

A STUDY OF PARTIAL CALCULUS WITH APPLICATIONS TO THE GENETIC SYSTEMS

Sarwar Sultana

Assistant Professor in Mathematics, Government women's first grade college, Jewargi colony kalaburagi.

Abstract:

In comparison to fractional-order differential equations, integer-order differential equations generally fail to properly explain a variety of phenomena in numerous branches of science and engineering. This article implements efficient analytical techniques within the Caputo operator to investigate the solutions of some fractional partial differential equations. The Adomian decomposition method, homotopy perturbation method, and Elzaki transformation are used to calculate the results for the specified issues. In the current procedures, we first used the Elzaki transform to simplify the problems and then applied the decomposition and perturbation methods to obtain comprehensive results for the problems. For each targeted problem, the generalized schemes of the suggested methods are derived under the influence of each fractional derivative operator. The current approaches give a series-form solution with easily computable components and a higher rate of convergence to the precise solution of the targeted problems. It is observed that the derived solutions have a strong connection to the actual solutions of each problem as the number of terms in the series solution of the problems increases. Graphs in two and three dimensions are used to plot the solution of the proposed fractional models. The methods used currently are simple and efficient for dealing with fractional-order problems. The primary benefit of the suggested methods is less computational time. The results of the current study will be regarded as a helpful tool for dealing with the solution of fractional partial differential equations.

Keywords: analytical techniques; Elzaki transform; fractional differential equations; Caputo operator

1. Introduction

Fractional calculus has been the subject of extensive investigation for a long time. In the subject of fractional calculus, new methods and mechanisms are constantly being developed which enable the discovery of significant, difficult insights and previously unknown connections across many fields of physics. The calculus theory was developed in the seventeenth century by Newton and Leibniz. Leibniz developed the integral and derivative notations that are still in use these days. Later, any real order was added to the definition of the terms' derivatives and integrals. Lacroix introduced the idea of a noninteger-order derivative for the first time in 1819. Abel examined the initial application in 1823 [1]. Therefore, the aforementioned field received considerable attention from Fourier, Liouville, Riemann, Grunwald, Letnikov, etc. [2,3]. Fractional order differentiation and integration do not have specific definitions, whereas arbitrary order differentiation and integration offer an expansion of the classical order. The definitions of arbitrary-order derivatives and

integrations have been introduced by many academics in different ways. The definitions of Caputo and Riemann–Liouville among all these are the most broadly applicable. All of these approaches are only expansions of the techniques already used to address the integer case models because the noninteger derivative simplifies the integer derivative to any order [4,5]. Thus, the memory effect is a term that is widely used to describe the study of the dynamics of the function fractionally. The classical derivatives are gradually shown using the nonlocality of the fractional operators. Fractional operators are used to define complex memories and a range of other objects that can be studied using standard mathematical methods like classical differential calculus. Although the application of the FC concept across several academic areas is still in the early stages, FC is currently a very promising tool due to its broadening use in the dynamics of complex nonlinear occurrences. The main reason for studying numerical methods for fractional differential equations (FDEs) is that fractional derivative models are becoming more and better liked by the entire scientific community. Nonlinear partial differential equations (PDEs) have been a popular subject in the field of nonlinear science and have been used to describe problems in a variety of areas, including ecology and economic systems, image processing, quantum physics, and epidemiology. PDEs are frequently utilized in a variety of physical applications, such as supersonic and turbulent flow, magnetohydrodynamic movement through pipes, wave dispersion and propagation, computational fluid dynamics, magnetic resonance imaging, population modeling, medical imaging, electrically signaling nerves, and others [6–8]. To learn more, consider the reference mentioned in [9]. The widespread nature of PDE has been confirmed by a fairly precise evaluation of the number of COVID patients [10,11]. PDE can be used to predict the shape of COVID-19, as seen in [12]. However, for several difficult problems in these domains, the fractional PDE is more precise than the integerorder partial differential equation. Therefore, establishing numerical solutions for fractional PDEs is important. The solution of PDEs can be made simpler by using symmetry, which is a fundamental idea in both mathematics and physics. Particularly, applying symmetry to fractional PDE solutions can substantially simplify mathematical analysis and provide accurate or approximative solutions [13,14]. In a fractional PDE, symmetry can be employed to limit the number of independent variables, which will make the solution process easier. In addition, solutions that are invariant under specific transformations like translations, rotations, and scaling can be found using symmetry. This may result in conclusions that are simpler to understand and have greater physical significance. In general, symmetry is an effective tool for solving fractional PDEs and can be very helpful in explaining the underlying physical phenomena [15,16]. A summary of FDEs and their applications may be found in various significant references. In [17,18], a full review of fractional calculus, FDEs, and their applications in a variety of domains is covered. In the study of viscoelasticity, the theory of FDEs and its applications have been examined [19,20]. Due to the complexity of fractional orders, it is difficult to find analytical solutions for the majority of FDEs. As a result, numerical solutions are found for the aforementioned FDEs. For this purpose, various numerical algorithms have been developed [21,22]. Spectral methods are numerical methods that use spectral representations of the solution to approximate the solution of a differential equation. The tau, collocation, and Galerkin methods are examples of spectral methods that have been used for solving FDEs. The tau

method is a spectral method that uses a shifted Legendre polynomial basis to approximate the solution of FDEs. This collocation method is a spectral method that uses a basis of orthogonal polynomials to approximate the solution of FDEs at collocation points. Several studies have demonstrated the effectiveness of spectral methods for solving FDEs [23–28]. Overall, spectral methods such as the tau, collocation, and Galerkin methods have proven to be effective for solving FDEs and time-fractional differential equations. These methods provide accurate solutions with high efficiency, making them valuable tools for modeling and simulation in various fields. It is becoming clear that fractional partial differential equations (FPDEs) are an effective modeling tool for complex multiscale occurrences, including those involving overlapping microscopic and macroscopic dimensions. The fractional order of the derivatives in FPDEs can be a function of space and time or even a distribution, in contrast to integer-order.

PDEs. It develops outstanding possibilities for modeling and simulating multi-physics phenomena, such as the seamless transition from wave propagation to diffusion or from local to nonlocal dynamics. Numerous well-known scholars have made contributions to this area due to the importance of analytically solving FPDEs in engineering and science. Several methods have been investigated in order to investigate approximate solutions to FPDEs, including the Yang transform decomposition method for fractional-order diffusion equations [29] and time-fractional phi-four equations [30], the reduced differential transform method for coupled time fractional nonlinear evolution equations [31], and the natural transform decomposition method for the solution of fractional Caudrey–Dodd–Gibbon equations [32] and fractional Kuramoto–Sivashinsky equations [33], fractional homotopy analysis method for solving the fractional epidemic model [34] and fractional KdV–Burgers–Kuramoto equation [35], homotopy perturbation transform method for solving fractional Noyes–Field model [36] and time-fractional Fisher’s equation [37], variational iteration transform method for fractional-order Newell–Whitehead–Segel equations [38] and for fractional-order Boussinesq equation [39], approximate analytical method for the solution of time-fractional telegraph equations [40] and the Adams–Bashforth method to study the time-fractional Tricomi equation with nonlocal and nonsingular kernel [41] and many more [42–49]. This study described two novel approaches: the Elzaki transform decomposition method (ETDM) and the homotopy perturbation transform method (HPTM). The Elzaki transform was introduced by Tarig Elzaki to make solving ordinary and partial differential equations in the time domain much easier. Since the 1980s, Adomian has developed a numerical approach for solving functional equations [50,51]. It gives analysis in the form of a series that converges quickly towards the exact solutions. It is considered a powerful method for solving both linear and nonlinear, homogeneous and nonhomogeneous partial and differential equations of integer and noninteger order. The homotopy perturbation method (HPM), which he first suggested in 1998 [52] and later advanced and enhanced [53,54], leads to a very fast convergence solution in the form of a series. To summarize, the Elzaki transform approach is initially utilized to approximate the Caputo-type temporal fractional derivative and turn the initial FPDE into its equivalent PDE. The resultant PDE is then solved using the adomian decomposition method and the homotopy perturbation approach, leading to quick and low-

cost methods for solving the original FPDE. Typically, only one iteration yields high precision in the solution, making it an effective and valuable mathematical tool for nonlinear equations.

Convergence Analysis

In this part, the suggested techniques for convergence analysis are discussed.

Theorem 1. Suppose the exact solution of (4) is $G(\rho, \zeta)$ and let $G(\rho, \zeta), G_n(\rho, \zeta) \in H$ and

$\alpha \in (0, 1)$, where H represents the Hilbert space. The solution obtained $\sum_{q=0}^{\infty} G_q(\rho, \zeta)$ will

converge $G(\rho, \zeta)$ if $G_q(\rho, \zeta) \leq G_{q-1}(\rho, \zeta)$ $A_q > A$, i.e., for any $\omega > 0 \exists A > 0$, such that $\|G_{q+n}(\rho, \zeta)\| \leq \beta, A_n, n \in \mathbb{N}$.

Proof. We take a sequence of $\sum_{q=0}^{\infty} C_0(\rho, \zeta) = G_0(\rho, \zeta)$,

$G_q(\rho, \zeta)$.

$C_1(\rho, \zeta) = G_0(\rho, \zeta) + G_1(\rho, \zeta)$,

$C_2(\rho, \zeta) = G_0(\rho, \zeta) + G_1(\rho, \zeta) + G_2(\rho, \zeta)$,

$C_3(\rho, \zeta) = G_0(\rho, \zeta) + G_1(\rho, \zeta) + G_2(\rho, \zeta) + G_3(\rho, \zeta)$,

.

$C_q(\rho, \zeta) = G_0(\rho, \zeta) + G_1(\rho, \zeta) + G_2(\rho, \zeta) + \dots + G_q(\rho, \zeta)$.

We must demonstrate that $C_q(\rho, \zeta)$ forms a “Cauchy sequence” in order to achieve the desired outcome. Additionally, let us take

$$\|C_{q+1}(\rho, \zeta) - C_q(\rho, \zeta)\| = \|G_{q+1}(\rho, \zeta)\| \leq \alpha \|G_q(\rho, \zeta)\| \leq \alpha^2 \|G_{q-1}(\rho, \zeta)\| \leq \alpha^3 \|G_{q-2}(\rho, \zeta)\| \dots$$

$\leq \alpha_{q+1} \|G_0(\rho, \zeta)\|$. For $q, n \in \mathbb{N}$, we have

$$\begin{aligned} \|C_q(\rho, \zeta) - C_n(\rho, \zeta)\| &= \|G_{q+n}(\rho, \zeta)\| = \|C_q(\rho, \zeta) - C_{q-1}(\rho, \zeta) + (C_{q-1}(\rho, \zeta) - C_{q-2}(\rho, \zeta)) \\ &+ (C_{q-2}(\rho, \zeta) - C_{q-3}(\rho, \zeta)) + \cdots + (C_{n+1}(\rho, \zeta) - C_n(\rho, \zeta))\| \\ &\leq \|C_q(\rho, \zeta) - C_{q-1}(\rho, \zeta)\| + \|(C_{q-1}(\rho, \zeta) - C_{q-2}(\rho, \zeta))\| \\ &+ \|(C_{q-2}(\rho, \zeta) - C_{q-3}(\rho, \zeta))\| + \cdots + \|(C_{n+1}(\rho, \zeta) - C_n(\rho, \zeta))\| \\ &\leq \alpha_q \|G_0(\rho, \zeta)\| + \alpha_{q-1} \|G_0(\rho, \zeta)\| + \cdots + \alpha_{q+1} \|G_0(\rho, \zeta)\| \\ &= \|G_0(\rho, \zeta)\| (\alpha_q + \alpha_{q-1} + \alpha_{q+1}) \\ &= \|G_0(\rho, \zeta)\| \frac{1 - \alpha_{q+1}}{1 - \alpha} \end{aligned}$$

As $0 < \alpha < 1$, and $G_0(\rho, \zeta)$ are bound, so take $\beta = \frac{1 - \alpha}{1 - \alpha_{q-n}} \alpha^{n+1} \|G_0(\rho, \zeta)\|$,

and we obtain

$$\|G_q(\rho, \zeta)\| \leq \beta, \forall q, n \in \mathbb{N}. \quad (26) \text{ Hence, } \{G_q(\rho, \zeta)\}_{q=0}^{\infty} \text{ makes a "Cauchy sequence" in } H. \text{ It proves that the sequence } \{G_q(\rho, \zeta)\}_{q=0}^{\infty} \text{ is a convergent sequence with the limit } \lim_{q \rightarrow \infty} G_q(\rho, \zeta) = G(\rho, \zeta) \text{ for}$$

$\exists G(\rho, \zeta) \in H$ which complete the proof.

Theorem 3. The result of (14) is unique when $0 < (\phi_1 + \phi_2) \alpha < 1$.

$$\frac{1}{2} \Gamma(\sigma+1)$$

Proof. Let $H = (C[J], \|\cdot\|)$ with the norm $\|\varphi(\zeta)\| = \max_{\zeta \in J} |\varphi(\zeta)|$ is Banach space, A continuous function on J . Let $I : H \rightarrow H$ is a nonlinear mapping, where

C

K_{l+1}

$$= KC + E^{-1} [u \sigma E[F_1(K_l(\rho, \zeta)) + G_1(K_l(\rho, \zeta))]], \quad l \geq 0.$$

Suppose that $|F_1(K) - F_1(K^*)| < \phi_1 |K - K^*|$ and $|G_1(K) - G_1(K^*)| < \phi_2 |K - K^*|$, where $K := (\rho K, \zeta)$ and $K^* := (\rho K^*, \zeta)$ are two different function values and ϕ_1, ϕ_2 are Lipschitz constants.

h

$$\|IK - IK^*\| \leq \max_{t \in J} |E^{-1} u \sigma E[F_1(K) - F_1(K^*)]|$$

$$\begin{aligned}
& + u\sigma E[G_1(K) - G_1(K^*)]] \\
& \leq \max_{\zeta \in J} \phi_1 E^{-1}[u\sigma E[|K - K^*|]] \\
& + \phi_2 E^{-1}[u\sigma E[|K - K^*|]] \\
& \leq \max_{\zeta \in J} (\phi_1 + \phi_2) E^{-1}[u\sigma E[|K - K^*|]] \\
& \leq (\phi_1 + \phi_2) E^{-1}[u\sigma E[|K - K^*|]] \\
& \zeta^\sigma = (\phi_1 + \phi_2)(\Gamma(\sigma)) \|K - K^*\| (\sigma + 1)
\end{aligned}$$

I is a contraction as $0 < (\phi_1 + \phi_2)(\zeta) < 1$. From the Banach fixed-point theorem
 $\frac{1}{2} \Gamma(\sigma+1)$
the result is unique.

Numerical Simulation Studies

The numerical analysis of nonlinear FDEs by implementing the ETDM and the HPTM is covered in this section of the work. The previously mentioned problems can be examined in tabular and graphic form with the aid of Maple. By using the suggested approaches, we demonstrated in Figure 1 the accurate and analytical behavior of the proposed methodolo-

gies at various fractional orders of $\sigma = 1, 0.9, 0.8, 0.7$, and $0 < \zeta \leq 0.10$. The behavior of the exact and suggested approach solutions at $\sigma = 1$ is depicted by the graphs in Figure 2a,b.

The accurate and approximate values of the equation $K(\rho, \zeta)$ are shown in Table 1 for various values of ζ in Problem 1 while the absolute error comparison is shown in Table 2 for different values of ρ and ζ with $\sigma = 0.97, 0.98, 0.99, 1$. The absolute error is calculated by the difference of exact

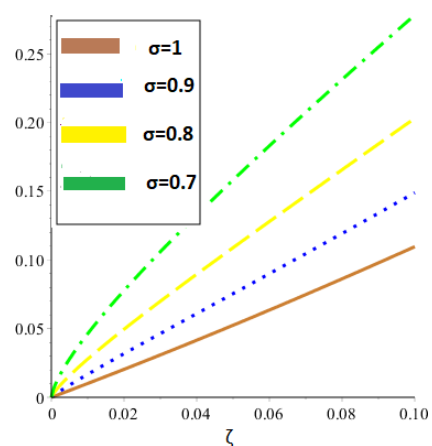


Figure 1. Graphical behavior of our method solution at several values of σ for problem 1.

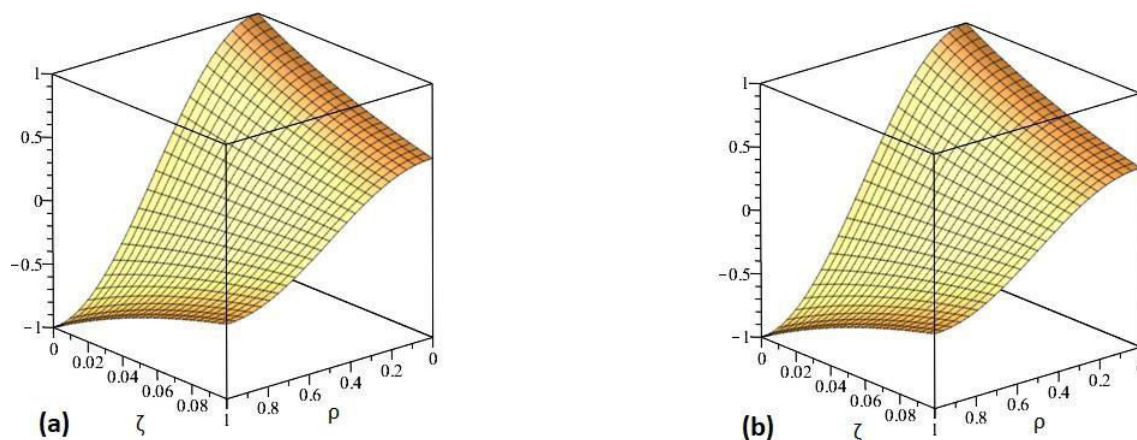


Figure 2. Graphs demonstrating the precise and our approximate solution for problem

Table 1. Solution of our methods at numerous values of σ in comparison with the exact solution for the problem 1.

ζ	$\sigma = 0.97$	$\sigma = 0.98$	$\sigma = 0.99$	$\sigma = 1$ (appro)	$\sigma = 1$ (exact)
0.01	0.0117633	0.0111807	0.0106265	0.0100996	0.0101003
0.02	0.0233010	0.0222901	0.02132284	0.0203973	0.0204026
0.03	0.0349031	0.0335108	0.0321742	0.0308910	0.0309087

0.04	0.0466235	0.0448768	0.0431960	0.0415786	0.0416204
0.05	0.0584848	0.0564017	0.0543938	0.0524583	0.0525394
0.06	0.0704987	0.0680924	0.0657699	0.0635280	0.0636673
0.07	0.0826713	0.0799523	0.0773249	0.0747856	0.0750055
0.08	0.0950063	0.0919829	0.0890584	0.0862293	0.0865554
0.09	0.1075054	0.1041847	0.1009698	0.0978570	0.0983183
0.10	0.1201692	0.1165574	0.1130578	0.1096666	0.1102951

Table 2. Our methods comparison in terms of absolute error at numerous values of σ for problem 1.

ζ	$\sigma = 0.97$	$\sigma = 0.98$	$\sigma = 0.99$	$\sigma = 1$ (Our Methods)
0.01	$1.6630475000 \times 10^{-03}$	$1.0803709000 \times 10^{-03}$	$5.2626840000 \times 10^{-04}$	$6.6320000000 \times 10^{-07}$
0.02	$2.8984310000 \times 10^{-03}$	$1.8874953000 \times 10^{-03}$	$9.2023270000 \times 10^{-04}$	$5.2784000000 \times 10^{-06}$
0.03	$3.9944519000 \times 10^{-03}$	$2.6021760000 \times 10^{-03}$	$1.2655523000 \times 10^{-03}$	$1.7718500000 \times 10^{-05}$
0.04	$5.0031309000 \times 10^{-03}$	$3.2563895000 \times 10^{-03}$	$1.5756267000 \times 10^{-03}$	$4.1764800000 \times 10^{-05}$
0.05	$5.9454599000 \times 10^{-03}$	$3.8623348000 \times 10^{-03}$	$1.8544374000 \times 10^{-03}$	$8.1101800000 \times 10^{-05}$
0.06	$6.8314089000 \times 10^{-03}$	$4.4251796000 \times 10^{-03}$	$2.1026386000 \times 10^{-03}$	$1.3931020000 \times 10^{-04}$
0.07	$7.6658381000 \times 10^{-03}$	$4.9468430000 \times 10^{-03}$	$2.3193742000 \times 10^{-03}$	$2.1986200000 \times 10^{-04}$
0.08	$8.4508834000 \times 10^{-03}$	$5.4275280000 \times 10^{-03}$	$2.5030175000 \times 10^{-03}$	$2.1986200000 \times 10^{-04}$
0.09	$9.1871024000 \times 10^{-03}$	$5.8664606000 \times 10^{-03}$	$2.6515311000 \times 10^{-03}$	$4.6130060000 \times 10^{-04}$
0.10	$9.8740913000 \times 10^{-03}$	$6.2622887000 \times 10^{-03}$	$2.7626624000 \times 10^{-03}$	$6.2853000000 \times 10^{-04}$

Conclusions:

The ETDM and the HPTM are two unique methodologies that have been thoroughly examined in this work for solving various types of fractional PDEs. The suggested approaches are the combined form of the Elzaki transformation with the Adomian decomposition method and the homotopy perturbation approach. Different dynamics for various fractional orders of the derivative are provided by the fractional-order solutions. In comparison to numerical studies, which require more complex computations, the task can be completed quite simply and

effectively using analytical solutions. After all, the researchers can now choose the fractional-order issue whose solution is comparable and extremely close to the experimental results of any physical problem. The identical solutions under the Caputo operator are seen, confirming the important dynamics of the offered problems. Numerical simulation and the graphical behavior of the model are presented to show the reliability of the implemented analytical technique. A comparative analysis of exact and approximate solutions is also presented. The calculated study results have been displayed in tables and graphs. The tables and graphs demonstrate that the approximate solution to the problems converges to the precise solution when the value of σ approaches the classical value 1 of the problems. The remarkable results show how simple and effective these approaches are and how they may be applied to other nonlinear problems. Thus, the expansion will be significantly valued to add other operators and approaches in the future, especially in light of the advantages of the current operator. The offered strategies were determined to be suitable to address any physical problem that arises in engineering and the sciences because of their straightforward operation.

References

1. Ross, B. (Ed.) Fractional Calculus and Its Applications: Proceedings of the International Conference Held at the University of New Haven, June 1974 (Vol. 457); Springer: Berlin, Germany, 2006.
2. Yavuz, M. Characterizations of two different fractional operators without singular kernel. *Math. Model. Nat. Phenom.* **2019**, 14, 302. [[CrossRef](#)]
3. Hilfer, R. Threefold introduction to fractional derivatives. In *Anomalous Transport: Foundations and Applications*; Wiley: New York, NY, USA, 2008; pp. 17–73.
4. Chaurasiya, V.; Rai, K.N.; Singh, J. Legendre wavelet residual approach for moving boundary problem with variable thermal physical properties. *Int. J. Nonlinear Sci. Numer. Simul.* **2022**, 23, 957–970.
5. Ganie, A.H.; Houas, M.; AlBaidani, M.M.; Fathima, D. Coupled system of three sequential Caputo fractional differential equations: Existence and stability analysis. *Math. Methods Appl. Sci.* **2023**. [[CrossRef](#)]
6. Harris, P.J. The mathematical modelling of the motion of biological cells in response to chemical signals. In *Computational and Analytic Methods in Science and Engineering*; Birkhäuser: Cham, Switzerland, 2020; pp. 151–171.
7. Yazgan, T.; Ilhan, E.; Çelik, E.; Bulut, H. On the new hyperbolic wave solutions to Wu-Zhang system models. *Opt. Quantum Electron.* **2022**, 54, 1–19. [[CrossRef](#)]
8. Tazgan, T.; Celik, E.; Gülnur, Y.E.L.; Bulut, H. On Survey of the Some Wave Solutions

- of the Non-Linear Schrödinger Equation (NLSE) in Infinite Water Depth. Gazi Univ. J. Sci. **2023**, 36, 819–843. [[CrossRef](#)]
9. Zhang, X.; Zhu, H.; Kuo, L.H. A comparison study of the LMAPS method and the LDQ method for time-dependent problems. Eng. Anal. Bound. Elem. **2013**, 37, 1408–1415. [[CrossRef](#)]
 10. Wang, H.; Yamamoto, N. Using a partial differential equation with Google Mobility data to predict COVID-19 in Arizona. arXiv **2020**, arXiv:2006.16928.
 11. Viguerie, A.; Lorenzo, G.; Auricchio, F.; Baroli, D.; Hughes, T.J.; Patton, A.; Reali, A.; Yankeelov, T.E.; Veneziani, A. Simulating the spread of COVID-19 via a spatially-resolved susceptible-exposed-infected-recovered-deceased (SEIRD) model with heterogeneous diffusion. Appl. Math. Lett. **2021**, 111, 106617. [[CrossRef](#)]
 12. Ahmed, J.J. Designing the shape of corona virus using the PDE method. Gen. Lett. Math. **2020**, 8, 75–82. [[CrossRef](#)]
 13. Mastoi, S.; Ganie, A.H.; Saeed, A.M.; Ali, U.; Rajput, U.A.; Mior Othman, W.A. Numerical solution for two-dimensional partial differential equations using SM's method. Open Phys. **2022**, 20, 142–154. [[CrossRef](#)]
 14. Al-Hababeh, A. Exact solution for commensurate and incommensurate linear systems of fractional differential equations. J. Math. Comput. Sci. **2023**, 28, 123–136. [[CrossRef](#)]
 15. He, H.M.; Peng, J.G.; Li, H.Y. Iterative approximation of fixed point problems and variational inequality problems on Hadamard manifolds. UPB Bull. Ser. A **2022**, 84, 25–36.
 16. Yuan, Q.; Kato, B.; Fan, K.; Wang, Y. Phased array guided wave propagation in curved plates. Mech. Syst. Signal Process. **2023**, 185, 109821. [[CrossRef](#)]
 17. Kilbas, A.A.; Srivastava, H.M.; Trujillo, J.J. Theory and Applications of Fractional Differential Equations; Elsevier: Amsterdam, The Netherlands, 2006; Volume 204.
 18. Podlubny, I. Fractional Differential Equations; Academic Press: San Diego, CA, USA; Boston, MA, USA, 1999; Volume 6.
 19. Mainardi, F. Fractional Calculus and Waves in Linear Viscoelasticity: An Introduction to Mathematical Models; World Scientific: Singapore, 2022.
 20. Liu, L.; Zhang, S.; Zhang, L.; Pan, G.; Yu, J. Multi-UUV Maneuvering Counter-Game for Dynamic Target Scenario Based on Fractional-Order Recurrent Neural Network. IEEE Trans. Cybern. **2022**. [[CrossRef](#)] [[PubMed](#)]
 21. Cao, Y.; Nikan, O.; Avazzadeh, Z. A localized meshless technique for solving 2D nonlinear integro-differential equation with multi-term kernels. Appl. Numer. Math. **2023**, 183,

140–156. [[CrossRef](#)]

22. Akram, T.; Abbas, M.; Ali, A. A numerical study on time fractional Fisher equation using an extended cubic B-spline approximation. J. Math. Comput. Sci **2021**, 22, 85–96. [[CrossRef](#)]
23. Srivastava, H.M.; Gusu, D.M.; Mohammed, P.O.; Wedajo, G.; Nonlaopon, K.; Hamed, Y.S. Solutions of general fractional-order differential equations by using the spectral Tau method. Fractal Fract. **2022**, 6, 7. [[CrossRef](#)]
24. Bonyadi, S.; Mahmoudi, Y.; Lakestani, M.; Jahangiri Rad, M. Numerical solution of space-time fractional PDEs with variable coefficients using shifted Jacobi collocation method. Comput. Methods Differ. Equ. **2023**, 11, 81–94.
25. Youssri, Y.H.; Abd-Elhameed, W.M.; Ahmed, H.M. New fractional derivative expression of the shifted third-kind Chebyshev polynomials: Application to a type of nonlinear fractional pantograph differential equations. J. Funct. Spaces **2022**, 2022, 3966135. [[CrossRef](#)]
26. Sadek, L.; Bataineh, A.S.; Talibi Alaoui, H.; Hashim, I. The Novel Mittag-Leffler-Galerkin Method: Application to a Riccati Differential Equation of Fractional Order. Fractal Fract. **2023**, 7, 302. [[CrossRef](#)]
27. Hakkar, N.; Dhayal, R.; Debbouche, A.; Torres, D.F. Approximate Controllability of Delayed Fractional Stochastic Differential Systems with Mixed Noise and Impulsive Effects. Fractal Fract. **2023**, 7, 104. [[CrossRef](#)]
28. Youssri, Y.H.; Atta, A.G. Spectral collocation approach via normalized shifted Jacobi polynomials for the nonlinear Lane-Emden equation with fractal-fractional derivative. Fractal Fract. **2023**, 7, 133. [[CrossRef](#)]
29. Sunthrayuth, P.; Alyousef, H.A.; El-Tantawy, S.A.; Khan, A.; Wyal, N. Solving fractional-order diffusion equations in a plasma and fluids via a novel transform. J. Funct. Spaces **2022**, 2022, 1899130. [[CrossRef](#)]
30. Mishra, N.K.; AlBaidani, M.M.; Khan, A.; Ganie, A.H. Numerical Investigation of Time-Fractional Phi-Four Equation via Novel Transform. Symmetry **2023**, 15, 687. [[CrossRef](#)]
31. Owyed, S.; Abdou, M.A.; Abdel-Aty, A.H.; Alharbi, W.; Nekhili, R. Numerical and approximate solutions for coupled time fractional nonlinear evolutions equations via reduced differential transform method. Chaos Solitons Fractals **2020**, 131, 109474. [[CrossRef](#)]
32. Fathima, D.; Alahmadi, R.A.; Khan, A.; Akhter, A.; Ganie, A.H. An Efficient Analytical Approach to Investigate Fractional Caudrey-Dodd-Gibbon Equations with Non-Singular Kernel Derivatives. Symmetry **2023**, 15, 850. [[CrossRef](#)]
33. Saad Alshehry, A.; Imran, M.; Khan, A.; Shah, R.; Weera, W. Fractional View Analysis of Kuramoto-Sivashinsky Equations with Non-Singular Kernel Operators. Symmetry **2022**, 14, 1463. [[CrossRef](#)]

34. Arqub, O.A.; El-Ajou, A. Solution of the fractional epidemic model by homotopy analysis method. J. King Saud Univ.-Sci. **2013**, *25*, 73–81. [[CrossRef](#)]
35. Song, L.; Zhang, H. Application of homotopy analysis method to fractional KdV-Burgers-Kuramoto equation. Phys. Lett. A **2007**, *367*, 88–94. [[CrossRef](#)]
36. Alaoui, M.K.; Fayyaz, R.; Khan, A.; Shah, R.; Abdo, M.S. Analytical investigation of Noyes-Field model for time-fractional Belousov-Zhabotinsky reaction. Complexity **2021**, *2021*, 3248376. [[CrossRef](#)]
37. Zidan, A.M.; Khan, A.; Shah, R.; Alaoui, M.K.; Weera, W. Evaluation of time-fractional Fisher's equations with the help of analytical methods. Aims Math. **2022**, *7*, 18746–18766. [[CrossRef](#)]
38. Areshi, M.; Khan, A.; Shah, R.; Nonlaopon, K. Analytical investigation of fractional-order Newell-Whitehead-Segel equations via a novel transform. AIMS Math. **2022**, *7*, 6936–6958. [[CrossRef](#)]
39. Alyobi, S.; Shah, R.; Khan, A.; Shah, N.A.; Nonlaopon, K. Fractional Analysis of Nonlinear Boussinesq Equation under Atangana- Baleanu-Caputo Operator. Symmetry **2022**, *14*, 2417. [[CrossRef](#)]
40. Das, S.; Vishal, K.; Gupta, P.K.; Yildirim, A. An approximate analytical solution of time-fractional telegraph equation. Appl. Math. Comput. **2011**, *217*, 7405–7411. [[CrossRef](#)]
41. Karaagac, B. Two step Adams Bashforth method for time fractional Tricomi equation with non-local and non-singular Kernel. Chaos Solitons Fractals **2019**, *128*, 234–241. [[CrossRef](#)]
42. Nonlaopon, K.; Alsharif, A.M.; Zidan, A.M.; Khan, A.; Hamed, Y.S.; Shah, R. Numerical investigation of fractional-order Swift- Hohenberg equations via a Novel transform. Symmetry **2021**, *13*, 1263. [[CrossRef](#)]
43. Sunthrayuth, P.; Ullah, R.; Khan, A.; Shah, R.; Kafle, J.; Mahariq, I.; Jarad, F. Numerical analysis of the fractional-order nonlinear system of Volterra integro-differential equations. J. Funct. Spaces **2021**, *2021*, 1537958. [[CrossRef](#)]
44. Salama, F.M.; Ali, N.H.M.; Abd Hamid, N.N. Fast O (N) hybrid Laplace transform-finite difference method in solving 2D time fractional diffusion equation. J. Math. Comput. Sci. **2021**, *23*, 110–123. [[CrossRef](#)]
45. Botmart, T.; Agarwal, R.P.; Naeem, M.; Khan, A.; Shah, R. On the solution of fractional modified Boussinesq and approximate long wave equations with non-singular kernel operators. AIMS Math. **2022**, *7*, 12483–12513. [[CrossRef](#)]
46. Alderremy, A.A.; Aly, S.; Fayyaz, R.; Khan, A.; Shah, R.; Wyal, N. The analysis of

fractional-order nonlinear systems of third order KdV and Burgers equations via a novel transform. Complexity **2022**, 2022, 4935809. [[CrossRef](#)]

47. Shah, N.A.; Hamed, Y.S.; Abualnaja, K.M.; Chung, J.D.; Shah, R.; Khan, A. A comparative analysis of fractional-order kaup- kupersmidt equation within different operators. Symmetry **2022**, 14, 986. [[CrossRef](#)]

48. Jassim, H.K.; Shareef, M.A. On approximate solutions for fractional system of differential equations with Caputo-Fabrizio fractional operator. J. Math. Comput. Sci. **2021**, 23, 58–66. [[CrossRef](#)]

49. Mishra, N.K.; AlBaidani, M.M.; Khan, A.; Ganie, A.H. Two Novel Computational Techniques for Solving Nonlinear Time- Fractional Lax's Korteweg-de Vries Equation. Axioms **2023**, 12, 400. [[CrossRef](#)]

50. Adomian, G. Nonlinear Stochastic System Theory and Applications to Physics; Kluwer Academic Publishers: Dordrecht, The Netherlands, 1989.

51. Adomian, G. Solving Frontier Problems of Physics: The Decomposition Method; Springer: Dordrecht, The Netherlands, 1994.

52. Homotopy, H.J. perturbation technique. Comput. Methods Appl. Mech. Eng. **1999**, 178.

53. He, J.H. A coupling method of a homotopy technique and a perturbation technique for non-linear problems. Int. J. Non-Linear Mech. **2000**, 35, 37–43. [[CrossRef](#)]

54. He, J.H. Homotopy perturbation method: A new nonlinear analytical technique. Appl. Math. Comput. **2003**, 135, 73–79. [[CrossRef](#)]

55. Elzaki, T.M. The new integral transform Elzaki transform. Glob. J. Pure Appl. Math. **2011**, 7, 57–64.

56. Alshikh, A.A.; Mahgob, M.M.A. A comparative study between Laplace transform and two new integrals “Elzaki” transform and “Aboodh” transform. Pure Appl. Math. J **2016**, 5, 145–150. [[CrossRef](#)]

57. Elzaki, T.M.; Alkhateeb, S.A. Modification of Sumudu transform “Elzaki transform” and Adomian decomposition method. Appl. Math. Sci. **2015**, 9, 603–611. [[CrossRef](#)]

58. Elzaki, T.M. On the connections between Laplace and Elzaki transforms. Adv. Theor. Appl. Math. **2011**, 6, 1–11.

59. Sedeeg, A.K.H. A coupling Elzaki transform and homotopy perturbation method for solving nonlinear fractional heat-like equations. Am. J. Math. Comput. Model **2016**, 1, 15–20.

60. Naem, M.; Yasmin, H.; Shah, R.; Shah, N.A.; Chung, J.D. A Comparative Study of Fractional Partial Differential Equations with the Help of Yang Transform. Symmetry **2023**, 15, 146. [[CrossRef](#)]