

METHODS OF SOLVING FIRST ORDER AND FIRST DEGREE EQUATION

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Abstract:

This paper can deal with the topic under Numerical Methods. In these methods, we analyzed under the topic of Methods to find result of first order and first degree equation using various single step method, like method of Taylor's series, Method of Rungekutta, and Predictor corrector method by Milne's Thomson and by Adam's Basforthmethod. From this we solve some problems with the same initial condition we get different solution for different methods. We compare this all and suggest which one the best method to find the solution of is given equation. Also we derive the formula for the above mentioned single step methods. A various issue in science and innovation can be figured into differential equations. The systematic approaches for addressing differential conditions are relevant just to a restricted class of conditions. It also used in solving the problems rose in medicine, business and also in engineering.

Keywords: Runge-Kutta Method, Taylor's Series method, Predictor- corrector method, Milne's Thomson, Adam's Basforth method

INTRODUCTION

A various issue in science and innovation can be figured into differential equations. The systematic approaches for addressing differential conditions are relevant just to a

restricted class of conditions. Quite regularly the differential equation acting in physical solving does not be a no longer belongs to those acquainted kinds and one is obliged to turn to mathematical techniques. These techniques are of much more significant whilst we recognize that the processing machines are presently promptly accessible which lesser numerical work significantly. Numerical methods for differential equations are of great importance to the engineers and physicists because practical problems often lead to differential equations that cannot be solved by analytical methods. We have already seen methods for finding of to any desired degree of accuracy the roots of any algebraic or transcendental equations having numerical coefficients. Now we will discuss on various methods for finding, to any desired degree of accuracy of the numerical solution of any ordinary differential equation with given initial conditions.

First order Initial Value Problem

Let us consider first order differential equation

$$\frac{dy}{dx} = f(x, y), y(x_0) = y_0$$

Then the solution of Initial value problem can be obtained in two methods.

(i) Single step method

In these methods we use information about the curve at one point and we do not iterate the solution. This method involves more evaluation of the function.

The numerical solution of Ordinary differential equation by

- a) Taylor series Method
- b) Euler Method
- c) Runge-kutta Method.

All require the information at a single point $x = x_0$

(ii) Multi-step Method

These methods required fewer. Evaluations of the function (Past 4 values) of the $f(x)$ to estimate the solution at a point and iterations are performed till sufficient accuracy is achieved. Estimation of error is possible and the methods are called Predictor-Corrector method.

In this type, we commonly use

- a) Milne's Method
- b) Adams-Bahforth Method and Adams-Moulton Method

1.1.Contributions of Numerical Methods

The standard purpose of numerical strategies is the layout and evaluation of strategies to offer rough, however correct answers to difficult sums, the type which might be given underneath.

- a. An innovative numerical strategy is important in creating mathematical climate forecast plausible.
- b. Computation in direction of an aircraft needs the accurate solution of numerical system on ODE.
- c. Some group of cars will develop their vehicles by utilizing the system of reproductions of vehicle crashes. Such recreations are basically comprised of deriving differential conditions partially in numerical method.
- d. The reserves of Hedge (private financial reserves) were used their fields by to determine some worth of stocks , as well as the other members of market are the derivatives by using the methods numerical.
- e. In airlines we used some modern advance calculation to choose the cost of tickets, planes, group tasks, and also the needs of fuel. This deflect is known as operation research.
- f. Some organizations like insurance companies utilize the projects in numerical for specializers examination. The entire requires a superior strategy that will limit the calculation of errors. The few strategies for deriving differential conditions having a coefficient of numerical with beginning or limit conditions.

2. Problems Using Single Step Method

Using the Single step method we can find the solution using Taylor's series method. Here we discussed the formula for Taylor's series, Runge-kutta method, Predictor- corrector formula using Basforth and Milne's method and the problems using above methods are discussed here.

Problems on Taylor's Series

Using Taylor series method, compute the value of $y(0.2)$ correct to 3 decimal places from

$$\frac{dy}{dx} = 1 - 2xy \text{ given that } y(0) = 0.$$

Solution:

$$\text{Given } y' = 1 - 2xy$$

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$$\text{and } y(0) = 0$$

$$\text{i.e., } x_0 = 0, y_0 = 0$$

To find y(0.1)

We know that

$$y_1 = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \dots \quad (1)$$

$$y' = 1 - 2xy$$

$$y_0' = 1 - 2x_0y_0$$

$$y_0' = 1 - 2(0)(0) = 1 - 0$$

$$y_0' = 1$$

$$y'' = -2[xy' + y]$$

$$y_0'' = -2[x_0y_0' + y_0]$$

$$y_0'' = -2[(0)(1) + 0] = -2(0)$$

$$y_0'' = 0$$

$$y''' = -2[xy'' + 2y']$$

$$y_0''' = -2[x_0y_0'' + 2y_0']$$

$$y_0''' = -2[(0)(0) + 2(1)] = -2(2)$$

$$y_0''' = -4$$

Taking $h = 0.1$ and substituting (2),(3) and (4) in (1), we get

$$y(0.1) = 1 + \frac{0.1}{1!}(1) + \frac{(0.1)^2}{2!}(0) + \frac{(0.1)^3}{3!}(-4) + \dots$$

$$y(0.1) = 1 + 0.1 - 0.000667$$

$$y(0.1) = 0.099333$$

To find y(0.2)

$$y_2 = y_1 + \frac{h}{1!}y_1' + \frac{h^2}{2!}y_1'' + \frac{h^3}{3!}y_1''' + \dots \quad (1)$$

$$y' = 1 - 2xy$$

$$y_1' = 1 - 2x_1y_1$$

$$y_1' = 1 - 2(0.1)(0.099333)$$

$$y_1' = 0.9801334$$

$$y'' = -2[xy' + y]$$

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$$y_0'' = -2[x_1 y_1' + y_1]$$

$$y_0'' = -2[(0.1)(0.9801334) + 0.09933]$$

$$y_0'' = 0 - 0.3926868$$

$$y_0''' = -2[xy'' + 2y']$$

$$y_0''' = -2[x_1 y_1'' + 2y_1']$$

$$y_0''' = -2[(0.1)(-0.3946868) + 2(0.9801334)]$$

$$y_0''' = -3.84159$$

Taking $h = 0.1$ and substituting (6),(7) and (8) in (5), we get

$$y_2 = y(0.2) = 0.09933 + \frac{0.1}{1!} (0.9801334) + \frac{(0.1)^2}{2!} (-0.3946868) + \frac{(0.1)^3}{3!} (-3.84159) + \dots$$

$$y(0.2) = 0.1954$$

Suppose if we take $h=0.2$ directly, we get

$$\begin{aligned} y_1 = y(0.2) &= y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \dots \\ &= 0 + (0.2)(1) + \frac{(0.2)^2}{2!} (0) + \frac{(0.2)^3}{3!} (-4) + \dots \end{aligned}$$

$$y(0.2) = 0.19467$$

By using Taylor's series we get the result of approximation

x	0	0.1	0.2
y	0	0.09933	0.1954

3. Using Fourth order Runge-Kutta Method**Formula**

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$\Delta y = \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4]$$

Here $x_0 = 0, y_0 = 0, h=0.1$

*Research Paper***To find $y(0.1)$ using Fourth order Runge-Kutta method**

$$k_1 = (0.1)(1 - 2x_0y_0)$$

$$= (0.1)(1 - 2(0)(0))$$

$$\boxed{k_1 = 0.1}$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_2 = (0.1)f\left(0 + \frac{0.1}{2}, 0 + \frac{0.1}{2}\right)$$

$$= (0.1)f(0.05, 0.05)$$

$$= (0.1)(1 - 2(0.05)(0.05))$$

$$= (0.1)(1 - 0.005)$$

$$= (0.1)(0.995)$$

$$\boxed{k_2 = 0.0995}$$

$$k_3 = (0.1)f\left(0 + \frac{0.1}{2}, 0 + \frac{0.0995}{2}\right)$$

$$= (0.1)f(0.05, 0.04975)$$

$$= (0.1)(1 - 2(0.05)(0.04975))$$

$$= (0.1)(1 - 0.004975)$$

$$= (0.1)(0.9950)$$

$$\boxed{k_3 = 0.09950}$$

$$k_4 = (0.1)f(0 + 0.1, 0 + 0.09950)$$

$$= (0.1)f(0.1, 0.09950)$$

$$= (0.1)(1 - 2(0.1)(0.09950))$$

$$= (0.1)(1 - 0.01990)$$

$$= (0.1)(0.9801)$$

$$\boxed{k_4 = 0.09801}$$

$$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= \frac{1}{6}[0.1 + 2(0.0995) + 2(0.0995) + 0.09801]$$

$$= \frac{1}{6}[0.1 + 0.199 + 0.199 + 0.09801]$$

$$= \frac{1}{6}(0.59601)$$

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$$\Delta y = 0.099335$$

$$y_1 = y_0 + \Delta y$$

$$y(0.1) = 0 + 0.099335$$

$$y_1 = 0.099335$$

To find $y(0.2)$ using Fourth order Runge-Kutta method

Here $x_1 = 0.1, y_1 = 0.099335$

$$k_1 = hf(x_1, y_1)$$

$$k_1 = (0.1)(1 - 2x_1y_1)$$

$$k_1 = (0.1)(1 - 2(0.1)(0.099335))$$

$$= (0.1)(1 - 0.019867)$$

$$k_1 = (0.1)(0.98013)$$

$$k_1 = 0.098013$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$k_2 = (0.1)f\left(0.1 + \frac{0.1}{2}, 0.099335 + \frac{0.098013}{2}\right)$$

$$k_2 = (0.1)f(0.1 + 0.05, 0.099335 + 0.04900)$$

$$= (0.1)f(0.15, 0.148335)$$

$$= (0.1)(1 - 2(0.15)(0.148335))$$

$$= (0.1)(0.97775)$$

$$k_2 = 0.097775$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= (0.1)f\left(0.1 + \frac{0.1}{2}, 0.099335 + \frac{0.097775}{2}\right)$$

$$= (0.1)f(0.1 + 0.05, 0.099335 + 0.04888)$$

$$= (0.1)f(0.15, 0.14822)$$

$$= (0.1)(1 - 2(0.15)(0.14822))$$

$$= (0.1)(1 - 0.044466)$$

$$= (0.1)(0.955534)$$

$$k_3 = 0.09555$$

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$$\begin{aligned}
 k_4 &= hf(x_1 + h, y_1 + k_3) \\
 &= (0.1)f(0.1 + 0.1, 0.099335 + 0.09555) \\
 &= (0.1)f(0.2, 0.194885) \\
 &= (0.1)(1 - 2(0.2)(0.194885)) \\
 &= (0.1)(1 - 0.077954) \\
 &= (0.1)(0.92204)
 \end{aligned}$$

$$k_4 = 0.09220$$

$$\begin{aligned}
 \Delta y &= \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] \\
 &= \frac{1}{6}[0.098013 + 2(0.097775) + 2(0.09555) + 0.09220] \\
 &= \frac{1}{6}[0.098013 + 0.19555 + 0.1911 + 0.09220] \\
 &= \frac{1}{6}[0.576863]
 \end{aligned}$$

$$\Delta y = 0.09614$$

$$\therefore y_2 = y_1 + \Delta y$$

$$y(0.2) = 0.09933 + 0.09614$$

$$y_2 = 0.19547$$

By using Fourth order Runge-Kutta method we get the result of the approximation

X	0	0.1	0.2
Y	0	0.09950	0.19458

4. Using Milne's Predictor Corrector Method

$$\text{Given } \frac{dy}{dx} = y' = 1 - 2xy \text{ and } h = 0.1$$

We know that from the method of Taylor's series

$$x_0 = 0, y_0 = 0$$

$$x_1 = 0.1, y_1 = 0.09933$$

$$x_2 = 0.2, y_2 = 0.1954$$

Now we use Taylor's series method to get y_3 value

$$y_3 = y_2 + \frac{h}{1!}y_2' + \frac{h^2}{2!}y_2'' + \frac{h^3}{3!}y_2''' + \dots$$

$$\text{Here } y' = 1 - 2xy$$

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$$\begin{aligned}y_2' &= 1 - 2x_2y_2 \\ &= 1 - 2(0.2)(0.1954) \\ &= 1 - 0.07816\end{aligned}$$

$$y_2' = 0.92184$$

$$\begin{aligned}y_2'' &= -2[xy' + y] \\ y_2'' &= -2[x_2y_2' + y_2] \\ &= -2[(0.2)(0.92184) + 0.1954] \\ &= -2[0.18436 + 0.1954] \\ &= -2[0.37976]\end{aligned}$$

$$y_2'' = -0.75952$$

$$\begin{aligned}y_2''' &= -2[x_2y_2'' + 2y_2'] \\ &= -2[(0.2)(-0.75952) + 2(0.92184)] \\ &= -2[-0.15190 + 1.84368] \\ &= -2[1.69178]\end{aligned}$$

$$y_2''' = 3.38356$$

Taking $h = 0.1$ and then find y_3

$$\begin{aligned}y_3 &= y(0.3) = 0.1954 + (0.1)(0.92184) + \frac{(0.1)^2}{2!}(-0.75952) + \frac{(0.1)^3}{3!}(-3.38356) + \dots \\ &= 0.1954 + (0.1)(0.92184) + \frac{0.01}{2}(-0.75952) + \frac{0.001}{6}(-3.38356) + \dots \\ &= 0.1954 + 0.092184 - 0.0037976 - 0.0005639 + \dots\end{aligned}$$

$$y_3 = 0.28322$$

Solve by Milne's Predictor-Corrector

We know that

$$x_0 = 0, y_0 = 0$$

$$x_1 = 0.1, y_1 = 0.09933$$

$$x_2 = 0.2, y_2 = 0.1954$$

$$x_3 = 0.3, y_3 = 0.28322$$

We use the formula of Milne's Predictor method

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}[2y_{n-2}' - y_{n-1}' + 2y_n'] \quad (4.4.1)$$

To get y_4 , put $n = 3$ in (4.4.1) we get

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$$y_{4,p} = y_0 + \frac{4h}{3} [2y'_1 - y'_2 + 2y'_3] \quad (4.4.2)$$

$$\begin{aligned} y'_1 &= 1 - 2x_1y_1 \\ &= 1 - 2(0.1)(0.09933) \\ &= 1 - 0.019866 \end{aligned}$$

$$y'_1 = 0.980134$$

$$\begin{aligned} y'_2 &= 1 - 2x_2y_2 \\ &= 1 - 2(0.2)(0.1954) \\ &= 1 - 0.07816 \end{aligned}$$

$$y'_2 = 0.92184$$

$$\begin{aligned} y'_3 &= 1 - 2x_3y_3 \\ &= 1 - 2(0.3)(0.28322) \\ &= 1 - 0.169932 \end{aligned}$$

$$y'_3 = 0.830068$$

Substituting $y'_1, y'_2,$ and y'_3 in (4.4.2) we get

$$\begin{aligned} y_{4,p} &= 0 + \frac{4(0.1)}{3} [2(0.980134) - 0.92184 + 2(0.83006)] \\ &= 0.13333[1.960268 - 0.92184 + 1.66012] \\ &= 0.13333[2.698548] \end{aligned}$$

$$y_{4,p} = 0.35979$$

5. Conclusion

we analyzed to find the ordinary differential equation of solution using various methods in numerical. we discussed the solution of approximations using Taylor's series method, also we discussed the same problem by using Runge-Kutta Method and we used Milne's Predictor-corrector Method & Adams Predictor-Corrector Method to find the approximations. From the above solving methods we conclude that

The Results of the approximations are shown in the table

Method	First approximation Y(0.1)	Second approximation Y(0.2)
Taylor's Series	0.09933	0.1954
Fourth order RungeKutta	0.09950	0.19458

method		
Milne's Predictor-corrector method	0.35979	0.35979

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