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ANTI-MAGIC LABELING FOR CERTAIN GRAPHS

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INTRODUCTION

Graph theory is the study of graphs, which are statistical structures, used to representation of pair wise relations between objects. Graph theory is applicable to examine artifact that is connected to other things. Some complicated problems formulate easy solutions when characterized as a graph. A graph in this statement is made up of nodes called as vertices and connections between the vertices called as edges. Generally vertices and edges of a graph are referred as graph elements. Graph theorists are progressively discovering that abundance of their problems can be resolved, or their research furthered by the use of computing techniques. Graph labeling problems are one among them. Numerous variations of labeling have been investigated in the literature. A complete survey on contemporary results, speculation and open problems in labeling graph is presented by J.A.Gallian[1].

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Magic graphs are related to the well known magic squares. Magic squares are defined as follows. Arrange numbers in a square such that it produces a same value over addition of numbers in each row, column and diagonal. These mathematical puzzles are famous to people for more than 4000 years, but no earlier than in the 1960s it was tried to concern this model to graphs.

The same idea is applied on graphs. Numbers are used to label edges and vertices of graph such that each vertex produces a constant weight where weight is the sum of edge label incident to that vertex. This is called as magic labeling. A graph labeling is a mapping of values to the elements of graph such as vertices or edges or both under certain conditions. Later on studies proved the importance of labeled graphs, by showing its applications in many research problems. In circuit designing, social psychology labeled graphs are enormously used. In computer science domain also they are applied in large scale of applications. Some of the areas are listed here.

- Coding Theory problems.
- Design of quality Radar locus codes.
- Synch set codes.
- Missile management codes.
- X ray Crystallographic analysis.
- Networks.
- Communication Networks.

1.1 Graph Labeling

Graph Labeling is the method of assigning a value or identification to graph elements. Primarily a label is used for identification purpose only. But afterward it is shifted to carry some appropriate information through it. For example, in traveling salesman problem, vertex labels indicate city name and edge labels specifies distance/cost in between two cities. A label can be subjective or measurable. There are basically two types of labeling of graph, namely quantitative labeling is assignment of some numbers to the elements of graph and qualitative labeling is assignment of qualitative nature to the elements of graph.

Quantitative Labeling: These labeling have outstanding investigation by broad diversity of applications in radio astronomy, development of missile guidance codes, and incorporeal characterization of medium using X ray crystallography etc. Probably the most popular of them all, called graceful labeling. In above example vertex labels are qualitative and edge labels are quantitative. In this thesis only quantitative labels are used.

Qualitative Labeling: These labeling are used in areas of human inquiry such as conflict resolution in social psychology, electrical circuit theory, energy crises etc. Consider a social group. A graph is associated as a mathematical model in which vertices represent the individuals and edge represents the relation between two individuals, when studied about the attitudinal behavior of the people in the social group.

Applications of Graph Labeling

Graph labeling is used in plenty of applications like coding theory, X ray crystallography, radar, astronomy, communication network addressing, circuit design, data base management etc. This thesis gives an overview of labeled graph applications to computer science in major areas like data mining, image processing, cryptography, software testing, information security, communication networks etc. The role of graph labeling in each area of computer science with applications is discussed in this session.

Structured mining

The procedure for discovering and removing applicable information from semi structured sets of data is known as Structure mining or structured data mining. Graph mining is one of the special cases of structured data mining. The output representation of data mining is constituted in graphs because the concept of data mining graphs can be widely used.

In Structured mining one approach called, B-AGM (Biased Apriori based Graph Mining), a bias for a particular category of the graph structure consists of the committed definitions of the canonical form and join operation. By picking a suitable bias on the platform of the AGM framework, the complete mining for the frequent sub graphs of the objective class are seeking for is defined.

Network Representation

Computer Science is having a vast number of domains and network representations can be used in every domain ranging from data structures and graph algorithms, to parallel and distributed computing and communication networks.

Dual layer representation can be used to represent networks logical view. This allows performing high level network analysis to identify characteristics of nodes and copes for identification of their topologies. Generally network modeling systems are described with weights and directions. Dual layer techniques for a directed graph allow more complex networks analysis and identify more inner and hidden details. This plays an important role for clustering analysis.

Database Management

Database management is widely used in many applications because of its efficiency. For storing information and this is done with tables. These tables can be considered as nodes and the connections that are drawn between them to illustrate the relationship can be represented as labels to the nodes.

The amount of applications calling for efficient large graph management is considerably growing. Social network analysis, Internet and bio computation are some examples of such applications. In these instances, the total concentration put on the structural analysis of the

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relationships between disparate entities organized in huge networks or graph like structures. Being capable to perfectly handle such graphs becomes crucial, keeping graph database management systems in the eye of the storm. Among the various challenging tasks produced by graph databases, discovering a perfect way to represent and manipulate huge graphs that do not entirely fit in memory is still an unresolved problem. DEX is a system which deals with this problem. It is a high performance graph works on bitmaps for managing database system. By using bitmap structures for graph representation it is feasible to better the production of a graph database system, permit for the perfect handling of very large graphs holding thousands of millions of nodes and edges.

Magic Labeling

Simple and undirected graphs are used in this thesis. Let *G*be a graph, *V*vertex set of *G*and *E*is the edge set of*G*. Let *m*and *n*denotes number of edges and number of vertices. General notations [2] are followed here.

Magic labeling belongs to the family of quantitative labeling which can be defined as a process of assigning labels i.e., integers in range {1,2...*m n* + }to graph elements such that weight of each element produces a constant sum called as magic constant *k*where weight of an element is defined as sum of labels applied to them. There are several variations of magic labeling like Vertex magic total labeling, Edge magic total labeling and Total magic labeling.

Vertex magic labeling can be expressed as a bijective function denoted by $\lambda_1 : \{1, 2 \ldots m+n\} \to V \cup E$ and if there is a vertex magic constant k_1 such that for any vertex v, the weight is calculated as $\lambda_1(v) + \sum_{u \in A(v)} \lambda_1(v, u) = k_1 \; \forall v \in V$ *and* $A(v)$ is

the set of adjacent vertices to given vertex *v*whereλ_1 (*v*)is the label applied on the vertex and λ_1 (*v u* ,)is the label applied on the edge(*v u u A v* ,) ∀ ∈ ( ). Figure 1.1 is an example for vertex magic total labeling with magic constant 19 on a graph having 10 graph elements i.e., 5 vertices and 5 edges.

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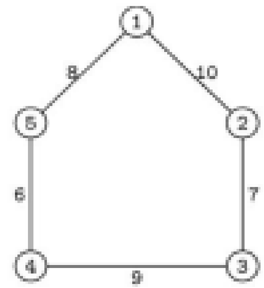

Figure 1.1: An Example for Vertex magic total labeling on graph with n k = = 10& 19

In Figure 1.1 the graph has 5 vertices and 5 edges. So, they are assigned labels starting from 1 to 10, where 10 is the number of graph elements $m=5$&$n=5$. The weight of each vertex is calculated as the sum of assigned to it and labels assigned incident edges. The weight of vertex with label 1 is 8+1+10=19. The weight of vertex with label 3 is 7+3+9=19 and is same for remaining all vertices. So, all vertices are assigned with labels such that each produces a constant weight called as magic constant. Here it is 19.

In similar fashion Edge magic total labeling can be defined. Edge magic total labeling can be expressed as a bijective function denoted by λ_2: $\{1,2... m+n\} \to V \cup E$ and if there is a edge magic constant $k_2$such that for any edge(v u,),the weight is calculated as $\lambda_2(v)+ \lambda_2(u)+ \lambda_2(v,u)=k_2 \forall e \in E$where $\lambda_2(v)$is the label applied on the vertex $v$and $\lambda_2(v\ u,\ )$is the label applied on the edge(v,u) $\forall u \in A(v)$. Figure.1.2 is an example for edge magic total labeling with magic constant 12 on a graph having 9 graph elements i.e., 4 vertices and 5 edges.

In Figure.1.2 the graph has 4 vertices and 5 edges. So, they are assigned labels starting from 1 to 9, where 9 is the number of graph elements$m$ n =4 & n= 5. The weight of each edge is calculated as the sum of assigned to it and labels assigned to the vertices of that edge. The weight of edge with label 5 is 1+5+6=12. The weight of edge with label 7 is 3+7+2=12 and is same for remaining all edges. So, all edges are assigned with labels such that each produces a constant weight called as magic constant. Here it is 12.

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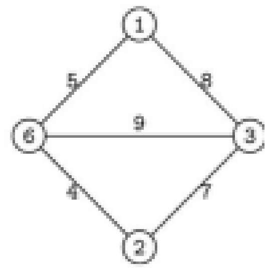

Figure 1.2: An example for Edge magic total labeling on graph with n k = = 9& 12

If the graph has both Vertex magic total labeling and Edge magic total labeling for magic constants k_1 and k_2, then such graph is said to be having Total magic labeling as shown in Figure 1.3.

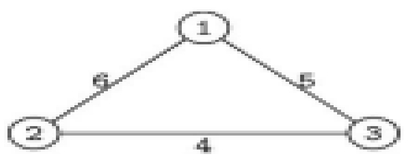

Figure 1.3: An Example for Total magic labeling on graph with $n= 6$ $k_1=12$& $k_2= 9$

In Figure 1.3 the graph has 3 vertices and 3 edges. So, they are assigned labels starting from 1 to 6, where 6 is the number of graph elements$m$ n = = 3& 3. The weight of edges is calculated as 9 and is same for all edges. The weight of vertices is calculated as 12 and is same for all vertices. So, this graph is an example for total magic labeling as it obeys both edge magic total labeling with magic constant 9 and vertex magic total labeling with magic constant 12.

1.4 Antimagic Labeling

In contrast to magic labeling, antimagic labeling requires different weights for each graph element. Antimagic labeling of a graph is defined as a process of assigning labels i.e., integers {1,2...m n + }to graph elements such that weight of each element is different. There are various types of antimagic labeling based on the nature of weights.

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For the given graph if the weight function applied on vertices is different in pair wise, then such labeling is called as Vertex antimagic edge labeling. If the weight function applied on edges and is different in pair wise, then such labeling is called as Edge antimagic vertex labeling. These antimagic labeling can be either strong or weak. If the graph elements are having different weights such antimagic labeling are said to be strong otherwise weak.

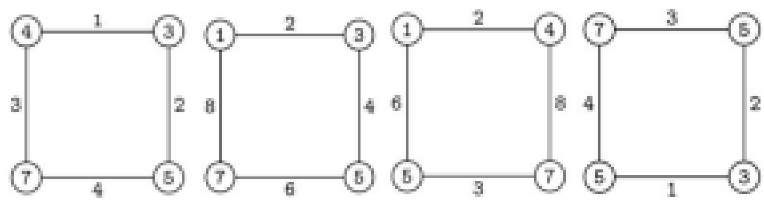

Figure 1.4: Examples for types of Vertex antimagic labeling on graph with $n = 4$

Vertex antimagic edge labeling can be expressed as a bijective function denoted by λ_3:{1,2 …m }→ E and the weight of each vertex is pair wise different where the weight of any vertex is calculated as $\sum_{u \in \forall(v)} \lambda_3 (v,u)$, $\forall v \in V$ and $A(v)$ is the set of adjacent vertices to given vertex v where $\lambda_3 (v ,u)$ is the label applied on the edge(v, u) $\forall u \in A(v)$. An observation is that the weight of vertex itself acts as label. This is again categorized as strong vertex antimagic edge labeling or a weak vertex antimagic edge labeling. Figure 1.4(a) the graph edges are labeled with numbers in the range {1,2,3,4}where 4 is number of edges of the graph. The label for a vertex is calculated as the sum of edge labels incident to that vertex.

The weight of each vertex is assigned as its label. All pair wise vertices having different weights and labels assigned to vertices have no duplicates. So it is considered as strong vertex antimagic edge labeling. Figure 1.4(b) consists of vertex labels are calculated in the similar fashion. The weights are 5, 3, 5 & 7. All these labels are pair wise different but had duplicates. So it is considered as weak vertex antimagic edge labeling.

A *friendship graph* is defined as the collection of n triangles (where a triangle is a cycle of size 3) with one common vertex called as hub and n is size of friendship graph. The friendship graph with n triangles is denoted as T_n. Each vertex in T_n has degree 2 except the hub.

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Hub has exactly degree$2n$. An example to friendship graph is shown in Figure 1.10. This is a friendship graph with size 3. The number of triangles gives the size of the friendship graph.

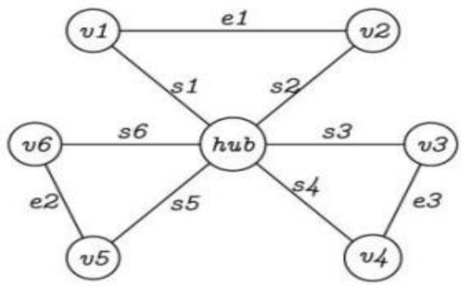

Figure 1.10: Example of friendship graph with n = 3

MAGIC LABELLING

Sedlacek introduced magic labeling with impetus of magic squares. From 1960‟s several authors are working on various types of magic labeling on divergent regional anatomies of graphs. Majority of them studied various properties of the graphs. Magic labeling is exhibited in terms of Vertex magic total labeling, Edge magic total labeling and Total magic labeling.

The graphs that scrutinized are limited, modest and undirected. The standard notations to denote the graph are followed. An imprecise remission for graph theoretic notations is [2]. The graphs used in this thesis are paths, cycles, wheels, fan graphs and friendship graphs. Imprecise definitions of path, cycle, wheel, fan graph, friendship graph are as follows.

Path graph: A path is a populated graph $P_n(V,E)$ of the form $V =\{ v_1v_2\ldots v_n\}$and $E =\{(v_0,v_1), (v_1,v_2)\ldots(v_{n-1} v_n)\}$where $\forall i v_i$are well defined and n is the path length.

Cycle graph: A cycle graph $C_n(V,E)$ where $n \geq 3$, $V =\{ v_1v_2\ldots v_n\}$and $E =\{(v_0v_1, v_1,v_2\ldots v_{n-1} v_n, v_n v_0,)\}$where $\forall\ i v_i$ are distinct and n is the cycle size.

Wheel graph: A wheel graph $W_n\ (V,E)$ is a cycle of size n with central hub and all vertices of cycle are adjacent to it.

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Fan graph: A Fan graph F_n (V,E) is a path of length n with a pivotal axis and all vertices of path adjacent to central hub.

Friendship graph: A Friendship graph $T_n(V,E)$ consists of n triangles with one common vertex called as hub where n size of friendship graph and each triangle is a cycle of size 3.

Labeled graphs are becoming an increasingly useful family of Mathematical Models for a extensive scope of applications such as Conflict resolution in social psychology, electrical circuit theory and energy crisis, Coding Theory problems, incorporating the design of good Radar location codes, Synch set codes, Missile guidance codes and helix codes with unsurpassed autocorrelation properties and in determining ambivalence in X ray Crystallographic dissection, to Design Communication Network addressing Systems, in determining Optimal Circuit Layouts and Radio Astronomy., etc. Designated graphs are replicating pre eminent role substantially in the field of computer science. This appraisal showed its consequence in networking channels, data mining, cryptography, SQL query solving, etc. This colossal range of applications imputed us towards labeled graphs.

Vertex Magic Total Labeling

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Vertex magic total labeling was instigated by McDougall et al. in 2002. Vertex magic labeling can be expressed as a bijective function denoted by $\lambda_1 : \{1, 2 \ldots m+n\} \rightarrow V \cup E$ and if there is a vertex magic constant K_1 such that for any vertex v, the weight is calculated as $\lambda_1(v) + \sum_{u \in A(v)} \lambda_1(v, u) = K_1 \forall v \in V$ and $A(v)$ is the set of adjacent vertices to given vertex v where $\lambda_1(v)$ is the label applied on the vertex and $\lambda_1(v,u)$ is the label applied on the edge $(v,u) \forall u \in A(v)$. They applied vertex magic total labeling on cycles, paths and for some complete graphs. They evinced some properties of these graphs with vertex magic total labeling. They proved that the wheel w_n has a vertex magic total labeling if and only if $n <= 11$. Consequently, outputs are found for wheels with multiple centers and several other generalizations of wheels. The total classification of vertex magic total labeling for the wheels $W_{3,} W_4$ and W_5 reveals unexpectedly large numbers.

NarsinghDeo[5] investigated wheel graphs, fan graphs, t-fold wheels, and friendship graphs and in all cases an upper limit for magic constant is found for the specific size of graphs which permit a vertex magic total labeling. As well, the spectrum of all wheels is investigated, and enumeration of all distinct labeling of wheels is provided. I.Gray, J. MacDougall, R. Simpson, and W. Wallis[6] said that vertex magic total labeling on a graph *G*is one to one map λ from *V (G) ∪ E(G)* onto the integers 1,2… |*V(G)* ∪ *E(G)*| with the property that given *x* ,λ(x)+ $\sum_{y\text{-}x}$ λ(y)=k for some magic constant. They completely determine which complete bipartite graphs have vertex magic total labeling. A lower bound for a VMTL is obtained by applying the largest |*V* | labels to the vertices, while an upper bound is formed by applying the smallest |*V* | labels to the vertices. The Equation 2.1 consigns lower and upper bound for vertex magic constant without taking into account the structure of the graph [7].

$$\frac{13n^2 + 11n + 2}{2(n+1)} \leq k \leq \frac{17n^2 + 15n + 2}{2(n+1)} \qquad 2.1$$

This is a vertex magic constant k limit equation. Once the structure of the graph is taken into account, auxiliary limits may be found. The set of integers which are delimited by these upper and lower bounds is the realistic range. The values which are the magic constant for some VMTL of a graph form the graph's spectrum. Therefore the spectrum is a subset of the sensible range. Equation 2.2 gives limit for a cycle given by H.R. Andersen et al. [8] in 2002.

$$\frac{5n+3}{2} \leq k \leq \frac{7n+3}{2} \qquad 2.2$$

Computational methods for solving labeling problem are described in [9]. In [10] James M. McQuillan1 and Dan McQuillan presented a new algorithm for finding vertex magic total labeling of disjoint unions of triangles. They use an exclusive algorithm described to find labeling with very restrictive properties and then ventured to produce other labeling from these. They show constructively that there exists a vertex magic total labeling, VMTL for each of the feasible values of $7c_3$,$9C_3$ and $11C_3$.

Mirka Miller, Chris Rodger, Rinovia Simanjuntak [14] introduced a new magic labeling whose evaluation is based on the neighborhood of a vertex. They completely solve the extant problem of 1 vertex magic vertex labeling for all complete bipartite, tripartite and regular multi partite graphs. Daisy Cunningham [15] addressed labeling graphs in such a way that the sum of the vertex labels and incident edge labels are the same for every vertex. Bounds on this so called magic number are found for cycle graphs. They exhibited some algorithms for finding vertex magic cycle graphs with a magic number that lies within the bounds.

ANTIMAGIC LABELING FOR CERTAIN FAMILIES OF GRAPHS

In magic labeling, every graph element can be given with a label such that the weight of each is same and is a magic constant. It may seem strange to term a graph as having “antimagic"labeling, but the term comes from its connection to magic labelings and magic squares. A magic square is an arrangement of numbers into a square such that the sum of each row, column and diagonal are equal. The term “antimagic"then comes from being the opposite

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of magic, or arranging numbers in a way such that no two sums in pair wise are equal. Antimagic labeling, every graph element can be given with a labeling such that the weight of each graph element is pair wise different.

This chapter consist detailed description of various antimagic algorithms applied on different classes of graphs. As the labeling sequence depends on the structure of graph, each graph requires its own algorithm even though the basic idea used is common. The concept of variations is used to generate labeling sequence. With help of heuristics all possible labeling sequences are generated and verified weather they forms any kind of antimagic labeling sequence or not. If it produces any kind of antimagic labeling, depending upon the graph structure these sequences are applied to components of graph in order to produce a particular kind of antimagic labeling.

Labeling is a process of assignment of labels to edges, vertices, or both edges and vertices of a graph. Based on the graph element on which labels will be applied, antimagic labeling can be categorized as vertex antimagic labeling and edge antimagic labeling. Vertex antimagic labeling is the process of assigning labels to edges of graph and weight of vertex is calculated based on edge labels and is pair wise distinct. The weight itself acts as a label to vertex. In similar way Edge antimagic labeling is the process of assigning labels to vertices of graph and weight of edge is calculated based on edge labels and is pair wise distinct.

Antimagic labeling can be strong or weak. If weight of each vertex/edge calculated is pair wise distinct it can be called as antimagic graph. But based on the weights, labeling can be categorized as strong or weak. If the weights have any duplicates it will be weak antimagic otherwise strong antimagic.

Figure 3.1 is an example of strong vertex antimagic labeling on complete graphK_4. Note that the weight of vertex itself acting as a label to it. The weight of vertex is the sum of the labels of edges incident to that vertex. In example the vertex labeled as 6 is its weight i.e., the sum of labels (2+1+3) assigned to its incident edges. In the similar way label 11 is 2+4+5, 12 is

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5+1+6 and 13 is 3+4+6. As all these weights are distinct, hence it is an example of strong vertex antimagic labeling.

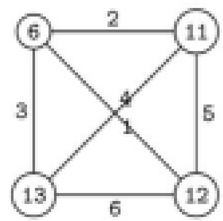

Figure 3.1: An example for Strong vertex antimagic labeling on complete graph K_4

Figure 3.2 is an example of weak vertex antimagic labeling on the complete graph K_4. Note that the weight of vertex itself acting as a label to it. The weight of vertex is the sum of the labels of edges incident to that vertex. In example the vertex labeled as 13 is its weight i.e., the sum of labels (3+6+4) assigned to its incident edges. In similar fashion 10 is 3+5+2, 8 is 2+5+1 and 10 is 1+5+4. As all these weights are pair wise distinct but has duplicates labels. So, it is an example of weak vertex antimagic labeling.

Conclusion

Introduction of graph labeling and its applications. Various kinds of graph labeling and graph structures used in this thesis have been introduced. a complete survey of magic and antimagic labeling algorithms is given briefly. Existing work and results are discussed. Chapter 3 consists of research methodology followed in this thesis. Problem definition, objectives, applications, proposed methodology and implementation details are discussed here. Several authors proposed various theorems to prove Antimagic labeling for particular structure of graph. But so far no generalized approach proposed to generate AMTLs for various structures of graph. This thesis made an attempt to address this problem. A new algorithm is designed to generate all possible non isomorphic vertex antimagic edge labelings, vertex antimagic total labeling, $(a\ d,\ )$ −vertex antimagic labelings, super vertex antimagic labelings, edge antimagic vertex labelings, edge antimagic total labeling, $(a\ d,\ )$ −edge antimagic labeling and super edge

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antimagic labelings for specific structures of graph like cycles, wheels, fan graphs and friendship graphs.

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