By Theorem 4.4 (iii), we have $Int(A) \subseteq \widehat{S}_p^*(Int(A))$ and so $Int(Int(A)) \subseteq Int(\widehat{S}_p^*Int(A))$ and hence $Int(A) \subseteq Int(\widehat{S}_p^*Int(A))$. This proves (iii).

(iv) Since $\text{Int}(A)$ is open, by Theorem 3.12, every α -open set is $\widehat{S}_p^*$ open set. $\text{Int}(A)$ is $\widehat{S}_p^*$ - open and hence by involving Theorem 4.3 (ii) $\widehat{S}_p^* \left(\text{Int}(\text{Int}(A)) \right) = \text{Int}(A)$. Thus (iv) is proved.

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