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The Total Edge-to-Vertex Steiner Number of a Graph

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ABSTRACT—An edge-to-vertex Steiner set W of a connected graph G is called a *total edge-to-vertex Steiner* set if $<W>$ has no isolated edges. The minimum cardinality of a total edge-to-vertex Steiner set of G is a *total edge-to-vertex Steinernumber* and is denoted by $s_{tev}(G)$.Some general properties satisfied by this concept are studied. Some of the standard graphs are determined.If p, a and b are positive integers such that $4 \leq a \leq b$ then there exists a connected graph G such that $s_{ev}(G) = a$ and $s_{tev}(G) = b$,where $s_{ev}(G)$is aedge-to-vertex Steiner number of a graph.

Keywords—total edge-to-vertex Steiner number, edge-to-vertex Steiner number, edge Steiner number Steiner number.

AMS Subject Classification: 05C12.

I. INTRODUCTION

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. The distance $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u - v$ path in G. An $u - v$ path of length $d(u, v)$ is called an $u - v$ geodesic. For basic graph theoretic terminology, we refer to Buckley F and Harary F [2].

For a non-empty set W of vertices in a connected graph G, the Steiner distance $d(W)$ of W is the minimum size of a connected subgraph of G containing W. Necessarily, each such subgraph is a tree and is called a Steiner tree with respect to W or a Steiner W-tree. It is to be noted that $d(W) = d(u, v)$,when $W = \{u, v\}$. The set of all vertices of G that lie on some Steiner W-tree is denoted by $S(W)$.If $S(W) = V$, then W is called a Steiner set for G. A Steiner set of minimum cardinality is a minimum Steiner set or simply a s-set of G and this cardinality is the Steiner number $s(G)$ of G. The Steinernumber of a graph was introduced and studied in [4]. Let G be a connected graph with at least 2 vertices. An *edge Steiner set* of G is a set $W \subseteq V(G)$ such that every edge of G is contained in a Steiner W-tree. The *edge Steiner number* $s_e(G)$ is the minimum cardinality of its edge Steiner sets and any edge Steiner set of cardinality$s_e(G)$ is a *minimum edge Steiner* set of G.This concept is introduced in [4].

Let $G = (V, E)$ be a connected graph with at least 3 vertices. For a non-empty set W of edges in a connected graph in G, the edge-to-vertex Steiner distanced_{ev}(W) of W is the minimum size of a tree containing $V(W)$ and is called an edge-to-vertex Steiner tree with respect to W or a Steiner W_{ev}-tree of G. For a given set $W \subseteq E(G)$, there may be more than one Steiner W_{ev} - tree of G. A set $W \subseteq E$ is called an edge-to-vertex Steiner set if every vertex of G lies on a Steiner W_{ev} -tree of G. The edge-to-vertex Steiner number $s_{ev}(G)$ of G is the minimum cardinality of its edge-to-vertex Steiner sets and any edge-to-vertex Steiner sets of cardinality$s_{ev}(G)$ is a minimum edge-to-vertex Steiner set of G.

The following theorems are used in the sequel.

Theorem 1.1. Each extreme vertex of a graph G belongs to every edge Steiner set of G.

II THE TOTAL EDGE-TO-VERTEX STEINER NUMBER OF A GRAPH

Definition 2.1. An edge-to-vertex Steiner set W of a connected graph G is called a *total edge-to-vertex*

Steiner set if <W > has no isolated edges. The minimum cardinality of a total edge-to-vertex Steiner set of G is a *total edge-to-vertex Steinernumber* and is denoted by $s_{tev}(G)$.

Example 2.2. For the graph G in Figure 2.1, Let $W = \{v_1v_2, v_1v_8, v_4v_5, v_5v_6\}$ is an edge-to-vertex Steiner set of G. Since <W> has no isolated edges, W is a total edge-to-vertex Steiner set of G and so $s_{tev}(G) \leq 4$. It is easily verified that no two element or three element subsets of G is a total edge-to-vertex Steiner set of G. Therefore $s_{tev}(G) = 4$.

Theorem 2.4. For a connected graph G, $2 \leq s_{ev}(G) \leq s_{tev}(G) \leq q$.

Proof. Since any edge-to-vertex Steiner set at least two edges $s_{ev}(G) \geq 2$. Let W be

a total edge-to-vertex Steiner set of G so that $s_{ev}(G) = |W|$. Since W is alsoanedge-to-vertex Steiner set of G, it is clear that $s_{ev}(G) \leq s_{tev}(G) = |W|$. Hence $s_{ev}(G) \leq s_{tev}(G)$. Since the edge $E(G)$ is a total edge-to-vertex Steiner set of G, $s_{tev}(G) \leq q$. Thus $2 \leq s_{ev}(G) \leq s_{tev}(G) \leq q$.

Theorem 2.5. If v is an extreme vertex of a connected graph G, then every total edge-to-vertex Steiner set contains at least one extreme edge that is incident with v.

Proof. This followsfrom Theorem 2.4.

Theorem 2.6. Eachend edge of a connected graph G lies in every total edge-to-vertex Steiner set of G.

Proof. This followsfrom Theorem 2.5.

Theorem 2.7. Let G be a connected graph and e be an end edge of G. Then every total edge-to-vertex Steiner set contains at least one adjacent edge e of G.

Proof. Let e be an end edge of G and $e_1, e_2, \ldots, e_k$ $(k \geq 1)$ be the adjacent edges of G. Let W be a total edge-to-vertex Steiner set of G. By Theorem 4.6, $e \in W$. We

prove that W contains at least one $e_i (1 \leq i \leq k)$. If at least one $e_i (1 \leq i \leq k)$ is an end edge, then by Theorem 2.6, $e_i \in W$ for $(1 \leq i \leq k)$ so let us assume that no $e_i (1 \leq i \leq k)$ is an end edge of G. We have to prove W contains at least one $e_i (1 \leq i \leq k)$. If not suppose W contains no $e_i (1 \leq i \leq k)$, then $< W >$ has an isolated edge, which is a contradiction. Therefore every total edge-to-vertex Steiner set contains at least one adjacent edge e of G.

Theorem 2.8. For the complete graph $G = K_p$ $(p \geq 5)$, $s_{tev}(K_p) = p - 1$.

Proof. Let v be a vertex of G and $v_1, v_2, \ldots, v_{p-1}$ be the adjacent vertices of v. Then $S = \{vv_1, vv_2, \ldots, vv_{p-1}\}$ is a total edge-to-vertex Steiner set of K_p so that $s_{tev}(K_p) \leq p - 1$. We prove that $s_{tev}(G) = p - 1$. Suppose that $s_{tev}(G) \leq p - 2$. Then there exists a total edge-to-vertex Steiner set W such that $|W| \leq p - 2$. Let $e = uv$ be an edge in $K_p$such that $e \notin W$. Then either u or v lies on any Steiner W_{ev} -tree of K_p and so W is not a total edge-to-vertex Steiner set of G so that $s_{tev}(K_p) = p - 1$.

Theorem 2.9. For a cycle C_p $(p \geq 6)$,

$$s_{tev}(C_p) = \begin{cases} 4 & \text{if } p \text{ is even} \\ 5 & \text{if } p \text{ is odd} \end{cases}$$

Proof. Let p be even. Let u and v be two antipodal vertices of C_p. Let u_1 and u_2 bethe adjacent vertices of u and let v_1 and v_2 be the two adjacent vertices of v. Then S = $\{uu_1, uu_2, vv_1, vv_2\}$ is a total edge-to-vertex Steiner set of G and so $s_{tev}(C_p) \leq 4$. It is easily verified that no two elements or three elements subset of C_p is a total edge-to- vertex Steiner set of G. Hence $s_{tev}(C_p)$= 4.

Suppose that p is odd. Let v and w be two antipodal vertices of u. Let u_1 and u_2 be the adjacent vertices of u and v_1 is the adjacent vertices of v and w_1 is adjacent to w such that $v_1 \neq$ w and $w_1 \neq v$. Then S = $\{uu_1, uu_2, vv_1, ww_1\}$ is a total edge-to-vertex Steiner set of G and so $s_{tev}(C_p) \leq 5$. It is easily verified that no two element or three element subset of C_p is a total edge-to-vertex Steiner set of G. Hence $s_{tev}(C_p)$ = 5.

Theorem 2.10. For a complete bipartite graph $G = K_{m,n}$ $(2 \leq m \leq n)$, $s_{tev}(G) = m + n - 2$.

Proof. Let $X = \{x_1, x_2, \ldots, x_m\}$, and $Y = \{y_1, y_2, \ldots, y_n\}$ be the bipartite sets of G. Let $W = \{x_1y_1, x_2y_2, \ldots, x_my_m, y_1x_2, y_1x_3, \ldots, y_1x_m, x_my_{m+1}, x_my_{m+2}, \ldots, x_my_n\}$. Then W is a total edge-to-vertex Steiner set of G so that $s_{tev}(G) \leq m + n - 2$ .We prove that $s_{tev}(G) = m + n - 2$. Suppose that $s_{tev}(G) \leq m + n - 3$. Then there exist a total edge-to-vertex Steiner set of W' such that $|W'| \leq m + n - 3$. Let $e = xy$ be an edge in $K_{m,n}$ such that e$\notin W'$. Then either x or y does lies on any Steiner W_{ev}-tree of $K_{m,n}$ and so W' is not a total edge-to-vertex Steiner set of G so that $s_{tev}(G) = m + n - 2$.

Theorem 2.11. For any connected graph G, $s_{tev}(G)$= 2 if and only if G is either $G = P_3$ or K_3.

Proof. Let G is either P_3 or K_3, then it is easily verified that $s_{tev}(G)$= 2. Conversely, let $s_{tev}(G)$= 2. Let $W = \{uv, wz\}$ be a total edge-to-vertex Steiner set of G. Since <W> has no isolated edges, there must be a vertex common to uv and wz. Let us assume that $v = w$. If $u$and w are not adjacent, then $= P_3$. So we have done. If u and w are adjacent, then $= K_3$. So we have done.

III. CONCLUSIONS

With the contribution of the edge-to-vertexSteiner number of a graph, we can introduce the total edge-to-vertexSteiner number $s_{tev}(G)$.the total edge-to-vertex Steiner number of certain graphs can be studied with some parameters.

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