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BI-MAGIC LABELLING FOR CERTAIN GRAPHS

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Abstract

Graph theory is one of the few branches of mathematics that may be said to have a precise starting date. In 1736, Leonhard Euler solved a celebrated problem, known as the Konigsberg bridges problem. The question had been posed whether it was possible to walk over all the seven bridges spanning the river pregel in Konigsberg just once without retracing one's footsteps. Euler reduced the question to a graph theoretical problem, and found an ingenious solution. Euler's solution marked not only the introduction of the discipline of graph theory but also the first applications of the discipline to a specific problem. Since its inception, graph theory has been exploited for the solution of numerous practical problems, and today still retains an applied character. In the early days, very important strides were made in the development of graph theory by the investigation of some very concrete problems, e.g. Kirchhoff's study of electrical circuits, and Cayley's attempts to enumerate chemical isomers. Also many branches of mathematics such as group theory, matrix theory, numerical analysis, probability and topology have their interaction with graph theory. It has also become more and more clear in recent years that the two disciplines of graph theory and computer science have much in common and thateach is capable of assisting significantly in the development of the other.

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Introduction to Magic Types Graph Labeling

Motivation:

A magic square is an arrangement of numbers in the form of a square so that the sum of the entries in the rows, in the columns and in the diagonals is always the same. This kind of puzzle of constructing a magic square is known to people for more than 4000 years and the earliest recorded appearance dating to about 2200 BC, in China. Magic graphs are related to the well-known magic squares. In 1960s, attempts were made to apply this concept to graphs. The idea is to label with positive integers, the vertices and edges of a graph in such a way that the sum of all the labels associated with the vertices or the edges is a constant and a graph possessing such a property is said to be magic.

Labeling of graphs is one of the most interesting areas of investigation in graph theory. A labeling of a graph G is an assignment of integers to the vertices or to the edges or to both, satisfying certain conditions. If the domain of a labeling is the vertex set alone or the edge set alone, then it is called a vertex or an edge labeling respectively. If the domain of a labeling is the set of vertices and edges, then such a labeling is called a total labeling. Further, if we consider the plane graphs, it is also possible to label the faces of these graphs. Labeled graphs have wide applications in many fields, such as x-rays crystallography, coding theory, cryptography, radar, astronomy, circuit design, and communication network addressing. Different types of graph labeling such as graceful, harmonious, magic, antimagic etc., have been studied by many authors. Numerous variations of labeling have been investigated in the literature. A survey on recent results, conjectures and open problems on graph labeling is presented by J.A. Gallian [24]. In this thesis, we study about vertex magic, vertex bimagic, edge magic, edge bimagic and face magic labeling of graphs.

Vertex magic, vertex antimagic and vertex bimagic labeling

In 2002, MacDougall et al. [31] introduced the notion of vertex magic total labeling. A one-to-one mapping λ from $V \cup E$ onto the integers $\{1, 2, ..., p + q\}$ is a vertex magic total labeling of a graph $G(V, E)$ if there is a constant k so

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that

$$\text{for every vertex } u, \text{ weight of the vertex under } \lambda,\ wt_\lambda(u) = \lambda(u) + \sum_{uv\in E} \lambda(uv) = k.$$

In [31] the authors have studied properties of vertex magic graphs and identified families of graphs having vertex magic total labeling and also classes of graphs which do not admit vertex magic total labeling. Vertex magic total labeling has also been extensively studied by many authors [11, 17, 33, 36]. MacDougall et al. [32] further introduced the concept of super vertex magic labeling. An vertex magic total labeling λ of a graph G is called super vertex magic labeling if $\lambda(V) = \{1, 2, ..., p\}$. Swaminathan and Jeyanthi [45] called a vertex magic total labeling λ of G as E-super vertex magic labeling if $\lambda(E) = \{1, 2, ..., q\}$. Recently Marimuthu and Balakrishnan [34] studied E-super vertex magic labeling extensively.

The definition of (a, d) vertex antimagic total labeling was introduced by Baca et al. in [16]. A graph G is said to be (a, d) vertex antimagic total graph if there exist positive integers a, d and a bijectionλ from $V \cup E$ on to the set of consecutive integers $\{1, 2, ..., p + q\}$ such that the induced mapping g_λ: $V \to W$ is also a bijection, where $W = \{wt_\lambda(x)/x \in V\} = \{a, a+d, a+2d, ..., a+(p-1)d\}$ is the set of weights of vertices in G. Such a labeling is called super if the smallest possible labels appear on the vertices. Yegnanarayanan [51] defined several variations of vertex magic labeling and vertex antimagic labeling namely, (1, 1) vertex magic labeling, (1, 0) vertex magic labeling, (0, 1) vertex magic labeling, (1, 1) − (a, d) vertex antimagic labeling, (1, 0) − (a, d) vertex antimagic labeling, (0, 1) − (a, d) vertex antimagic labeling and also investigated the existence of such labeling on a number of classes of graphs. Further (1, 1) vertex bimagic labeling was introduced byBaskarBabujee in [19].

Face magic labeling

The notion of face magic labeling of plane graphs was introduced by Ko Wei Lih [30] in 1983. However, the subject of face magic labeling can be traced back to the 13th century when similar notions were investigated by the Chinese Mathematician Yang Hui (1275). This concept was further developed by Chang Chhao (1670). Lih [30] extended this concept to face magic labeling as follows: Let $G = (V, E, F)$ be a finite plane graph where V, E and F are its vertex set,

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edge set and set of faces respectively with $|V| = p$, $|E| = q$ and $|F| = f$. A labeling of type (a, b, c) of G assigns labels from the set $\{1, 2, ..., ap+ bq+ cf\}$ to vertices, edges and faces of G such that each vertex receives a-label, each edge receives b-label and each face receives c-label and each label is used exactly once. a, b and c are restricted to the values of $\{0, 1\}$. Labeling of type (1, 0, 0),(0, 1, 0) and (0, 0, 1) are called vertex labeling, edge labeling and face labeling respectively. The weight of a face $wt(f)$ under a labeling is the sum of labels of a face together with labels of vertices and edges forming that face. A labeling is said to be magic, if for every positive integer s, all s-sided faces have the same weight and we allow different weights for different s.

Lih [30] described face magic labeling of type (1, 1, 0) for wheel, friendship graphs, prisms and some of platonic polyhedra. Baca [12] has described magic and consecutive labeling for fans, planar pyramids and ladders. The face magic labeling of type (1, 1, 1) for Mobius ladder, grid graphs and honeycomb are proved in [13, 14, 15]. In [27] face magic labeling of type (1, 1, 1) for special families of planar graphs with 3-sided faces, 5-sided faces, 6-sided faces and one external infinite face are shown. In [28], magic labeling of type (a, b, c) for families of wheels are proved.

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In this chapter, we deal with vertex bimagic labeling for new families of graphs. In section 2.1, we discuss new techniques of generating (1, 1) vertex bimagic, (1, 0) vertex bimagic and (0, 1) vertex bimagic graphs using operations on vertex magic and vertex antimagic graphs [9]. In section 2.2, we consider graph families which admit E-super vertex bimagic labeling for our discussion and establish interesting results. In section 2.3, we identify some families of graphs which admit 1-vertex bimagic vertex labeling.

2.1 Vertex Bimagic Graphs from Magic and Antimagic Graphs Theorem 2.1.1 If G has $(1, 1) - (a, 1)$ vertex antimagic labeling, then $G + K_1$ admits (1, 1) vertex bimagic labeling.

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*Proof.*Let for $a > 0$, $G(V, E)$ be a $(1, 1) - (a, 1)$ vertex antimagic graph with p vertices $\{u_i: 1 \leq i \leq p\}$ and q edges. Consider the bijection $\lambda : V \cup E \to \{1, 2, ..., p+ q\}$ on G with the antimagic property that vertex weight $wt_\lambda(u_i) = a+p-i;\ 1 \leq i \leq p$.

Let $G^0(V^0, E^0) = G + K_1$ be the graph with $V^0 = V \cup \{w\}$ and $E^0 = E \cup \{wu_i: 1 \leq i \leq p\}$. Consider the bijection $\mu : V^0 \cup E^0 \to \{1, 2, ..., p + q, p + q + 1, ..., 2p + q + 1\}$ defined as follows:

$\mu(u_i) = \lambda(u_i);\ 1 \leq i \leq p,\ \mu(e) = \lambda(e)$, for all $e \in E$.

$\mu(w) = 2p + q + 1$ and $\mu(wu_i) = p + q + i$, for $1 \leq i \leq p$.

To complete the proof, it is necessary to show that the constants are k_1 and k_2 as given below.

For $u_i \in V, 1 \leq i \leq p$ we have

$$wt_\mu(u_i) = \mu(wu_i) + wt_\lambda(u_i) = (p+q+i) + (a+p-i) = 2p+q+a = k_1.$$

$$\text{For } w \in V',\ wt_\mu(w) = \sum \mu(wu_i) + \mu(w) = \sum_{i=1}^{p}(p+q+i) + (2p+q+1)$$
$$= (p+q)p + \frac{p(p+1)}{2} + 2p + q + 1$$
$$= (p+1)(q+1+\frac{3p}{2}) = k_2.$$

Therefore, G'admits (1, 1) vertex bimagic labeling for $G + K_1$ with the weights

$k_1 = 2p + q + a$ and $k_2 = (p + 1)(q + 1 + \frac{3p}{2})$.

Theorem 2.1.2 Let G be a graph of odd order. If G has (1, 1) vertex magic labeling, then $G + 2K_1$ admits (1, 1) vertex bimagic labeling.

*Proof.*Let $G(V, E)$ be a graph that admits (1, 1) vertex magic labeling. Then there exists a mapping $\lambda : V \cup E \to \{1, 2, ..., p + q\}$ such that the weights at each vertex $wt_\lambda(u_i) = a$, a constant.

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Consider the new graph $G'(V', E') = G + 2K_1$ with $V' = V \cup \{v, w\}$ and $E' = E \cup \{vu_i, wu_i: 1 \leq i \leq p\}$. Define a bijection $\mu : V' \cup E' \to \{1, 2, ..., 3p + q + 2\}$ as given below:

$\mu(x) = \lambda(x),\ \forall x \in V \cup E.$

Case(i) : When $n = 4k + 1$, $k \in N$

For $i = 2, 4, 6, ..., \frac{p-1}{2}$; $\mu(vu_{2i}) = p + q + 2i$, $\mu(vu_{2i+1}) = p + q + 2i + 1$,

$\mu(wu_{2i}) = 3p + q + 1 - 2i$, $\mu(wu_{2i+1}) = 3p + q - 2i$.

For $i = 1, 3, 5..., \frac{p-3}{2}$; $\mu(vu_{2i}) = 3p + q + 1 - 2i$, $\mu(vu_{2i+1}) = 3p + q - 2i$,

$\mu(wu_{2i}) = p + q + 2i$, $\mu(wu_{2i+1}) = p + q + 1 + 2i$.

Case(ii) : When $n = 4k - 1$, $k \in N$

For $i = 2, 4, 6, ..., \frac{p-3}{2}$; $\mu(vu_{2i}) = p + q + 2i$, $\mu(vu_{2i+1}) = p + q + 2i + 1$,

$\mu(wu_{2i}) = 3p + q + 1 - 2i$, $\mu(wu_{2i+1}) = 3p + q - 2i$.

For $i = 1, 3, 5..., \frac{p-3}{2}$; $\mu(vu_{2i}) = 3p + q + 1 - 2i$, $\mu(vu_{2i+1}) = 3p + q - 2i$,

$\mu(wu_{2i}) = p + q + 2i$, $\mu(wu_{2i+1}) = p + q + 1 + 2i$.

$\mu(v) = 3p + q + 2$, $\mu(vu_1) = p + q + 1$, $\mu(w) = 3p + q + 1$ and $\mu(wu_1) = 3p + q$.

We verify the vertex weights in G'

$wt_{\mu}(u_1) = \mu(u_1) + \mu(vu_1) + \mu(wu_1) = a + (p + q + 1) + (3p + q)$

$= 4p + 2q + a + 1 = k_1.$

When $n = 4k - 1$, $k \in N$ we have

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For $i = 1, 3, 5\ldots,\frac{p-1}{2}$; $wt_{\mu}(u_{2i}) = wt_{\lambda}(u_{2i}) + \mu(vu_{2i}) + \mu(wu_{2i})$

$= a + (3p + q + 1 - 2i) + (p + q + 2i)$

$= 4p + 2q + a + 1 = k_1$.

$wt_{\mu}(u_{2i+1}) = wt_{\lambda}(u_{2i+1}) + \mu(vu_{2i+1}) + \mu(wu_{2i+1})$

$= a + (3p + q - 2i) + (p + q + 1 + 2i)$

$= 4p + 2q + a + 1 = k_1$.

For $i = 2, 4, 6, \ldots,\frac{p-3}{2}$; $wt_{\mu}(u_{2i}) = wt_{\lambda}(u_{2i}) + \mu(vu_{2i}) + \mu(wu_{2i})$

$= a + (p + q + 2i) + (3p + q + 1 - 2i)$

$= 4p + 2q + a + 1 = k_1$.

$wt_{\mu}(u_{2i+1}) = wt_{\lambda}(u_{2i+1}) + \mu(vu_{2i+1}) + \mu(wu_{2i+1})$

$= a + (p + q + 2i + 1) + (3p + q - 2i)$

$= 4p + 2q + a + 1 = k_1$.

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$$\begin{aligned}
wt_\mu(v) &= \mu(v)+\mu(vu_1)+\sum_{i\ odd}\mu(vu_{2i})+\sum_{i\ odd}\mu(vu_{2i+1})+\sum_{i\ even}\mu(vu_{2i})+\sum_{i\ even}\mu(vu_{2i+1})\\
&= (3p+q+2)+(p+q+1)+\sum_{i\ odd}(3p+q+1-2i)+\sum_{i\ odd}(3p+q-2i)+\\
&\quad \sum_{i\ even}(p+q+2i)+\sum_{i\ even}(p+q+1+2i)\\
&= 4p+2q+3+(3p+q+1)(\frac{p+1}{4})-2(1+3+...+\frac{p-1}{2})+(3p+q)(\frac{p+1}{4})-\\
&\quad 2(1+3+...+\frac{p-1}{2})+(p+q)(\frac{p-3}{4})+2(2+4+...+\frac{p-3}{2})+\\
&\quad (p+q+1)(\frac{p-3}{4})+2(2+4+...+\frac{p-3}{2})\\
&= 4p+2q+3+(6p+2q+1)(\frac{p+1}{4})-4(1+3+...+\frac{p-1}{2})+\\
&\quad (2p+2q+1)(\frac{p-3}{4})+4(2+4+...+\frac{p-3}{2})\\
&= 4p+2q+3+(6p+2q+1)(\frac{p+1}{4})-(\frac{p+1}{2})^2+\\
&\quad (2p+2q+1)(\frac{p-3}{4})+(p-3)(\frac{p+1}{4})\\
&= 2p^2+pq+q+\frac{1}{2}(7p+3)=k_2
\end{aligned}$$

$$\begin{aligned}
\text{and } wt_\mu(w) &= \mu(w)+\mu(wu_1)+\sum_{i\ odd}\mu(wu_{2i})+\sum_{i\ odd}\mu(wu_{2i+1})+\\
&\quad \sum_{i\ even}\mu(wu_{2i})+\sum_{i\ even}\mu(wu_{2i+1})\\
&= (3p+q+2)+(p+q+1)+\sum_{i\ odd}(3p+q+1-2i)+\sum_{i\ odd}(3p+q-2i)+\\
&\quad \sum_{i\ even}(p+q+2i)+\sum_{i\ even}(p+q+1+2i)\\
&= 4p+2q+3+(3p+q+1)(\frac{p-1}{4})-2(1+3+...+\frac{p-3}{2})+(3p+q)(\frac{p-1}{4})-\\
&\quad 2(1+3+...+\frac{p-3}{2})+(p+q)(\frac{p-1}{4})+2(2+4+...+\frac{p-1}{2})+
\end{aligned}$$

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$$(p+q+1)(\frac{p-1}{4}) + 2(2+4+\dots+\frac{p-1}{2})$$
$$= 4p+2q+3+(8p+4q+2)(\frac{p-1}{4}) - 4(1+3+\dots+\frac{p-3}{2}) +$$
$$(2p+2q+1)(\frac{p-1}{4}) + 4(2+4+\dots+\frac{p-1}{2})$$
$$= 4p+2q+3+(8p+4q+2)(\frac{p-1}{4}) - (\frac{p-1}{2})^2 + (p-1)(\frac{p+3}{4})$$
$$= 2p^2+pq+q+\frac{1}{2}(7p+3) = k_2.$$

When $n = 4k + 1$, $k \in N$ we have

For $i = 1, 3, 5\dots, \frac{p-3}{2}$; $wt_\mu(u_{2i}) = wt_\lambda(u_{2i}) + \mu(vu_{2i}) + \mu(wu_{2i})$

$= a + (3p + q + 1 - 2i) + (p + q + 2i)$

$= 4p + 2q + a + 1 = k_1.$

$wt_\mu(u_{2i+1}) = wt_\lambda(u_{2i+1}) + \mu(vu_{2i+1}) + \mu(wu_{2i+1})$

$= a + (3p + q - 2i) + (p + q + 1 + 2i)$

$= 4p + 2q + a + 1 = k_1.$

For $i = 2, 4, 6, \dots, \frac{p-1}{2}$; $wt_\mu(u_{2i}) = wt_\lambda(u_{2i}) + \mu(vu_{2i}) + \mu(wu_{2i})$

$= a + (p + q + 2i) + (3p + q + 1 - 2i)$

$= 4p + 2q + a + 1 = k_1.$

$wt_\mu(u_{2i+1}) = wt_\lambda(u_{2i+1}) + \mu(vu_{2i+1}) + \mu(wu_{2i+1})$

$= a + (p + q + 2i + 1) + (3p + q - 2i)$

$= 4p + 2q + a + 1 = k_1$.

$$wt_\mu(v) = \mu(v)+\mu(vu_1)+\sum_{i\ odd}\mu(vu_{2i})+\sum_{i\ odd}\mu(vu_{2i+1})+\sum_{i\ even}\mu(vu_{2i})+\sum_{i\ even}\mu(vu_{2i+1})$$
$$= (3p+q+2)+(p+q+1)+\sum_{i\ odd}(3p+q+1-2i)+\sum_{i\ odd}(3p+q-2i)+$$
$$\sum_{i\ even}(p+q+2i)+\sum_{i\ even}(p+q+1+2i)$$
$$= 4p+2q+3+(3p+q+1)(\frac{p-1}{4})-2(1+3+...+\frac{p-3}{2})+(3p+q)(\frac{p-1}{4})-$$
$$2(1+3+...+\frac{p-3}{2})+(p+q)(\frac{p-1}{4})+2(2+4+...+\frac{p-1}{2})+$$
$$(p+q+1)(\frac{p-1}{4})+2(2+4+...+\frac{p-1}{2})$$
$$= 4p+2q+3+(8p+4q+2)(\frac{p-1}{4})-(\frac{p-1}{2})^2+(p-1)(\frac{p+3}{4})$$
$$= 2p^2+pq+p+\frac{1}{2}(7p+3) = k_2$$

and $wt_\mu(w) = \mu(w)+\mu(wu_1)+\sum_{i\ odd}\mu(wu_{2i})+\sum_{i\ odd}\mu(wu_{2i+1})+$
$$\sum_{i\ even}\mu(wu_{2i})+\sum_{i\ even}\mu(wu_{2i+1})$$
$$= (3p+q+1)+(3p+q)+\sum_{i\ odd}(p+q+2i)+\sum_{i\ odd}(p+q+1+2i)+$$
$$\sum_{i\ even}(3p+q+1-2i)+\sum_{i\ even}(3p+q-2i)$$
$$= 6p+2q+1+(p+q)(\frac{p-1}{4})-2(1+3+...+\frac{p-3}{2})+(p+q+1)(\frac{p-1}{4})+$$
$$2(1+3+...+\frac{p-3}{2})+(3p+q+1)(\frac{p-1}{4})-$$
$$2(2+4+...+\frac{p-1}{2})+(3p+q)(\frac{p-1}{4})-2(2+4+...+\frac{p-1}{2})$$
$$= 6p+2q+1+(8p+4q+2)(\frac{p-1}{4})+(\frac{p-1}{2})^2-(p-1)(\frac{p+3}{4})$$
$$= 2p^2+pq+q+\frac{1}{2}(7p+3) = k_2.$$

Hence, the resultant graph G' has (1, 1) vertex bimagic labeling.

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Theorem 2.1.3 If an odd order graph G has $(1, 1)-(a, 2)$ vertex antimagic labeling, then simultaneous vertex by edge duplication at all vertices on G admits $(1, 1)$ vertex bimagic labeling.

*Proof.*Let $G(V, E)$ be a $(1, 1) - (a, 2)$ vertex antimagic graph with $\lambda : V \cup E \rightarrow \{1, 2, ..., p + q\}$ such that the vertex weights are $wt_\lambda(u_i) = a + 2p - 2i$, for $1 \leq i \leq p$. Consider the graph obtained by simultaneous vertex by edge duplication at all vertices on G with $V' = V \cup \{u'_i, u''_i: 1 \leq i \leq p\}$ and $E' = E \cup \{e'_i, e''_i, e'''_i: 1 \leq i \leq p\}$, where e''_i, e'''_i are adjacent to u'_i. Similarly, e'_i, e'''_i are adjacent to u''_i.A bijective 21

mapping $\lambda': V' \cup E' \rightarrow \{1, 2, ...,6p + q\}$ is given below:

$$\lambda'(e'_i) = p + q + i, \text{ for } 1 \leq i \leq p.$$

$$\lambda'(e''_i) = 2p + q + i, \text{ for } 1 \leq i \leq p.$$

$$\lambda'(e'''_i) = \begin{cases} 3p + q + \frac{p+3}{2} - \frac{i+1}{2}, & \text{if } i \equiv 1 \ (mod\ 2), \text{ for } 1 \leq i \leq p \\ 4p + q + 1 - \frac{i}{2}, & \text{if } i \equiv 0 \ (mod\ 2), \text{ for } 2 \leq i \leq p-1. \end{cases}$$

$$\lambda'(u'_i) = \begin{cases} 5p + q + 1 - \frac{i+1}{2}, & \text{if } i \equiv 1 \ (mod\ 2), \text{ for } 1 \leq i \leq p \\ 4p + q + \frac{p+1}{2} - \frac{i}{2}, & \text{if } i \equiv 0 \ (mod\ 2), \text{ for } 2 \leq i \leq p-1. \end{cases}$$

$$\lambda'(u''_i) = \begin{cases} 6p + q + 1 - \frac{i+1}{2}, & \text{if } i \equiv 1 \ (mod\ 2), \text{ for } 1 \leq i \leq p \\ 5p + q + \frac{p+1}{2} - \frac{i}{2}, & \text{if } i \equiv 0 \ (mod\ 2), \text{ for } 2 \leq i \leq p-1. \end{cases}$$

The vertex weights under the labeling λ' are

$wt_{\lambda'}(u_i) = wt_\lambda(u_i) + \lambda'(e'_i) + \lambda'(e''_i)$

$= (a + 2p - 2i) + (p + q + i) + (2p + q + i) = a + 5p + 2q, 1 \leq i \leq p.$

When $i \equiv 1 \ (mod\ 2), 1 \leq i \leq p,$ we have $wt_\lambda'(u'_i) = \lambda'(u'_i) + \lambda'(e''_i) + \lambda'(e'''_i)$

$= (5p + q + 1 - \frac{i+1}{2}) + (2p + q + i) + (3p + q + \frac{p+3}{2} - \frac{i+1}{2})$

$= 10p + 3q + \frac{p+3}{2}.$

$wt_\lambda'(u''_i) = \lambda'(u''_i) + \lambda'(e'_i) + \lambda'(e'''_i)$

$= (6p + q + 1 - \frac{i+1}{2}) + (p + q + i) + (3p + q + \frac{p+3}{2} - \frac{i+1}{2})$

$= 10p + 3q + \frac{p+3}{2}.$

When $i \equiv 0 \ (mod\ 2), 1 \leq i \leq p,$ we have $wt_\lambda'(u'_i) = \lambda'(u'_i) + \lambda'(e''_i) + \lambda'(e'''_i)$

$= (4p + q + \frac{p+1}{2} - \frac{i}{2}) + (2p + q + i) + (4p + q + 1) = 10p + 3q + \frac{p+3}{2}$

$wt_\lambda'(u''_i) = \lambda'(u''_i) + \lambda'(e'_i) + \lambda'(e'''_i)$

$= (5p + q + \frac{p+1}{2} - \frac{i}{2}) + (p + q + i) + (4p + q + 1 - \frac{i}{2})$

$= 10p + 3q + \frac{p+3}{2}$

Hence the resultant graph admits (1, 1) vertex bimagic labeling with $k_1 = a+5p+2q$ and $k_2 = 10p + 3q + \frac{p+3}{2}$

Theorem 2.1.4 If G has $(1, 1) - (a, d)$ vertex antimagic labeling, then $G \ \Theta \ dK_1$ admits (1, 1) vertex bimagic labeling.

*Proof.*Let $G(V, E)$ be a $(1, 1) - (a, d)$ vertex antimagic graph with $V = \{u_1, u_2, ..., u_n\}$. Let λ be the vertex antimagic labeling of G such that $\lambda(u_i) = i$, for $1 \leq i \leq p$. The vertex weights are

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$wt_\lambda(u_i) = a + (i - 1)d$, for $1 \leq i \leq p$. Consider the graph $G^0(V^0, E^0) = G \quad dK_1$ with $V^0 = V \cup \{u^j_i$: $1 \leq i \leq p, 1 \leq j \leq d\}$ and $E^0 = E \cup \{e^j_i$: $1 \leq i \leq p, 1 \leq j \leq d\}$. Define a vertex bimagic labeling λ^0: $V^0 \cup E^0 \rightarrow \{1, 2, ..., p + q + 2dp\}$ on G^0 as given below:

For $x \in V \cup E$, $\mu(x) = \lambda(x)$.

For $1 \leq i \leq p$, $1 \leq j \leq d$;

$\lambda'(u^j_i) = p + q + 2dp - jp + i$, $\lambda'(e^j_i) = p + q + j - (i - 1)$.

The vertex weights in G' under λ' is calculated in the following.

For each i, $1 \leq i \leq p$

$$\begin{aligned} wt_{\lambda'}(u_i) &= wt_\lambda(u_i) + \sum_{j=1}^{d} \lambda'(e_i^j) = (a + id - d) + \sum_{j=1}^{d}(p + q + jp - i + 1) \\ &= (a + id - d) + (p + q - i + 1)d + \frac{pd(d+1)}{2} \\ &= \tfrac{1}{2}(2a + pd^2 + 3pd + 2qd) = k_1. \end{aligned}$$

For u_i^j, $i = 1, 2, ..., p, j = 1, 2, ..., d$, we have

$$wt_{\lambda'}(u_i^j) = \lambda'(u_i^j) + \lambda'(e_i^j) = (p + q + 2dp - jp + i) + p + q + jp - (i - 1)$$

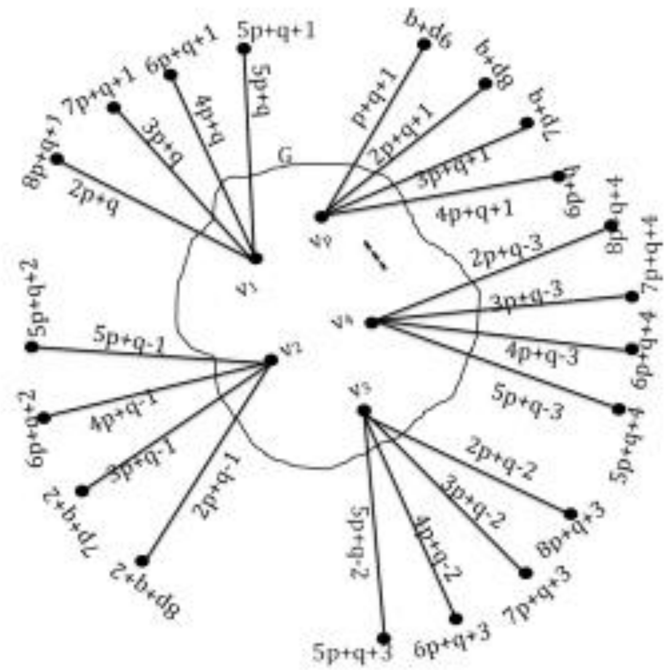

Figure 2.1.Vertex bimagic labeling of G ʘ4K₁

$= 2p + 2q + 2dp + 1 = k_2$.

Thus, the graph $G \; \Theta \; dK_1$ admits (1, 1) vertex bimagic labeling.

the proof of theorem 2.3.7 reveals that the vertex weights in $G_1 + G_2$ are $k'_1 = k_1 + k$ and $k'_2 = k_2 + k$. In both cases the graph $G_1 + G_2$ admits 1-vertex bimagic vertex labeling.

Conclusion

In the thesis the notions of magic and bimagic labeling are investigated for several families of graphs. First, different types of vertex bimagic labeling have been studied; namely vertex bimagic total labeling, E-super vertex bimagic labeling and 1-vertex bimagic vertex labeling. New techniques of generating (1, 1) vertex bimagic, (1, 0) vertex bimagic and (0, 1) vertex bimagic graphs have been discussed using some operations on vertex magic and vertex antimagic graphs. E-super vertex bimagic labeling is identified for new families of graphs and 1-vertex bimagic vertex labeling has been established. Then edge magic and edge bimagic labeling are studied for variant operation of graphs. Edge magic graceful labeling motivated to introduce the notion of edge bimagic graceful labeling. Finally face magic labeling is studied by using duplication operation on trees and cyclic graphs.

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