

Heat Transfer Analysis Of Carbon Nanofluids Under Stretch-Induced Flow

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Abstract:

In this paper we have considered Two-dimensional flow and heat transfer of carbon-nano fluids which contain SWCNT and MWCNT. The governing equations of flow and heat transfer partial differential equations are transferred into ordinary differential equations, which are solved numerically.

The effects of physical parameters on flow and heat transfer are analyzed with aid of graphs. The wall temperature gradient and skin friction quantities are numerically calculated and tabulated. Our results are in good agreement with earlier studies for some limitations cases.

Keywords: MWCNT, SWCNT, Heat transfer, nano-fluid, stretching sheet

1. Introduction:

The applications of low thermal conductivity of heat transfer fluids used in power generation, micro electronics cooling, chemical production, refrigeration and air-conditioning, transportation and many other applications. To enhance thermal conductivity of these fluids to increase heat transfer rate. The best of one technique. To enhance effective thermal conductivity of these heat transfer fluids is to add nanoparticles or nanotubes in the base fluid.

In recent era, the researcher concentrates on flow over a stretching sheet has enormous interest because its most abundant industrialized applications like, the sweptback extraction of soft sheet crystal, polymer fiber, preservation of bath, paper production, aerodynamic extrusion of plastic sheets, glass blowing, metal spinning and drawing, plastic films. Hence the quality of the product depends on the rate of heat transfer of the stretching sheet and nano particles.

Carbon nanotubes (CNTs) due to cylindrical carbon molecules origin and found to have special thermal properties with very high thermal conductivities. The diameter of CNTs ranges from ~ 1 to ~ 100 nm and has lengths in micrometer. Fluids which are concern with heat transfer were used in several applications to enhance effective thermal conductivity of the fluids to increase heat transfer rate. Y Li, et al [2014] studies quantitative evaluation of the underdetermined properties of CNT nanofluid as a heat transfer medium from a materials engineering viewpoint.

Choi [1995] given experimental & theoretical by many researches , [6-12] that during flushing with small solid volumes fraction of nano fluid particles i.e less than 5% the thermal conductivity of fluid heat can be enhanced by 10-50%. The thermal conductivity of single

wall, CNT up to 6,600 W/mk and for multiwall CNT upto 3,000 W/mk has been repeated by Hone (2004) & et al [2012]. Sakiadis (1961) was the first to analyze the boundary layer flow over a continuous stretching surface. Then crane [1970] studies the flow over a stretching plate.

Bui Hung Thang et al [2015] studies carbon nanotubes (CNTs) are one of the most valuable materials with high thermal conductivity (above) 1750 W/mk compared to thermal conductivity of Ag 4/9 W/mk). They also showed that the most suitable nanoadditives in fabricating the nanofluid with thermal conductivities that the significantly higher than those of the Prandtl liquids even when the CNT's concentrations are negligible.

Today more than ever, cooling is one of the most pressing needs of many industrial technologies because of the ever increasing heat generation rates at both micro-level (such as computer chips) and macro-level (such as Car Engines) however conventional heat transfer fluids such as air, water, ethylene glycol and oil shows very low thermal conductivity compared to solids.

P.K. Nagarajah, et al (2014) investigate A review on Nanofluids for solar collector applications found that Nanofluids plays vital role in various thermal applications such as automotive industries heat exchangers, solar power generation etc. mostly heat transfer augmentation in solar collectors is one of the key issues in energy saving compact designs and different operational temperatures.

Meyer et al (2013) investigated experimentally the convective heat transfer enhancement of a aqueous suspensions of multi wall CNTs flowing through a straight horizontal tube, they determined the heat transfer coefficient & friction factors as function of Reynolds number. They found that heat transfer was enhanced when comparing the data on a Reynolds Nusselt graph and the increase in Viscosity was four times in the increase in the thermal conductivity.

The objective of the present study to analyze the flow and heat transfer of CNT nanofluids due to stretching sheet. Which contains SWCNT and MWCNT. The governing equations of flow and heat transfer partial differential equations are transferred into nonlinear ordinary differential equations by means of similarity transformation and or solved numerically .

2. Theoretical models for effective thermal conductivity

Several theoretical models, capable of predicting the effective thermal conductivity enhancement of CNT suspensions, are available in open literature. All of these models are based on Fourier's law of heat conduction. Maxwell (1904) proposed an explicit relation for the effective thermal conductivity in terms of the thermal conductivity ratio,

$$u = \frac{xy}{x+y} \text{ show that } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = u \quad \text{and the volume fraction } \phi$$

$$\frac{k_{nf}}{k_f} = 1 + \frac{3(\alpha - 1)\phi}{(\alpha + 2) + (\alpha - 1)\phi} \quad (1)$$

Considering higher orders of volume fraction, Jeffery (1973) and Davis (1986) proposed the following theoretical models

$$\frac{k_{nf}}{k_f} = 1 + 3\lambda\phi + \left(3\lambda^2 + \frac{3\lambda^2}{4} + \frac{9\lambda^2(\alpha+2)}{16(2\alpha+3)} + \dots \dots \dots \right) \phi^2 \quad (2)$$

$$\frac{k_{nf}}{k_f} = 1 + \frac{3(\alpha-1)\phi}{(\alpha+2) + (\alpha-1)\phi} \{ \phi + \phi(\alpha)\phi^2 + o(\phi^3) \} \quad (3)$$

respectively, where $\lambda = \frac{(\alpha-1)}{(\alpha+2)}$. The higher order terms represent pair interactions of randomly dispersed spheres. For low-volume fractions, the above three models result nearly identical predictions of the effective thermal conductivity. Furthermore, these models do not compensate for the shape factor of the particles. Hamilton and Crosser (1962) developed a theoretical model which accounts for the shape factor of the particles

$$\frac{k_{nf}}{k_f} = \frac{\alpha + (n-1)\phi}{\alpha + (n-1)\phi + (1-\alpha)\phi^n} \quad (4)$$

where n is the shape factor of a particle given by $n = \frac{3}{\phi^x}$ where $\phi=1$ for spheres and 0.5 for cylinders and x ranges from 1 to 2. When $\phi=1$; the Hamilton and Crosser (HC) model reduces to Maxwell model. Choi et al. (2001) shows that for cylindrical shape particles the HC model ($n = 6$) significantly underestimated the experimental data by showing a very minor increase of about 10 % for 1.0 vol% nanotubes in oil in comparison to the 160 % experimental enhancement in the thermal conductivity.

Xue (2005) noticed that the existing models are only valid for spherical or rotational elliptical particles with small axial ratio. Furthermore, these models do not account for the effect of the space distribution of the CNTs on thermal conductivity. Xue (2005) proposed a theoretical model based on Maxwell theory considering rotational elliptical nanotubes with very large axial ratio and compensating the effects of the space distribution on CNTs

$$\frac{k_{nf}}{k_f} = \frac{1 - \phi + 2\phi \frac{k_{CNT}}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}}{1 - \phi + 2\phi \frac{k_f}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}} \quad (5)$$

In this study, we have employed Xue (2005) model to determine thermal conductivity and the dimensionless heat transfer rates of nanofluids. In the next section, we will develop the mathematical model to investigate the skin friction and the dimensionless heat transfer rates of different nanofluids. In “Numerical method” we will discuss the numerical approach to solve the proposed model. Finally, we will present the physical interpretation and significance of our results.

3. Mathematical model

We consider the two-dimensional flow over a stretching sheet with heat transfer in water/oil-based nanofluids containing single and multi-wall CNTs. The flow is assumed to be laminar, steady, and incompressible. The base fluids and the CNTs are assumed to be in thermal

equilibrium. The ambient temperature is assumed to be constant. Using order of magnitude analysis, the standard boundary layer equations for this problem can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{nf} \frac{\partial^2 u}{\partial y^2} \quad (7)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} \quad (8)$$

where u and v are the velocity components along the x - and y -axes, T is the temperature, ν_{nf} and α_{nf} are the effective kinematic viscosity and thermal diffusivity of nanofluids, respectively. The effective properties of nanofluids may be expressed in terms of the properties of base fluid and carbon nanotubes, and the solid volume fraction of CNTs in the base fluids as follows

$$\left. \begin{aligned} \rho_{nf} &= (1 - \phi)\rho_s + \phi\rho_{CNT} \\ (\rho c_p)_{nf} &= (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_{CNT} \\ \nu_{nf} &= \frac{\mu_{nf}}{\rho_{nf}}, \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \end{aligned} \right\} \quad (9)$$

where k_{nf} is the thermal conductivity of the nanofluid (see Eq.5), $(\rho c_p)_{nf}$ is the heat capacity of nanofluid and ϕ is the solid volume fraction of nanofluid. The hydrodynamic and thermal boundary conditions for the problem can be written

$$\left. \begin{aligned} u = cxv = 0, T = T_w \text{ at } y = 0 \\ u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (10)$$

where c is the stretching parameter and U_∞ is the free stream velocity. We look for a similarity solution of Eqs. (6), (7), (8), (9) and (10) of the following form

$$\eta = \sqrt{\frac{c}{\nu_f}} y, u = cx f'(\eta), v = -\sqrt{c\nu_f} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (11)$$

where η is the similarity variable and $Re_x = \frac{U_\infty x}{\nu_f}$ is the local Reynolds number based on the free stream velocity and the kinematic viscosity of the base fluid. The stream function

ψ is defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$: Employing the similarity variables (11), Eqs. (6), (7) and (8) reduce to the following nonlinear system of ordinary differential equations:

$$f''' + (1 - \phi)^{2.5} \left[\left(1 - \phi + \frac{\rho_{CNT}}{\rho_f} \right) \{ f f' - f'^2 \} \right] = 0 \quad (12)$$

$$\left(\frac{k_{nf}}{k_f} \right) \theta'' + Pr \left(1 - \phi + \phi \frac{(\rho c_p)_{CNT}}{(\rho c_p)_f} \right) (f \theta') = 0 \quad (13)$$

subject to the boundary condition (10) which become

$$\begin{aligned} f(0) &= 0, f'(0) = 1 \text{ at } f'(\infty) = 0 \\ \theta(0) &= 1 \text{ at } \theta(0) = 0 \text{ as } \theta(\infty) \rightarrow 0 \end{aligned} \quad (14)$$

Here, primes denote differentiation with respect to η , $P_r = \frac{(\mu c_p)_f}{k_f}$ is the Prandtl number of the base fluid.

4. Numerical technique

The system of coupled nonlinear differential Eqns. (12) and (13) along with the boundary conditions Eqn. (14) are solved numerically using fourth order Runge-Kutta Fehlberg method with shooting technique for different values of governing parameters including Prandtl number P_r , and CNT volume fraction ϕ . The boundary value problem is first transformed into an initial value problem (IVP). Then the IVP is solved by a systematic guessing for $f'(0)$ and $\theta'(0)$ until the boundary conditions have been at ∞ , asymptotically decay to zero is to obtained numerical solution. The step size and the convergence criteria were taken as $\Delta\eta = 0.001$ and 10^{-6} respectively.

5. Results

The flow and heat transfer of single & multi wall CNTs fluids have been investigated. The governing partial differential equations and the corresponding boundary conditions are converted into set of non linear ordinary differential equations and these equations are solved numerically.

Fig (1) shows the velocity profile for different values of volume fraction in the case of SWCNT we observed that as ϕ increases the velocity $f(\eta)$ decreases.

Fig (2) depict the velocity profile for different values of volume fraction in the case of MWCNT here we conclude that as we increases the volume fraction ϕ then there is decrease in the velocity profile $f(\eta)$. It is also seen that for each case the increase in solid volume fraction of nano particles velocity profile decreases.

Fig (3 a) describes the influence of solid volume fraction for temperature profile. It is found that the temperature increases with the increase of nano particles fraction increases for the case of SWCNT.

Fig (3 b) shows the influence of governing parameters solid volume fraction for heat transfer rate for the case of MWCNT, the temperature profile increases for increasing value of ϕ .

Fig (4 a) it is shows that temperature profile decreases with the increase of Prandtl number where as nonparticles fraction increases for the case of SWCNT.

Fig (4 b) depicts that again decrease of temperature profile for the increase of Prandtl numbers for the case of MWCNT.

List of symbols

C_p Specific heat, J/kg K

C_f	Friction coefficient
f	Dimensionless stream function
k	Thermal conductivity, W/m K
Nu	Local Nusselt number
Pr	Prandtl number of base fluid
Re_x	Local Reynolds number
T	Local fluid temperature, K
T_∞	Free stream temperature, K
u	x-Component of velocity, m/s
U	Free stream velocity, m/s
v	y-Component of velocity, m/s
x	Distance along the plate, m
y	Distance normal to the plate, c

Subscripts

nf	Nanofluid
CNT	Carbon nanotube

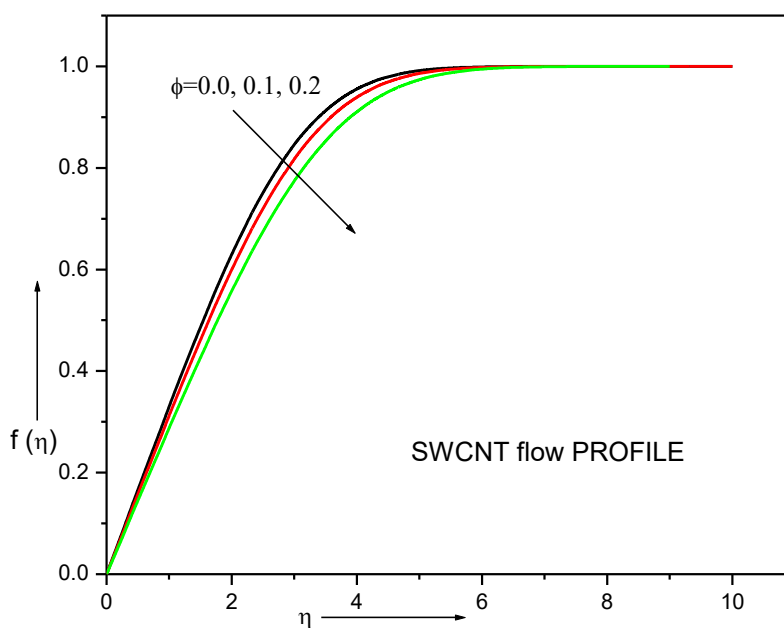
f	Base fluid
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Greek symbols

α	Thermal diffusivity, m ² /s
ϕ	Volume fraction of ferrofluid
η	Similarity variable
μ	Absolute viscosity, N s/m ²
ν	Kinematic viscosity, m ² /s
ρ	Density, kg/m ³
ρc_p	Heat capacity, kg/m ³ K
θ	Dimensionless temperature
ψ	Stream function.

Abbreviations

CNT	carbon nanotube
SWCNT	Single-wall carbon nanotube
MWCNT	Multi-wall carbon nanotube



Fig(1).Variation of flow profile for different values of ϕ

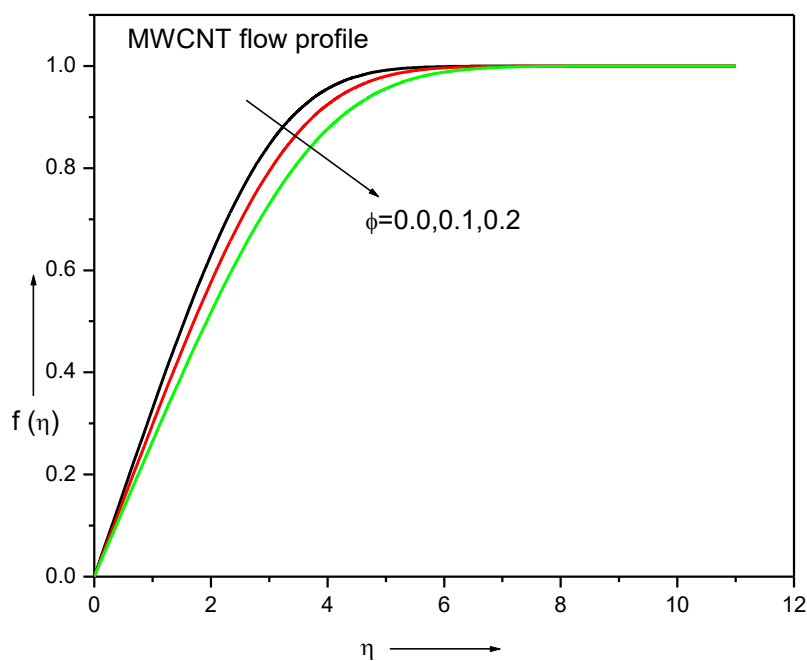


Fig.2. flow profile for different values of ϕ

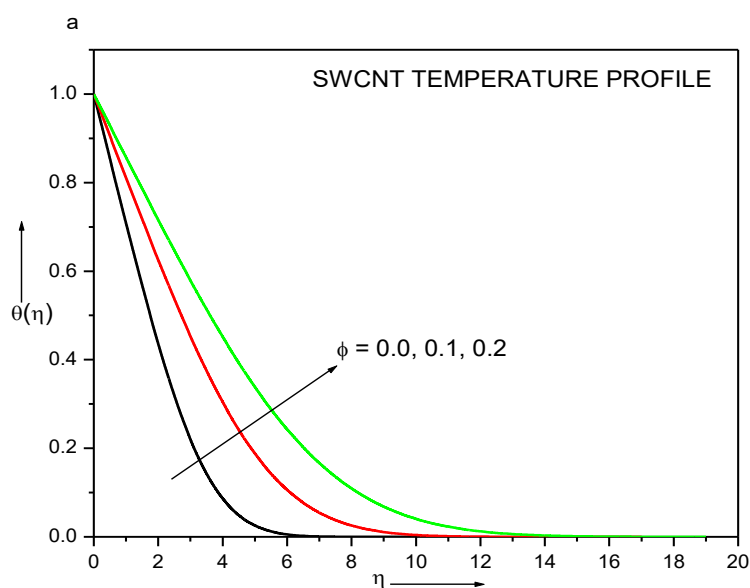


Fig.3 (a). Temperature Profile for different values of solid volume fraction

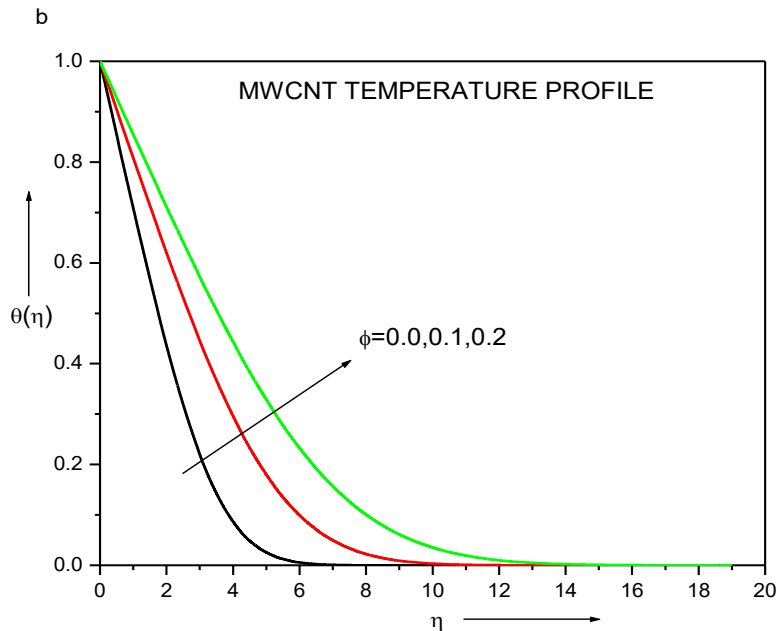


Fig.3 (b). Temperature Profile for different values of solid volume fraction

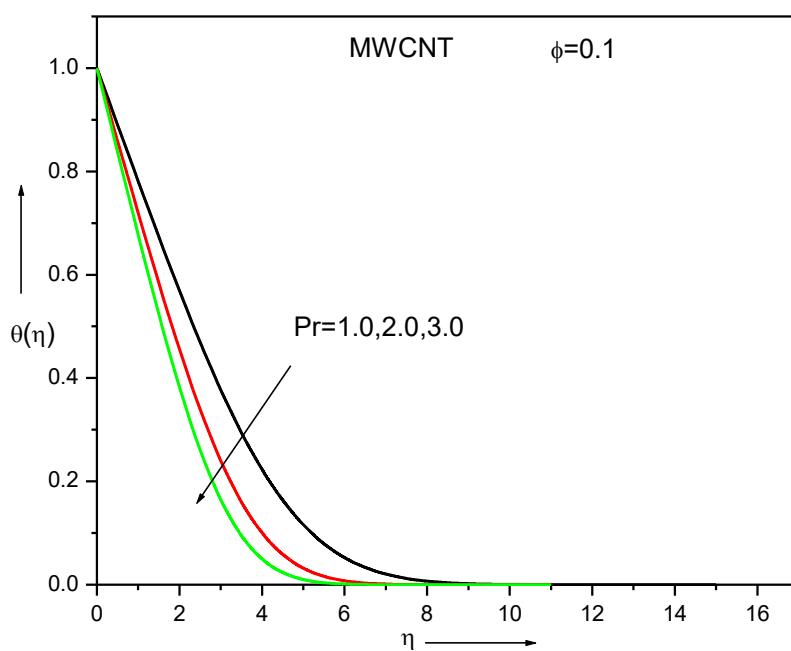


Fig.4(a). Temperature Profile for different values of Pr.

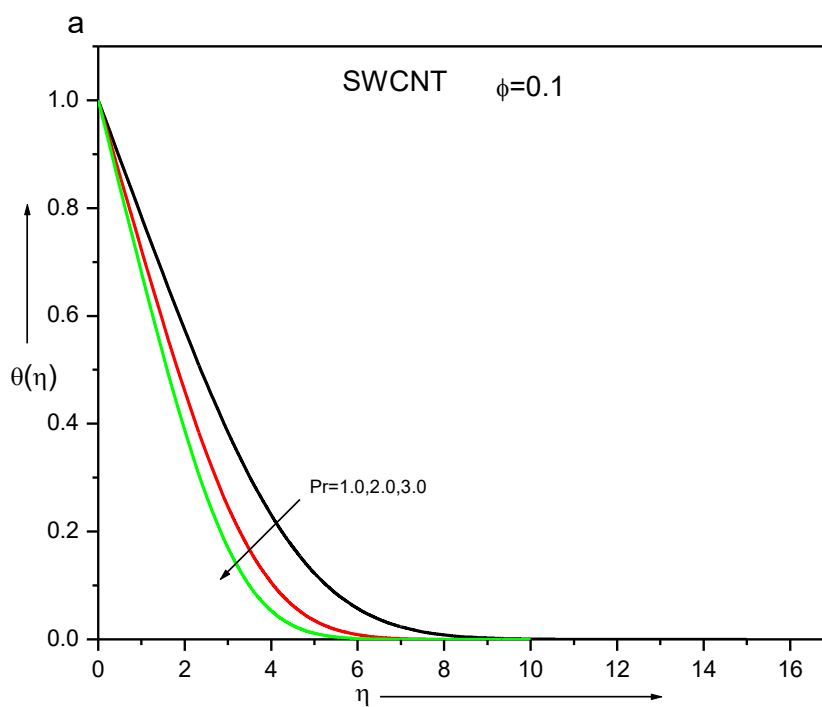


Fig.4.(b). Temperature Profile for different values of Pr.

6. Conclusion:

1. The present study investigates the flow and heat transfer of carbon nano fluids along the stretching sheets with the boundary conditions. It is concluded that
2. The effect of solid volume fraction reduce velocity for both the case SWCNT and MWCNT and the boundary layer thickness also decreases.
3. The heat transfer rate increases with the increase of nano particle fraction for both cases SWCNT MWCNT.
4. The heat transfer profile decreases with the increase of Prandtl number for fix value of solid volume fraction for both cases SWCNT and MWCNT .
5. It is observed that the influence of Pr for temperature profile and nano particle fraction, as Pr has decreasing behavior for $\theta(\eta)$.

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