e-ISSN 2320 –7876 www.ijfans.org
Vol.11, S Iss 1, 2022
Research paper , UGC CARE Listed (Group -I) Journal

Construction of fuzzy multinomial statistical control charts for the air quality index of major cities in India

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Abstract:

Control charts are used as the most important statistical tools to recognize the irregular situation in a process. The p-control chart is the widely used tool to monitor the attribute characteristics of more than one variable simultaneously. It classifies each unit into defective or non-defective according to the quality characteristics. But in practical situation, there are many categories exist such as excellent, good, medium and bad. In this paper, Fuzzy Multinomial Control (FMC) chart is constructed to handle the vague situations of data and also to identify the quality of Air Quality Index (AQI) for the major cities in India.

Key words: Fuzzy multinomial chart, p chart, linguistic variable, Air quality Index.

1. Introduction

Control charts are used to monitor the ongoing processes. It consists of three lines, called Upper Control Limit (UCL), Central Line (CL) and Lower Control limit (LCL). When one or more sample points are outside the control limits, the chart indicates the presence of a change in the process and the quality checker should check for the cause of the out-of-control process. The control chart method was introduced by Dr Walter A. Shewhart. It is one of the most important quality checking techniques used to detect adverse causes. But the real-life data are difficult to interpret since they involve a certain level of uncertainty that may be connected to human subjectivity. Fuzzy set theory can apply with such vagueness and can be applied to traditional p-control chart. The use of fuzzy control charts instead of the traditional control charts to analyze the variability in many processes is necessary. Fuzzy logic, proposed by Zadeh (1965), is a branch of mathematics that allows handling the vagueness situation. It provides a simple way of reasoning with vague, ambiguous and imprecise information. The use of fuzzy logic to the traditional control chart is interesting since observations close to control limits can cause false alarms with traditional control charts. Fuzzy control charts may provide more flexibility to control a process, since quality experts can vary the degree of uncertainty inserted in the process and use α to vary the degree of requirement for inspection. The Fuzzy Multinomial Control chart was constructed based on the multinomial distribution. Fuzzy Multinomial control chart is used when there are more than two classifications.

2. Review of Literature

An approach based on fuzzy set theory to construct attribute control charts from linguistic data is proposed in [17]. Four transformation methods that convert fuzzy sets associated with linguistic values into scalars were presented in their work. Control charts for attributes using α-cut is developed in [6]. By selecting an appropriate level, this approach enables determining tightness in inspection. The construction of control charts using fuzzy multinomial quality is introduced in [3]. They have utilized the concept in the food processing industry for packing of a frozen food. They have classified the packed product as bad, medium, good and excellent. [14] illustrated fuzzy regression control chart based on α- cut approximation. [15] used transformation methods to the α-level fuzzy midrange to construct $\bar{X}-R$ and $\bar{X}-S$ using α-cut. [13] Utilizing α-cuts, a triangular fuzzy multinomial control chart with variable sample size was built. An aluminum die-cast lighting component was used in a production line. [8] Have conducted a comparison of the control charts for fuzzy multinomial and exponential weighted moving average. They discovered that the cost of inspection was reduced by the Fuzzy Multinomial Control Chart, which was based on variable sampling sizes. [4] Introduced low-cost fuzzy logic control for green house environments and web monitoring.[18] discussed about the performance of Fuzzy Multinomial control chart. They have demonstrated the procedure for the performance of FMCC and used the concept of ARL to identify the out-of-control situation. [2] applied the fuzzy control chart for monitoring mean and range of univariate process. In this research work, the data values classified as more quality characteristics and fuzzy control chart implemented to the given data. This paper demonstrates the application of the FMC chart to identify the quality of AQI for the major cities in India.

3. Differences between traditional Control Chart and Fuzzy Control Chart

Fuzzy control charts are more reliable and flexible than the traditional control chart. The important contribution of fuzzy control chart is its ability to represent vague data. Control charts can be used when the data are accurate, when the data are not accurate it is better to use fuzzy control charts. Fuzzy control charts shows lesser out of limit lots than the traditional control charts since the former possess larger amplitude of their control limits. The important advantage of fuzzy control chart is that it does not need to fulfill the assumption of normality where it is necessary condition for traditional control charts. The traditional control charts are most typically used when the data obtained are well known and the exact data is available. In this type of peculiar situation it is better to use fuzzy control charts. The usage of the traditional control chart leads to the conclusion whether to accept or reject the lot while using fuzzy control chart we can classify into different categories such as poor quality, medium quality, good quality and excellent quality. Fuzzy Cumulative Sum Control chart (CUSUM) and Exponential Weighted Moving Average Control Chart (EWMA) are used to detect small shift in the process.

4. Fuzzy Transformation Techniques

As the data are fuzzy, there is a requirement to modify the linguistic data in the sample to represent the fuzzy sets that are attached to them. The four most popular fuzzy transformation methods are fuzzy mean, fuzzy median, fuzzy mode and fuzzy midrange.

4.1.Fuzzy mode:

In the fuzzy mode, which is symbolized by, the membership function of the value of a fuzzy set's base variable equals 1

$$f_{mode} = \{x|\mu F(x)=1\} \text{ for all } x \in F. \quad (1)$$

4.2.Fuzzy Mean:

The formula to calculate the fuzzy mean is defined by

$$F_{avg} = \frac{\int_0^1 x\mu F(x)}{\int_0^1 \mu F(x)}. \quad (2)$$

4.3.Fuzzy Median:

The point that separates the curve under a fuzzy set's membership function into two equal rejoins and satisfies the following equation is that point.

$$\int_a^{F_{med}} \mu F(x)dx = \int_{F_{med}}^{c} \mu F(x)dx = \frac{1}{2}\int_a^{c} \mu F(x)dx. \quad (3)$$

4.4.Fuzzy Midrange:

The fuzzy midrange is defined by

$$F_{mr} = ½ (a_\alpha + c_\alpha) \quad (4)$$

Where $a_\alpha = \min\{F_\alpha\}$

and $c_\alpha = \max\{F_\alpha\}$.

5. Fuzzy Multinomial Control Chart

In attribute control chart p chart is the most widely used control chart. When there are more than two classifications then fuzzy multinomial control chart is the most representative chart to be used. The fuzzy multinomial control chart is based on the degrees of membership and it can be represented in the form of fuzzy membership function. The statistical principles underlying the fuzzy multinomial control chart are based on the multinomial distribution. In FMCC we define L as a linguistic variable which take k mutually exclusive members $\{l_1,l_2,l_3,l_4,\ldots,l_k\}$. Then we assign the weight w_i to each term l_i and the fuzzy set is defined as

$$\tilde{\mathcal{L}}=\{(l_1,m_1),(l_2,m_2),(l_3,m_3),\ldots,(l_k,m_k)\}. \quad (5)$$

In order to monitor the out-of-control signal in the production process, take independent samples of different sizes. The size of the sample to be drawn each time is decided by choosing a member randomly from
$\{n_1, n_2, \ldots, n_s\}$.

The statistical principles underlying the FMC with variable sample size are based on the multinomial distribution. $\tilde{\mathcal{L}}$ is a linguistic variable which can take k mutually exclusive members $\{l_1, l_2, \ldots, l_k\}$. Assume that the production process is operating in a stable manner and p_i is the probability that an item is l_i, i = 1, 2,…, k. and the independent consecutive items are produced. Let us consider n be the random sample size, then the product units are selected. Let X_i, i = 1, 2 …k, be the number of items of the product that are l_i, i = 1, 2 …k. Then $X_1, X_2, \ldots, X_k$ has a multinomial distribution with parameters n_r and $p_1, p_2, \ldots, p_k$. It is known that each X_i, i = 1, 2 …k, marginally has a binomial distribution with the mean $n_r\, p_i$ and variance $n_r p_i(1-p_i)$, i= 1, 2 …k, respectively. The weighted average of the linguistic variable $\tilde{\mathcal{L}}$ with sample size n_r is defined by

$$\bar{\bar{L}} = \frac{\sum_{i=1}^{k} X_i m_i}{\sum_{i=1}^{k} X_i} = \frac{\sum_{i=1}^{k} X_i m_i}{n_r}, \qquad n_r \epsilon \{n_1, n_2, \ldots n_s\} \quad (6)$$

The control limits for FMC chart are

$$UCL = E\left[\ \bar{\bar{L}}\ \right] + d\sqrt{var(\bar{\bar{L}})} \quad CL = E(\bar{\bar{L}}) \quad \text{and}$$

$$LCL = E\left[\ \bar{\bar{L}}\ \right] - d\sqrt{var(\bar{\bar{L}})} \quad (7)$$

where d is considered as the distance of the control limits from the center line

$$E\left[\bar{\bar{L}}\right] = \sum_{i=1}^{k} p_i\, m_i \quad \text{and}$$

$$var\left(\bar{\bar{L}}\right) = \frac{1}{n_r}[\sum_{i=1}^{k} m^2{}_i p_i\,(1 - p_i) - 2\sum_{i=1 i<j}^{k} \sum_{j=1}^{k} m_i\, m_j p_i p_j, \; n_r \epsilon \{n_1, n_2, \ldots n_s\} \quad (8)$$

where n_1, n_2 *and* n_s are pre-determined sample sizes.

6.Construction of Fuzzy Multinomial Statistical Control charts for Air Quality Index

The air quality index is used for reporting daily air quality. It tells us how clean or polluted our air is and what associated health effects might be a concern for us. The AQI focuses on health effects we may experience within a few hours or days after breathing polluted air. Environmental Protection Agency (EPA) calculates the AQI for five major pollutants. For each of the pollutants, EPA has established national air quality standards to protect public health. Ground level ozone and airborne particles are the two pollutants that pose the greatest threat to human health. Air quality index is used by government agencies to communicate to the public how polluted the air currently is or how polluted it is forecast to become. AQI information is obtained by averaging readings from an air quality sensor which can increase due to vehicle traffic, forest fire or anything that can increase air pollution. Public health risks increase as the AQI increases, especially affecting children, the elderly and individuals with respiratory or cardiovascular issues. During these times, government bodies generally encourage people to reduce physical activity outdoors or even avoid for going at altogether. The various effects of different pollutant particles are discussed below briefly.

Sulphur Dioxide: Short term exposure can harm the respiratory system, making breathing difficult. It can affect visibility by reacting with other air particles to form haze and stain culturally important objects such as statues and monuments.

Nitrogen Dioxide: Aggravates respiratory illness, causes haze to form by reacting with other air particles, causes acid rain, pollutes coastal waters.

Carbon monoxide: High concentration in air reduces oxygen supply to critical organs like the heart and brain. At very high levels, it can cause dizziness, confusion, unconsciousness and death.

PM2.5 & PM10: Particulate matter pollution can cause irritation of the eyes, nose and throat, coughing, chest tightness and shortness of breath, reduced lung function, irregular heartbeat, asthma attacks, heart attacks and premature death in people with heart or lung disease.

Table 1. Air Quality Index Data

Sample No	Good	Satisfactory	Moderate	Poor	Very Poor	Sample Size
1	1	5	14	2	1	23
2	1	5	16	2	1	25
3	1	7	13	1	2	24
4	2	11	9	1	2	25
5	2	12	7	1	2	24
6	2	10	7	4	2	25
7	2	10	9	3	1	25
8	2	7	9	3	3	24
9	2	6	9	3	4	24
10	3	8	9	2	2	24
11	2	8	6	2	6	24
12	1	7	7	5	4	24
13	2	7	10	3	2	24
14	2	6	6	3	7	24
15	1	6	8	3	5	23
16	2	9	6	4	3	24
17	2	6	7	3	6	24
18	1	10	7	2	5	25
19	4	7	8	1	3	23
20	6	5	9	2	2	24

Air Quality Index is the system which is used to warn the public when air pollution is dangerous. The AQI tracks ozone (smog) and particle pollution as well as four other widespread air pollutants. AQI is classified into six different types according to its values. When AQI lies between 0 and 50 it is considered as good, when the value lies between 51 and 100 it is considered as satisfactory, when the value lies between 101-200 it is considered as moderate, when AQI lies 201 and 300 it is considered as poor, when AQI lies between 301 and 400 it is considered as very poor, when it lies between 401 and 500 it is considered as severe. The five major pollutants are ground level ozone, particle pollution (also known as particulate matter, including PM2.5 and PM 10), Carbon monoxide, sulfur dioxide and nitrogen dioxide.

In order to check the quality of the air, here we have collected the data for the major metropolitan cities in India during the month of February 2022. The data is classified as such that the number of cities comes under the different categories are arranged for the first 20 days for the month of February. The data has been collected from the website of Central Pollution Control Board. Here the membership value is assigned to all the classifications: Good as 0, Satisfactory as 0.25, Moderate as 0.5, Poor as 0.75 and Very Poor as 1.

Table 2. Control limits of FMC chart

S.No	UCL	LCL	CL	$\overline{L}$
1	0.704083	0.361135	0.532609	0.510870
2	0.688518	0.371482	0.530000	0.510000
3	0.713068	0.370266	0.541667	0.531250
4	0.786869	0.413131	0.600000	0.590000
5	0.804563	0.424604	0.614583	0.604167
6	0.771452	0.348548	0.560000	0.520000
7	0.785346	0.394654	0.590000	0.560000
8	0.739769	0.301897	0.520833	0.489583
9	0.713754	0.265413	0.489583	0.458333
10	0.807231	0.359436	0.583333	0.562500
11	0.711274	0.247060	0.479167	0.458333
12	0.671239	0.245428	0.458333	0.406250
13	0.753422	0.329911	0.541667	0.510417
14	0.663962	0.190204	0.427083	0.395833
15	0.660022	0.231283	0.445652	0.413043
16	0.757228	0.305272	0.531250	0.489583
17	0.681335	0.214498	0.447917	0.416667
18	0.703293	0.296707	0.500000	0.480000
19	0.836782	0.337131	0.586957	0.576087
20	0.887835	0.341332	0.614583	0.593750

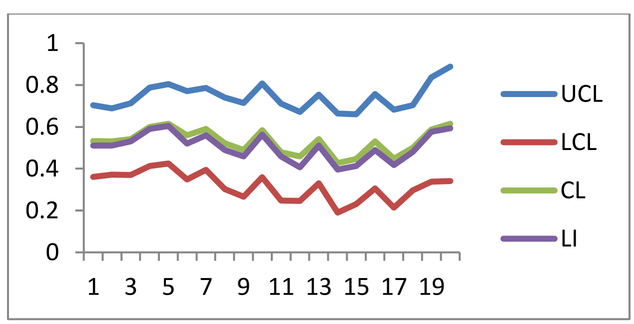

Figure 1. Fuzzy Multinomial Control Chart for AQI Data

Table 1 shows the 20 samples of FMC. Using the collected samples, LCL, UCL, CL and $\bar{\bar{L}}$ are calculated and tabulated in Table 2. Using the Table 2 values FMC chart is drawn. Figure 1 represents the FMC chart for AQI. This chart shows there is no out of control points which indicates the process is in control.

7. Conclusion

Traditional control chart is applicable for real valued data and it used for deciding whether the process is in control or out of control. In practical life due to human subjectivity the data cannot be collected accurately. In this situation traditional control chart cannot be applicable. In this paper, fuzzy logic was applied to traditional p-control chart. Each sample was transformed into linguistic values. FMC chart was developed for AQI data set. The results shows that the process is in control.

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