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The radio D-distance in harmonic mean number of a Quadrilateral snake graph and Doublequadrilateral snake graph.

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Abstract:

A radio D-distance in harmonic mean labelling of a connect graph G is an injective map f from the vertex set $V(G)$ to the N such that for two distinct vertices u and v of G, $d^D(u,v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq diam^D(G) + 1$. The radio D-distance harmonic mean number of $f, rh^D n(f)$ is the maximum number assigned to any vertex of G On Radio D-distance harmonic mean number of quadrilateral snake graph and doublequaderilateral snake graph.

Key words: Quadrilateral snake graph, Doublequadrilateral snake graph

Introduction:

A graph $G = (V, E)$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. Graph labelling was introduced by Alexander Rosa in 1967. Radio mean labelling was introduced by S.Somasundaram and R.Ponraj in 2004. Harmonic mean labelling was introduced by S.Somasundaram and S.S.Sandhya in 2012.

The concept of D-distance was introduced by D.Reddy Babu et al. The concept of radio D-distance was introduced by T.Nicholas and K.John Bosco in 2017.

Radio labelling (multi-level distance labelling)can be regarded as an extension of distance-two labelling which is motivated by the channel assignment problem introduced by Hale[6]. Chartrand et al. [2] introduced the concept of radio labelling of graph. Chartrand et al.[3] gave the upper bound for the radio number of path. The exact value for the radio number of path and cycle was given by Liu and Zhu [10]. However Chartrand et al.[2] obtained different values than Liu and Zhu [10]. They found the lower and upper bound for the radio number of cycle. Liu [9] gave the lower bound for the radio number of tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O. Togni [8]. M. M. Rivera et al. [20].

The concept of D-distance was introduced by D. Reddy Babu et al. [17,18,19]. If u, v are vertices of connected graph G, the D-length of a connected $u - v$ path s is defined as $l^D(s) = l(s) + \deg(v) + \deg(u) + \sum \deg(w)$ where the sum runs over all international vertices w of s and $l(s)$ is the length of the path. The D-distance $d^D(u,v)$ between two vertices u, v of a connected graph. G is defined a $d^D(u,v) = \min\{l^D(s)\}$ where the minimum is taken over all $u - v$ path s in G. In other words, $d^D(u,v) = \min\{l(s) + \deg(v) + \deg(u) + \sum \deg(w)$ where the sum runs over all intermediate vertices w in s and minimum is taken over all $u - v$ path s in G.

In this paper we introduce the concept the radio D-distance in harmonic mean number. A radio D-distance in harmonic labelling is a one to one mapping f from $V(G)$ to $\mathbb{N}$ satisfying condition.

$$d^D(u,v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq diam^D(G) + 1.$$

For every $u, v \epsilon V(G)$. The span of a labelling f is the maximum integer that f maps to a vertex of G. The radio D-distance harmonic mean number of G, $rh^D n(G)$ is the lowest span taken over all radio D-distance mean labelling of the graph G. The condition is called radio D-distance mean condition. In this paper we determine the radio harmonic mean number of some well-known graphs. The function $f: V(G) \to \mathbb{N}$ always represents injective map unless otherwise stated.

Theorem: 2.1

The radio D-distance in harmonic mean number of a quadrilateral snake $rh^D n(Q_n) = 8n - 8, n \geq 3$.

Proof:

Let $V(Q_n) = \{v_i, u_j \quad i = 1,2, \dots, n, j = 1,2, \dots, 2n - 2\}$ be the vertex set and $E(Q_n)\{v_i v_{i+1}, v_i u_j, u_j u_{j+1} \quad i = 1,2, \dots, n - 1, j = 1,2, \dots, 2n - 3\}$ be the edge set. Some distances are $d^D(v_1,v_2)=7, d^D(v_2,v_3)=9,\ d^D(v_1,u_1)=5,\ d^D(u_2,u_3)=10, d^D(u_1,u_{2n-2})=5n+1$. It is obvious that $diam^D(Q_{n)}=5n+1$. The radio d-distance in harmonic mean condition implies that

$$d^D(u,v)+\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil \geq diam^D(Q_{n)}+1$$

For every pair of $u \neq v$.

If vertex are adjacent

Fix $f(v_1)=5n-5$

For $d^D(v_1,v_2)$

$$d^D(v_1,v_2)+\left\lceil\frac{2f(v_1)f(v_2)}{f(v_1)+f(v_2)}\right\rceil \geq 5n+2$$

$$\Rightarrow \left\lceil\frac{2(5n-5)f(v_2)}{(5n-5)+f(v_2)}\right\rceil \geq 5n-5$$

$$f(v_2)=5n-4$$

For $d^D(v_2,v_3)$

$$d^D(v_2,v_3)+\left\lceil\frac{2f(v_2)f(v_3)}{f(v_2)+f(v_3)}\right\rceil \geq 5n+2$$

$$\Rightarrow \left\lceil\frac{2(5n-4)f(v_3)}{(5n-4)+f(v_3)}\right\rceil \geq 5n-7$$

$$f(v_3)=5n-3$$

$$f(v_i)=5n-6+i, \qquad 4 \leq i \leq n$$

Therefore $f(v_n)=6n-6$

If vertex are nonadjacent

For $d^D(v_1,u_1)$

$$d^D(v_1,u_1)+\left\lceil\frac{2f(v_1)f(u_1)}{f(v_1)+f(u_1)}\right\rceil \geq 5n+2$$

$$\Rightarrow \left\lceil\frac{2(5n-5)f(u_1)}{(5n-5)+f(u_1)}\right\rceil \geq 5n-3$$

$$f(u_1)=6n-5$$

For $d^D(u_1,u_2)$

$$d^D(u_1,u_2)+\left\lceil\frac{2f(u_1)f(u_2)}{f(u_1)+f(u_2)}\right\rceil \geq 5n+2$$

$\Rightarrow \left\lceil \frac{2(6n-5)f(u_2)}{(6n-5)+f(u_2)} \right\rceil \geq 5n-3$

$f(u_2) = 6n-4$

For $d^D(u_2, u_3)$

$d^D(u_2, u_3) + \left\lceil \frac{2f(u_2)f(u_3)}{f(u_2)+f(u_3)} \right\rceil \geq 5n+2$

$\Rightarrow \left\lceil \frac{2(6n-4)f(u_3)}{(6n-4)+f(u_3)} \right\rceil \geq 5n-8$

$f(u_3) = 6n-3$

$f(u_i) = 6n-6+i, \quad 4 \leq i \leq n.$

Therefore $f(u_n) = 7n-6.$

$f(u_i) = 6n-6+i, \quad n+1-2 \leq i \leq n+n-2.$

Therefore $f(u_{2n-2}) = 8n-8.$

That is, $rh^Dn(Q_n) \leq 8n-8$ ……….(1)

Since Q_n has $3n-2$ vertices, it requires $3n-2$ distinct labels. Also by the radio d-distance harmonic mean condition, $(5n-6)$ labels between 1 and $(5n-5)$ are forbidden.

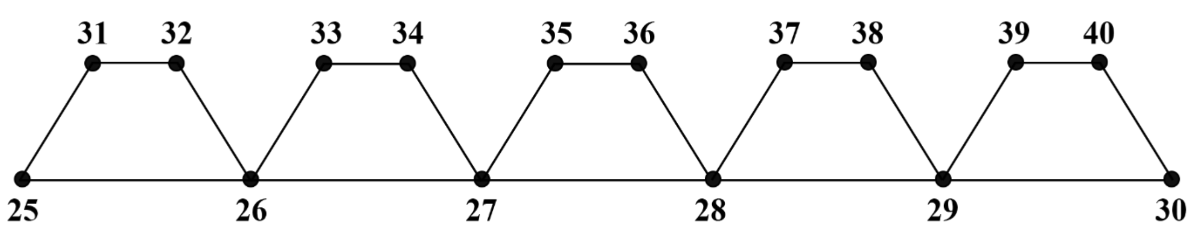
Hence $rh^Dn(Q_n) \geq 8n-8$ …………(2)

From (1) and (2), the result follows

Hence $rh^Dn(Q_n) = 8n-8 \; if \; n \geq 3$

Hence the proof.

Example:



$Figure$ 5.14 $rh^Dn(Q_6) = 40$

Theorem: 1.2

The radio D-distance in hrmonic mean number of a double quadrilateral snake graph $rh^D n[D(Q_n)] = 12n - 13,\ if\ n \geq 3$.

Proof:

Let $V(DQ_n) = \{v_i, u_j, w_k \quad i = 1,2, \dots, n; j, k = 1,2, \dots, 2n-2\}$ be the vertex set and $E[D(Q_n)] = \{v_i v_{i+1}, v_i u_j, v_i w_k, v_i u_j, v_i w_j \quad i = 1,2, \dots, n\, ; j, k = 1,2, \dots, 2n\}$ be the edge set. Some distances are $d^D(v_1, v_2) = 10, d^D(v_2, v_3) = 13$, $d^D(v_1, u_1) = 6,\ d^D(u_1, u_2) = 5, d^D(u_2, u_3) = 12,\ d^D(v_1, w_1) = 6,\ d^D(w_1, w_2) = 5,\ d^D(w_2, w_3) = 12, d^D(u_1, w_{2n-2}) = 7n - 1$. It is obvious that $diam^D[D(Q_n)] = 7n - 1$.

The radio D-distance in harmonic mean condition implies that

$$d^D(u, v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq diam^D[D(Q_n)] + 1.$$

Fix $f(v_1) = 7n - 8$

For $d^D(v_1, v_2)$

$$d^D(v_1, v_2) + \left\lceil \frac{2f(v_1)f(v_2)}{f(v_1)+f(v_2)} \right\rceil \geq 7n$$

$$\Rightarrow \left\lceil \frac{2(7n-8)f(v_2)}{(7n-8)+f(v_2)} \right\rceil \geq 7n - 10$$

$$f(v_2) = 7n - 7$$

For $d^D(v_2, v_3)$

$$d^D(v_2, v_3) + \left\lceil \frac{2f(v_2)f(v_3)}{f(v_2)+f(v_3)} \right\rceil \geq 7n$$

$$\Rightarrow \left\lceil \frac{2(7n-7)f(v_3)}{(7n-7)+f(v_3)} \right\rceil \geq 7n - 13$$

$$f(v_3) = 7n - 6$$

$$f(v_i) = 7n - 9 + i, \qquad 4 \leq i \leq n$$

Therefore $f(v_n) = 8n - 9$

For $d^D(v_1, u_1)$

$$d^D(v_1, u_1) + \left\lceil \frac{2f(v_1)f(u_1)}{f(v_1)+f(u_1)} \right\rceil \geq 7n$$

$$\Rightarrow \left\lceil \frac{2(7n-8)f(u_1)}{(7n-8)+f(u_1)} \right\rceil \geq 7n - 6$$

$f(u_1) = 8n - 8$

For $d^D(u_1, u_2)$

$d^D(u_1, u_2) + \left\lceil \frac{2f(u_1)f(u_2)}{f(u_1)+f(u_2)} \right\rceil \geq 7n$

$\Rightarrow \left\lceil \frac{2(8n-8)f(u_2)}{f(u_1)+f(u_2)} \right\rceil \geq 7n - 5$

$f(u_2) = 8n - 7$

For $d^D(u_2, u_3)$

$d^D(u_2, u_3) + \left\lceil \frac{2f(u_2)f(u_3)}{f(u_2)+f(u_3)} \right\rceil \geq 7n$

$\Rightarrow \left\lceil \frac{2(8n-7)f(u_3)}{(8n-7)+f(u_3)} \right\rceil \geq 7n - 12$

$f(u_3) = 8n - 7$

$f(u_i) = 8n - 9 + i, \quad 4 \leq i \leq n$

$f(u_n) = 9n - 9$

$f(u_i) = 8n - 9 + i, \quad n + 1 - 2 \leq i \leq n + n - 2$

$f(u_{2n-2}) = 10n - 11.$

For $d^D(v_1, w_1)$

$d^D(v_1, w_1) + \left\lceil \frac{2f(v_1)f(w_1)}{f(v_1)+f(w_1)} \right\rceil \geq 7n$

$\Rightarrow \left\lceil \frac{2(7n-8)f(w_1)}{(7n-8)+f(w_1)} \right\rceil \geq 7n - 6$

$f(w_1) = 10n - 10$

For $d^D(w_1, w_2)$

$d^D(w_1, w_2) + \left\lceil \frac{2f(w_1)f(w_2)}{f(w_1)+f(w_2)} \right\rceil \geq 7n$

$\Rightarrow \left\lceil \frac{2(10n-10)f(w_2)}{(10n-10)+f(w_2)} \right\rceil \geq 7n - 5$

$f(w_2) = 10n - 9$

For $d^D(w_2, w_3)$

$d^D(w_2, w_3) + \left\lceil \frac{2f(w_2)f(w_3)}{f(w_2)+f(w_3)} \right\rceil \geq 7n$

$\Rightarrow \left\lceil \frac{2(10n-9)f(w_3)}{(10n-9)+f(w_3)} \right\rceil \geq 7n-12$

$f(w_3) = 10n-8$

$f(w_i) = 10n-11+i, \quad 4 \leq i \leq n.$

$f(w_n) = 11n-11.$

$f(w_i) = 10n-11+i, \quad n+1-2 \leq i \leq n+n-2.$

$f(w_{2n-2}) = 12n-13.$

That is, $rh^D n(D(Q_n)) \leq 12n-13$……….(1)

Since $D(Q_n)$ has $5n-4$ vertices, it requires $5n-4$ distinct labels. Also by the radio d-distance harmonic mean condition, $(7n-9)$ labels between 1 and $(7n-8)$ are forbidden.

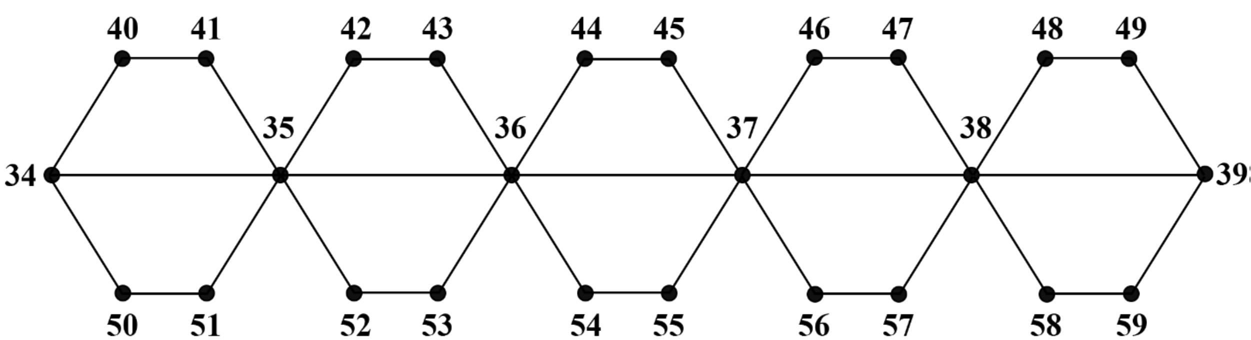
Hence $rh^D n(D(Q_n)) \geq 12n-13$ ………….(2)

From (1) and (2), the result follows

Hence $rh^D n[D(Q_n)] = 12n-13,\ if\ n \geq 3.$

Hence the proof.

Example:



$Figure\ 5.15\ \ rh^D n[D(Q_6)] = 59$

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