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Intuitionistic Pre * Open Maps in Intuitionistic Topological Spaces

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Abstract

The major goal of this work is to introduce the concepts of Intuitionistic Pre * Open maps, Intuitionistic Pre * Closed maps and their contra versions in ITS using the concepts of Intuitionistic Pre * Open and Intuitionistic Pre * Closed sets. Further we give characterization for these maps and discuss the relationship with other known intuitionistic maps. Also we find the equivalent conditions for Intuitionistic Pre * Open maps. We continue to look into the connection to Intuitionistic Pre open maps and Intuitionistic Regular * open maps in ITS.

Keywords: Intuitionistic Pre * Open map, Intuitionistic Pre * Closed map, Contra Intuitionistic Pre * Open map, Contra Intuitionistic Pre * Closed map.

AMS subject classification (2010): 54C05.

1. Introduction

In 1996, D. Coker [1] introduced the concept of intuitionistic sets and also he has introduced the concept of intuitionistic topological spaces. In 2016 G. Sasikala and M. Navaneethakrishnan [4] defined intuitionistic Pre open sets in intuitionistic topological spaces. We [5] gives the definition of intuitionistic pre * open sets in intuitionistic topological spaces.

In this study, we define intuitionistic pre * Open maps, intuitionistic pre * Closed maps and their contra versions. We also demonstrate that the intuitionistic pre * open map is intermediate between intuitionistic open and intuitionistic pre open maps.

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2. Preliminaries

Definition - 2.1. Let $\grave{X}$ be a non-empty set, an *intuitionistic set* (IS in short) $\ddot{A}$ is an object having the form $\ddot{A} = <\grave{X},\ddot{A}_1,\ddot{A}_2>$, where $\ddot{A}_1$ and $\ddot{A}_2$ are subsets of $\grave{X}$ satisfying $\ddot{A}_1 \cap \ddot{A}_2 =\phi$. The set $\ddot{A}_1$ and $\ddot{A}_2$ are called the set of members of $\ddot{A}$ and set of non-members of $\ddot{A}$ respectively.

Definition - 2.2. Let $\grave{X}$ be a non-empty set, $\ddot{A} = <\grave{X},\ddot{A}_1,\ddot{A}_2>$ and $\ddot{B} = <\grave{X},\ddot{B}_1,\ddot{B}_2>$ be an IS's and let $\{\ddot{A}_i : i \in J\}$ be arbitrary family of IS's, where $\ddot{A} = <\grave{X},\ddot{A}_1,\ddot{A}_2>$. Then the followings are hold.

a) $\ddot{A} \subseteq \ddot{B}$ iff $\ddot{A}_1 \subseteq \ddot{B}_1$ and $\ddot{A}_2 \supseteq \ddot{B}_2$.

b) $\ddot{A} = \ddot{B}$ iff $\ddot{A} \subseteq \ddot{B}$ and $\ddot{A} \supseteq \ddot{B}$.

c) $\ddot{A}^c = <\grave{X},\ddot{A}_2,\ddot{A}_1>$ is called the complement of $\ddot{A}$ and $\ddot{A}^c$ is also denoted by $\grave{X} - \ddot{A}$.

d) $\cup \ddot{A}_i = <\grave{X},\cup \ddot{A}_{i1},\cap \ddot{A}_{i2}>$.

e) $\cap \ddot{A}_i = <\grave{X},\cap \ddot{A}_{i1},\cup \ddot{A}_{i2}>$.

f) $\ddot{A} - \ddot{B} = \ddot{A} \cap \ddot{B}^c$.

g) $\ddot{\phi}_I = <\grave{X},\phi,\grave{X}>$ and $\ddot{X}_I = <\grave{X},\grave{X},\phi>$.

Definition - 2.3. Let $\grave{X}$ be a non-empty set and τ_{IT} be the family of intuitionistic sets of $\grave{X}$ then τ_{IT} is called an *intuitionistic topology* (IT in short) on $\grave{X}$ if it is satisfying the following axioms:

1) $\ddot{X}_I, \ddot{\phi}_I \in \tau_{IT}$.

2) $\ddot{A} \cap \ddot{B} \in \tau_{IT}$ for every $\ddot{A}, \ddot{B} \in \tau_{IT}$.

3) $\cup \ddot{A}_i \in \tau_{IT}$ for any arbitrary family $\{\ddot{A}_i : i \in J\} \subseteq \tau_{IT}$.

The pair $(\grave{X},\tau_{IT})$ is called *intuitionistic topological space* (ITS in short) and IS in τ_{IT} is known as the intuitionistic open set (IOS in short) in $\grave{X}$, the complement of the IOS is called the intuitionistic closed set (ICS in short) in $\grave{X}$.

Definition - 2.4. $(\grave{X},\tau_{IT})$ be an ITS and $\ddot{A}$ be a IS in $\grave{X}$ then $\ddot{A}$ is said to be *intuitionistic generalized closed* (Ig- closed in short) set if $Icl(\ddot{A}) \subseteq \ddot{U}$ whenever $\ddot{A} \subseteq \ddot{U}$ and $\ddot{U}$ is IOS in $\grave{X}$. The complement of the Ig – closed set is called the *Ig- open set* in $\grave{X}$.

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Definition - 2.5. Let ($\grave{X}$,τ_{IT}) be an ITS and Ä be a IS in $\grave{X}$ then *intuitionistic generalized closure of* Ä is defined as the intersection of all Ig – closed sets in $\grave{X}$ containing Ä and is denoted by Icl*(Ä). (i.e) Icl*(Ä) = ∩ {$\ddot{G}$: $\ddot{G}$ is an Ig- closed set in $\grave{X}$ and Ä ⊆ $\ddot{G}$}.

Definition - 2.6. Let ($\grave{X}$,τ_{IT}) be an ITS and Ä be an intuitionistic set then

a) Ä is intuitionistic pre open (IPO) set in $\grave{X}$ if A ⊆ Iint(Icl(A)).

b) Ä is intuitionistic pre * open (IP*O) set in $\grave{X}$ if A ⊆ Iint(Icl*(A)).

c) Ä is intuitionistic regular * open (IR*O) set in $\grave{X}$ if A = Iint(Icl*(A)).

The complement of the IPO, IP*O and IR*O sets are called the IPC, IP*C and IR*C sets in $\grave{X}$.

Definition - 2.7. Let ($\grave{X}$,τ_{IT}) be an ITS and Ä be a IS in $\grave{X}$ then the intuitionistic interior operator of Ä (Iint(Ä) in short) and intuitionistic closure operator of Ä (Icl(Ä) in short) are defined by:

Iint(Ä) = ∪ {$\ddot{G}$: $\ddot{G}$ is an IOS in $\grave{X}$ and Ä ⊇ $\ddot{G}$}.

Icl(Ä) = ∩ {$\ddot{G}$: $\ddot{G}$ is an ICS in $\grave{X}$ and Ä ⊆ $\ddot{G}$}.

Theorem - 2.8. Let ($\grave{X}$,τ_{IT}) be an ITS then the followings are hold.

a) Every IO set is IP*O set.
b) Every IP*O set is IPO set.
c) Every IR*O set is IP*O set.

Theorem - 2.9. Let ($\grave{X}$,τ_{IT}) be an ITS and Ä and $\ddot{B}$be a IS of $\grave{X}$ then the followings are hold.

a) Iint($\ddot{\phi}_I$) = $\ddot{\phi}_I$ and Iint($\ddot{X}_I$) = $\ddot{X}_I$.

b) Ä is an IOS iff Ä = Iint(Ä).

c) Ä ⊆ $\ddot{B}$ then Iint(Ä) ⊆ Iint($\ddot{B}$).

d) Iint(Ä ∩ $\ddot{B}$) = Iint(Ä) ∩ Iint($\ddot{B}$).

e) Iint(Ä ∪ $\ddot{B}$) ⊇ Iint(Ä) ∪ Iint($\ddot{B}$).

f) Icl($\ddot{\phi}_I$) = $\ddot{\phi}_I$ and Icl($\ddot{X}_I$) = $\ddot{X}_I$.

g) Ä is an ICS iff Ä = Icl(Ä).

h) Ä ⊆ $\ddot{B}$ then Icl(Ä) ⊆ Icl($\ddot{B}$).

i) Icl(Ä ∩ $\ddot{B}$) ⊆ Icl(Ä) ∩ Icl($\ddot{B}$).

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j) $Icl(\ddot{A} \cup \ddot{B}) = Icl(\ddot{A}) \cup Icl(\ddot{B})$.

Theorem - 2.10. Let $(\grave{X},\tau_{IT})$ be an ITS and $\ddot{A}$ and $\ddot{B}$ be a IS of $\grave{X}$ then the followings are hold.

a) $IP^*int(\ddot{\phi}_I) = \ddot{\phi}_I$ and $IP^*int(\ddot{X}_I) = \ddot{X}_I$.

b) If $\ddot{A}$ is IP*- open set then $\ddot{A} = IP^*int(\ddot{A})$.

c) $\ddot{A} \subseteq \ddot{B}$ then $IP^*int(\ddot{A}) \subseteq IP^*int(\ddot{B})$.

d) $IP^*cl(\ddot{\phi}_I) = \ddot{\phi}_I$ and $IP^*cl(\ddot{X}_I) = \ddot{X}_I$.

e) If $\ddot{A}$ is IP*- closed set then $\ddot{A} = IP^*cl(\ddot{A})$.

f) $\ddot{A} \subseteq \ddot{B}$ then $IP^*cl(\ddot{A}) \subseteq IP^*cl(\ddot{B})$.

Theorem - 2.11. Let $(\grave{X},\tau_{IT})$ be an ITS and $\ddot{A}$ be a IS of $\grave{X}$ then the followings are hold.

a) $Iint(\grave{X} - \ddot{A}) = \grave{X} - Icl(\ddot{A})$ and $Icl(\grave{X} - \ddot{A}) = \grave{X} - Iint(\ddot{A})$.

b) $IP^*int(\grave{X} - \ddot{A}) = \grave{X} - IP^*cl(\ddot{A})$ and $IP^*cl(\grave{X} - \ddot{A}) = \grave{X} - IP^*int(\ddot{A})$.

Theorem - 2.12. Let $f : \grave{X} \rightarrow \grave{Y}$ is said to be

a) I- continuous map if $f^{-1}(V)$ is IO set in $\grave{X}$ for every IO set V in $\grave{Y}$.

b) IP*- continuous map if $f^{-1}(V)$ is IP*O set in $\grave{X}$ for every IO set V in $\grave{Y}$.

c) IP*- irresolute map if $f^{-1}(V)$ is IP*O set in $\grave{X}$ for every IP*O set V in $\grave{Y}$.

d) I- open map if f(U) is IO set in $\grave{Y}$ for every IO set U in $\grave{X}$.

e) IP- open map if f(U) is IPO set in $\grave{Y}$ for every IO set U in $\grave{X}$.

f) IR*- open map if f(U) is IR*O set in $\grave{Y}$ for every IO set U in $\grave{X}$.

g) I- closed map if f(U) is IC set in $\grave{Y}$ for every IC set U in $\grave{X}$.

h) IP- closed map if f(U) is IPC set in $\grave{Y}$ for every IC set U in $\grave{X}$.

i) IR*- closed map if f(U) is IR*C set in $\grave{Y}$ for every IC set U in $\grave{X}$.

j) Contra I- open map if f(U) is IC set in $\grave{Y}$ for every IO set U in $\grave{X}$.

k) Contra IP- open map if f(U) is IPC set in $\grave{Y}$ for every IO set U in $\grave{X}$.

l) Contra IR*- open map if f(U) is IR*C set in $\grave{Y}$ for every IO set U in $\grave{X}$.

m) Contra I- closed map if f(U) is IO set in $\grave{Y}$ for every IC set U in $\grave{X}$.

n) Contra IP- closed map if f(U) is IPO set in $\grave{Y}$ for every IC set U in $\grave{X}$.

o) Contra IR*- closed map if f(U) is IR*O set in $\grave{Y}$ for every IC set U in $\grave{X}$.

3. Intuitionistic Pre * Open Maps

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Definition – 3.1. A map f from ITS $(\grave{X},\tau_{IT})$ into another ITS $(\grave{Y},\sigma_{IT})$ is called *Intuitionistic Pre * Open (in short IP*- Open) Map* if f(M) is IP*O set in $\grave{Y}$ for each IO set M in $\grave{X}$.

Definition – 3.2. A map f from ITS $(\grave{X},\tau_{IT})$ into another ITS $(\grave{Y},\sigma_{IT})$ is called *Contra Intuitionistic Pre * Open Map* if f(M) is IP*C set in $\grave{Y}$ for each IO set M in $\grave{X}$.

Example – 3.3. Let $\grave{X}$ = {a,b,c} and $\grave{Y}$ = {1,2,3}. Consider the IT's $\tau_{IT} = \{X_I,\phi_I,<\grave{X},\{a\},\{b,c\}>, <\grave{X},\{a,c\},\{b\}>\}$ and $\sigma_{IT} = \{Y_I,\phi_I,<\grave{Y},\{1\},\{3\}>,<\grave{Y},\{3\},\{1,2\}>,<\grave{Y},\{1,3\},\phi>\}$ then $IP^*O(\grave{Y}) = \{Y_I,\phi_I,<\grave{Y},\{1\},\{3\}>,<\grave{Y},\{1\},\phi>,<\grave{Y},\{1\},\{2,3\}>,<\grave{Y},\{3\},\{1,2\}>,<\grave{Y},\{1,3\},\phi>,<\grave{Y},\{1,3\},\{2\}>\}$.
Let $f : (\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ be a map defined by, f(a) = 1, f(b) = 2, f(c) = 3. Here, $f(X_I) = Y_I$, $f(\phi_I) = \phi_I$, $f(<\grave{X},\{a\},\{b,c\}>) = <\grave{Y},\{1\},\{2,3\}>$ and $f(<\grave{X},\{a,c\},\{b\}>) = <\grave{Y},\{1,3\},\{2\}>$ are IP*O sets in $\grave{Y}$. Therefore, f is IP*- open map.

Example – 3.4. Let $\grave{X}$ = {a,b,c} and $\grave{Y}$ = {1,2,3}. Consider the IT's $\tau_{IT} = \{X_I,\phi_I,<\grave{X},\{a\},\{b,c\}>\}$ and $\sigma_{IT} = \{Y_I,\phi_I,<\grave{Y},\{2\},\{1,3\}>,<\grave{Y},\{3\},\{1,2\}>,<\grave{Y},\{2,3\},\{1\}>\}$ then $IP^*C(\grave{Y}) = \{Y_I,\phi_I, <\grave{Y},\{1,3\},\{2\}>,<\grave{Y},\{1,2\},\{3\}>,<\grave{Y},\{1\},\{2,3\}>\}$. Let $f : (\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ be a map defined by, f(a) = 1, f(b) = 3, f(c) = 2. Here, $f(X_I) = Y_I$, $f(\phi_I) = \phi_I$ and $f(<\grave{X},\{a\},\{b,c\}>) = <\grave{Y},\{1\},\{2,3\}>$ are IP*C sets in $\grave{Y}$. Therefore, f is Contra IP*- open map.

Theorem – 3.5. Let $(\grave{X},\tau_{IT})$ and $(\grave{Y},\sigma_{IT})$ be an ITS then the followings are hold.

a) Every I- Open map is IP*- Open map.
b) Every IR*- Open map is IP*- Open map.
c) Every IP*- Open map is IP- Open map.
d) Every Contra I- open map is Contra IP*- open map.
e) Every Contra IR*- open map is Contra IP*- open map.

Proof: (a) Suppose a map $f : (\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ is I- open map. Let M be any IO set in $\grave{X}$ then f(M) is IO set in $\grave{Y}$. Since, every IO set is IP*O set. Therefore, f(M) is IP*O in $\grave{Y}$. Hence, f is IP*- Open map.
(b) Suppose a map $f : (\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ is IR*- open map. Let M be any IO set in $\grave{X}$ then f(M) is IR*O set in $\grave{Y}$. Since, every IR*O set is IP*O set. Therefore, f(M) is IP*O in $\grave{Y}$. Hence, f is IP*- Open map.
(c) Suppose a map $f : (\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ is IP*- open map. Let M be any IO set in $\grave{X}$ then f(M) is IP*O set in $\grave{Y}$. Since, every IP*O set is IPO set. Therefore, f(M) is IPO in $\grave{Y}$. Hence, f is IP- Open map.

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(d) Suppose a map f : $(\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ is Contra I- open map. Let M be any IO set in $\grave{X}$ then f(M) is IC set in $\grave{Y}$. Since, every IC set is IP*C set. Therefore, f(M) is IP*C in $\grave{Y}$. Hence, f is Contra IP*- open map.
(e) Suppose a map f : $(\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ is Contra IP*- open map. Let M be any IO set in $\grave{X}$ then f(M) is IP*C set in $\grave{Y}$. Since, every IP*C set is IPC set. Therefore, f(M) is IPC in $\grave{Y}$. Hence, f is Contra IP- open map.

The converse of the above theorem need not be true as shows in the following example.

Example – 3.6. In example – 3.3, f is IP*- open map. But $f(<\grave{X},\{a\},\{b,c\}>) = <\grave{Y},\{1\},\{2,3\}>$ and $f(<\grave{X},\{a,c\},\{b\}>) = <\grave{Y},\{1,3\},\{2\}>$ are does not belongs to σ_{IT}. Therefore, f is not a I- open map.

Example – 3.7. In example – 3.3, $IR^*O(\grave{Y}) = \{Y_I,\phi_I,<\grave{Y},\{1\},\{3\}>,<\grave{Y},\{3\},\{1,2\}>\}$. f is IP*- open map. But $f(<\grave{X},\{a\},\{b,c\}>) = <\grave{Y},\{1\},\{2,3\}>$ and $f(<\grave{X},\{a,c\},\{b\}>) = <\grave{Y},\{1,3\},\{2\}>$ are does not belongs to $IR^*O(\grave{Y})$. Therefore, f is not an IR*- open map.

Example – 3.8. Let $\grave{X} = \{1,2,3\}$ and $\grave{Y} = \{a,b,c\}$. Consider the IT's $\tau_{IT} = \{X_I,\phi_I,<\grave{X},\{1,3\},\phi>,<\grave{X},\{1,3\},\{2\}>\}$ and $\sigma_{IT} = \{Y_I,\phi_I,<\grave{Y},\{a\},\{b\}>,<\grave{Y},\{b\},\{c\}>,<\grave{Y},\{a,b\},\phi>,<\grave{Y},\phi,\{b,c\}>\}$ then $IP^*O(\grave{Y}) = \{Y_I,\phi_I,<\grave{Y},\phi,\{b,c\}>,<\grave{Y},\{a,b\},\phi>,<\grave{Y},\{a\},\{b\}>,<\grave{Y},\{b\},\{c\}>,<\grave{Y},\{a\},\{b,c\}>,<\grave{Y},\{a,b\},\{c\}>\}$ and $IPO(\grave{Y}) = \{Y_I,\phi_I, <\grave{Y},\phi,\{b,c\}>,<\grave{Y},\{a,b\},\phi>,<\grave{Y},\{a\},\{b\}>,<\grave{Y},\{b\},\{c\}>,<\grave{Y},\{a\},\{c\}>,<\grave{Y},\{a\},\{b,c\}>,<\grave{Y},\{a,b\},\{c\}>,<\grave{Y},\{a,c\},\{b\}>,<\grave{Y},\phi,\{c\}>,<\grave{Y},\{a\},\phi>,<\grave{Y},\{a,c\},\phi>\}$. Let f : $(\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ be a map defined by, f(1) = c, f(2) = b, f(3) = a. Here, $f(X_I) = Y_I$, $f(\phi_I) = \phi_I$, $f(<\grave{X},\{1,3\},\phi>) = <\grave{Y},\{a,c\},\phi>$ and $f(<\grave{X},\{1,3\},\{2\}>) = <\grave{Y},\{a,c\},\{b\}>$ are IPO sets in $\grave{Y}$ but $f(<\grave{X},\{1,3\},\phi>)$ and $f(<\grave{X},\{1,3\},\{2\}>)$ are not a IP*O set in $\grave{Y}$. Therefore, f is IP- open map but not IP*- open map.

Example – 3.9. Let $\grave{X} = \{1,2,3\}$ and $\grave{Y} = \{a,b,c\}$. Consider the IT's $\tau_{IT} = \{X_I,\phi_I,<\grave{X},\{3\},\{1,2\}>\}$ and $\sigma_{IT} = \{Y_I,\phi_I,<\grave{Y},\{a\},\{c\}>,<\grave{Y},\{c\},\{a,b\}>,<\grave{Y},\{a,c\},\phi>\}$ then $IC(\grave{Y}) = \{Y_I,\phi_I,<\grave{Y},\{c\},\{a\}>,<\grave{Y},\{a,b\},\{c\}>,<\grave{Y},\phi,\{a,c\}>\}$ and $IP^*C(\grave{Y}) = \{Y_I,\phi_I,<\grave{Y},\phi,\{a\}>,<\grave{Y},\phi,\{a,c\}>,<\grave{Y},\{c\},\{a\}>,<\grave{Y},\{b,c\},\{a\}>,<\grave{Y},\{b\},\{a,c\}>,<\grave{Y},\{a,b\},\{c\}>\}$. Let f : $(\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ be a map defined by, f(1) = a, f(2) = c, f(3) = b. Here, $f(X_I) = Y_I$, $f(\phi_I) = \phi_I$ and $f(<\grave{X},\{3\},\{1,2\}>) = <\grave{Y},\{b\},\{a,c\}>$ are IP*C sets in $\grave{Y}$ but $f(<\grave{X},\{3\},\{1,2\}>)$ is not a IC set in $\grave{Y}$. Therefore, f is Contra IP*- open map but not a Contra I- open map.

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Example – 3.10. In example – 3.9, IR*C($\grave{Y}$) = {Y_I,ϕ_I,<$\grave{Y}$,{c},{a}>,<$\grave{Y}$,{a,b},{c}>}. Here, f(X_I) = Y_I, f(ϕ_I) = ϕ_I and f(<$\grave{X}$,{3},{1,2}>) = <$\grave{Y}$,{b},{a,c}> are IP*C sets in $\grave{Y}$ but f(<$\grave{X}$,{3},{1,2}>) is not a IR*C set in $\grave{Y}$. Therefore, f is Contra IP*- open map but not a Contra IR*- open map.

Theorem – 3.11. A map f : ($\grave{X}$,τ_{IT}) → ($\grave{Y}$,σ_{IT}) is a IP*- open map iff f(Iint(A)) ⊆ IP*int(f(A)) for every IS A in $\grave{X}$.

Proof: Let f be IP*- open map and A be any IS of $\grave{X}$. Since, Iint(A) is IO set in $\grave{X}$ then f(Iint(A)) is IP*O set in $\grave{Y}$. Therefore, f(Iint(A)) = IP*int(f(Iint(A))) ⊆ IP*int(f(A)). Conversely, Let A be any IO set in $\grave{X}$ then A = Iint(A). By our assumption, f(A) = f(Iint(A)) ⊆ IP*int(f(A)). Also IP*int(f(A)) ⊆ f(A). Therefore, f(A) is IP*O set in $\grave{Y}$. Hence f is IP*- open map.

Theorem – 3.12. Let ($\grave{X}$,τ_{IT}) and ($\grave{Y}$,σ_{IT}) be an ITS in which every IP*O set is IOS. Then f : ($\grave{X}$,τ_{IT}) → ($\grave{Y}$,σ_{IT}) is a IP*- open map iff f(IP*int(A)) ⊆ IP*int(f(A)) for every IS A in $\grave{X}$.

Proof: Let f be IP*- open map and A be any IS of $\grave{X}$. Since, IP*int(A) is IP*O set in $\grave{X}$. By hypothesis, IP*int(A) is IO set in $\grave{X}$ then f(IP*int(A)) is IP*O set in $\grave{Y}$. Therefore, f(IP*int(A)) = IP*int(f(IP*int(A))) ⊆ IP*int(f(A)). Conversely, Let A be any IO set in $\grave{X}$ then A is IP*O set in $\grave{X}$. Therefore, A = IP*int(A). By our assumption, f(A) = f(IP*int(A)) ⊆ IP*int(f(A)). Also IP*int(f(A)) ⊆ f(A). Therefore, f(A) is IP*O set in $\grave{Y}$. Hence f is IP*- open map.

Theorem – 3.13. Let ($\grave{X}$,τ_{IT}), ($\grave{Y}$,σ_{IT}) and ($\grave{Z}$,μ_{IT}) be three ITS, f : ($\grave{X}$,τ_{IT}) → ($\grave{Y}$,σ_{IT}) be a surjection map and g : ($\grave{Y}$,σ_{IT}) → ($\grave{Z}$,μ_{IT}) be any map then the followings are hold,

a) If g∘f is IP*- open map and f is I- continuous map then g is IP*- open map.
b) If g∘f is I- continuous map and f is IP*- open map then g is IP*- continuous map.
c) If g∘f is IP*- continuous map and g is I- open map then f is IP*- continuous map.

Proof: (a) Let M be any IO set in $\grave{Y}$. Since, f is I- continuous then f^{-1}(M) is IO set in $\grave{X}$. Since g∘f is IP*- open map then (g∘f)(f^{-1}(M)) is IP*O set in $\grave{Z}$. Therefore g(f(f^{-1}(M))) = g(M) is IP*O set in $\grave{Z}$. Hence, g is IP*- open map.

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(b) Let M be any IO set in $\grave{Z}$. Since, g∘f is I- continuous map then $(g\circ f)^{-1}(M)$ is IO set in $\grave{X}$. Since f is IP*- open map then $f((g\circ f)^{-1}(M)) = f(f^{-1}(g^{-1}(M))) = g^{-1}(M)$ is IP*O set in $\grave{Y}$. Hence, g is IP*- continuous map.

(c) Let M be any IO set in $\grave{Y}$. Since, g is I- open map then g(M) is IO set in $\grave{Z}$. Since g∘f is IP*- continuous map then $(g\circ f)^{-1}(g(M)) = f^{-1}(g^{-1}(g(M))) = f^{-1}(M)$ is IP*O set in $\grave{X}$. Hence, f is IP*- continuous map.

Theorem – 3.14. Let $(\grave{X},\tau_{IT})$, $(\grave{Y},\sigma_{IT})$ and $(\grave{Z},\mu_{IT})$ be three ITS, $f : (\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ and $g : (\grave{Y},\sigma_{IT}) \rightarrow (\grave{Z},\mu_{IT})$ be any map then the followings are hold,

a) If f is I- open map and g is IP*- open map then g∘f is IP*- open map.
b) If f and g are I- open map then g∘f is IP*- open map.
c) If g∘f is IP*- open map and g is injective IP*- irresolute map then f is IP*- open map.

Proof: (a) Let M be any IO set in $\grave{X}$. Since f is I- open map then f(M) is IO set in $\grave{Y}$. Since, g is IP*- open map. Therefore g(f(M)) = g∘f(M) is IP*O set in $\grave{Z}$. Hence, g∘f is IP*O map.

(b) We know that, the composition of two I- open maps is again I- open map. Therefore, g∘f is I- open map. By theorem – 3.5. (a), g∘f is IP*- open map.

(c) Let M be any IO set in $\grave{X}$. Since, g∘f is IP*- open map then (g∘f)(M) is IP*O set in $\grave{Z}$. Since, g is IP*- irresolute map then $g^{-1}(g(f(M))) = g(M)$ is IP*O set in $\grave{Y}$. Hence, f is IP*- open map.

4. Intuitionistic Pre * Closed Maps

Definition – 4.1. A map f from ITS $(\grave{X},\tau_{IT})$ into another ITS $(\grave{Y},\sigma_{IT})$ is called *Intuitionistic Pre * Closed Map* if f(M) is IP*C set in $\grave{Y}$ for each IC set M in $\grave{X}$.

Definition – 4.2. A map f from ITS $(\grave{X},\tau_{IT})$ into another ITS $(\grave{Y},\sigma_{IT})$ is called *Contra Intuitionistic Pre * Closed Map* if f(M) is IP*O set in $\grave{Y}$ for each IC set M in $\grave{X}$.

Example – 4.3. Let $\grave{X}$ = {a,b,c} and $\grave{Y}$ = {1,2,3}. Consider the IT's $\tau_{IT} = \{X_I,\phi_I,<\grave{X},\{a\},\{b,c\}>, <\grave{X},\{a,c\},\{b\}>\}$ and $\sigma_{IT} = \{Y_I,\phi_I,<\grave{Y},\{1\},\{3\}>,<\grave{Y},\{3\},\{1,2\}>,<\grave{Y},\{1,3\},\phi>\}$ then $IC(\grave{X}) = \{X_I,\phi_I,<\grave{X},\{b\},\{a,c\}>,<\grave{X},\{b,c\},\{a\}>\}$ and $IP^*C(\grave{Y}) = \{Y_I,\phi_I,<\grave{Y},\{3\},\{1\}>,<\grave{Y},\phi,\{1\}>,$

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$<\grave{Y},\{2,3\},\{1\}>,<\grave{Y},\{1,2\},\{3\}>,<\grave{Y},\phi,\{1,3\}>,<\grave{Y},\{2\},\{1,3\}>\}$. Let $f : (\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ be a map defined by, f(a) = 1, f(b) = 2, f(c) = 3. Here, $f(X_I) = Y_I$, $f(\phi_I) = \phi_I$, $f(<\grave{X},\{b,c\},\{a\}>) = <\grave{Y},\{2,3\},\{1\}>$ and $f(<\grave{X},\{b\},\{a,c\}>) = <\grave{Y},\{2\},\{1,3\}>$ are IP*C sets in $\grave{Y}$. Therefore, f is IP*- closed map.

Example – 4.4. Let $\grave{X} = \{a,b,c\}$ and $\grave{Y} = \{1,2,3\}$. Consider the IT's $\tau_{IT} = \{X_I,\phi_I,<\grave{X},\{a\},\{b,c\}>\}$ and $\sigma_{IT} = \{Y_I,\phi_I,<\grave{Y},\{2\},\{1,3\}>,<\grave{Y},\{3\},\{1,2\}>,<\grave{Y},\{2,3\},\{1\}>\}$ then $IC(\grave{X}) = \{X_I,\phi_I, <\grave{X},\{b,c\},\{a\}>\}$ and $IP^*O(\grave{Y}) = \sigma_{IT}$ Let $f : (\grave{X},\tau_{IT}) \rightarrow (\grave{Y},\sigma_{IT})$ be a map defined by, f(a) = 1, f(b) = 3, f(c) = 2. Here, $f(X_I) = Y_I$, $f(\phi_I) = \phi_I$ and $f(<\grave{X},\{b,c\},\{a\}>) = <\grave{Y},\{2,3\},\{1\}>$ are IP*O sets in $\grave{Y}$. Therefore, f is Contra IP*- closed map.

Theorem – 4.5. Let $(\grave{X},\tau_{IT})$ and $(\grave{Y},\sigma_{IT})$ be an ITS then the followings are hold.

a) Every I- Closed map is IP*- Closed map.
b) Every IR*- Closed map is IP*- Closed map.
c) Every IP*- Closed map is IP- Closed map.
d) Every Contra I- closed map is Contra IP*- closed map.
e) Every Contra IR*- closed map is Contra IP*- closed map.

Proof: Proof is similar to Theorem – 3.5.

The converse of the above theorem need not be true as shows in the following example.

Example – 4.6. In example – 4.3, $IC(\grave{Y}) = \{Y_I,\phi_I,<\grave{Y},\{3\},\{1\}>,<\grave{Y},\{1,2\},\{3\}>,<\grave{Y},\phi,\{1,3\}>\}$. Clearly, f is IP*- closed map. But $f(<\grave{X},\{b,c\},\{a\}>) = <\grave{Y},\{2,3\},\{1\}>$ and $f(<\grave{X},\{b\},\{a,c\}>) = <\grave{Y},\{2\},\{1,3\}>$ are does not belongs to $IC(\grave{Y})$. Therefore, f is not a I- closed map.

Example – 4.7. In example – 4.3, $IR^*C(\grave{Y}) = \{Y_I,\phi_I,<\grave{Y},\{3\},\{1\}>,<\grave{Y},\{1,2\},\{3\}>\}$. Clearly, f is IP*- closed map. But $f(<\grave{X},\{b,c\},\{a\}>) = <\grave{Y},\{2,3\},\{1\}>$ and $f(<\grave{X},\{b\},\{a,c\}>) = <\grave{Y},\{2\},\{1,3\}>$ are does not belongs to $IR^*C(\grave{Y})$. Therefore, f is not an IR*- closed map.

Example – 4.8. Let $\grave{X} = \{1,2,3\}$ and $\grave{Y} = \{a,b,c\}$. Consider the IT's $\tau_{IT} = \{X_I,\phi_I,\grave{X},\{1,3\},\phi>, <\grave{X},\{1,3\},\{2\}>\}$ and $\sigma_{IT} = \{Y_I,\phi_I,<\grave{Y},\{a\},\{b\}>,<\grave{Y},\{b\},\{c\}>,<\grave{Y},\{a,b\},\phi>,<\grave{Y},\phi,\{b,c\}>\}$ then $IC(\grave{X}) = \{X_I,\phi_I,<\grave{X},\phi,\{1,3\}>,<\grave{X},\{2\},\{1,3\}>\}$, $IP^*C(\grave{Y}) = \{Y_I,\phi_I,<\grave{Y},\{b,c\},\phi>,<\grave{Y},\phi,\{a,b\}>, <\grave{Y},\{b\},\{a\}>,<\grave{Y},\{c\},\{b\}>,<\grave{Y},\{b,c\},\{a\}>,<\grave{Y},\{c\},\{a,b\}>\}$ and $IPC(\grave{Y}) = \{Y_I,\phi_I,<\grave{Y},\{b,c\},\phi>, <\grave{Y},\phi,\{a,b\}>,<\grave{Y},\{b\},\{a\}>,<\grave{Y},\{c\},\{b\}>,<\grave{Y},\{c\},\{a\}>,<\grave{Y},\{b,c\},\{a\}>,<\grave{Y},\{c\},\{a,b\}>,<\grave{Y},\{b\},\{a,c\}$

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>,<$\grave{Y}$,{c},ф>,<$\grave{Y}$,ф,{a}>,<$\grave{Y}$,ф,{a,c}>} . Let f : ($\grave{X}$,τ_{IT}) → ($\grave{Y}$,σ_{IT}) be a map defined by, f(1) = c, f(2) = b, f(3) = a. Here, f(X_I) = Y_I, f(ϕ_I) = ϕ_I, f(<$\grave{X}$,ф,{1,3}>) = <$\grave{Y}$,ф,{a,c}> and f(<$\grave{X}$,{2},{1,3}>) = <$\grave{Y}$,{b},{a,c}> are IPC sets in $\grave{Y}$ but f(<$\grave{X}$,ф,{1,3}>) and f(<$\grave{X}$,{2},{1,3}>) are not a IP*C set in $\grave{Y}$. Therefore, f is IP- closed map but not IP*- closed map.

Example – 4.9. Let $\grave{X}$ = {1,2,3} and $\grave{Y}$ = {a,b,c}. Consider the IT's τ_{IT} = {X_I,ϕ_I,<$\grave{X}$,{3},{1,2}>} and σ_{IT} = {Y_I,ϕ_I,<$\grave{Y}$,{a},{c}>,<$\grave{Y}$,{c},{a,b}>,<$\grave{Y}$,{a,c},ф>} then IP*O($\grave{Y}$) = {Y_I,ϕ_I,<$\grave{Y}$,{a},ф>, <$\grave{Y}$,{a,c},ф>,<$\grave{Y}$,{a},{c}>,<$\grave{Y}$,{a},{b,c}>,<$\grave{Y}$,{a,c},{b}>,<$\grave{Y}$,{c},{a,b}>}. Let f : ($\grave{X}$,τ_{IT}) → ($\grave{Y}$,σ_{IT}) be a map defined by, f(1) = a, f(2) = c, f(3) = b. Here, f(X_I) = Y_I, f(ϕ_I) = ϕ_I and f(<$\grave{X}$,{1,2},{3}>) = <$\grave{Y}$,{a,c},{b}> are IP*O sets in $\grave{Y}$ but f(<$\grave{X}$,{1,2},{3}>) is not a IO set in $\grave{Y}$. Therefore, f is Contra IP*- closed map but not a Contra I- closed map.

Example – 4.10. In example – 4.9, IR*C($\grave{Y}$) = {Y_I,ϕ_I,<$\grave{Y}$,{a},{c}>,<$\grave{Y}$,{c},{a,b}>}. Here, f(X_I) = Y_I, f(ϕ_I) = ϕ_I and f(<$\grave{X}$,{1,2},{3}>) = <$\grave{Y}$,{a,c},{b}> are IP*O sets in $\grave{Y}$ but f(<$\grave{X}$,{1,2},{3}>) is not a IR*O set in $\grave{Y}$. Therefore, f is Contra IP*- closed map but not a Contra IR*- closed map.

Theorem – 4.11. A map f : ($\grave{X}$,τ_{IT}) → ($\grave{Y}$,σ_{IT}) is a IP*- closed map iff IP*cl(f(A)) ⊆ f(Icl(A)) for every IS A in $\grave{X}$.

Proof: Let f be IP*- closed map and A be any IS of $\grave{X}$. Since, Icl(A) is IC set in $\grave{X}$ then f(Icl(A)) is IP*C set in $\grave{Y}$. Therefore, IP*(f(Icl(A))) = f(Icl(A)). i.e), IP*cl(f(A)) ⊆ f(Icl(A)). Conversely, Let A be any IC set in $\grave{X}$ then A = Icl(A). By our assumption, IP*(cl(f(A)) ⊆ f(Icl(A)) = f(A). Also, f(A) ⊆ IP*cl(f(A)). Therefore, f(A) = IP*cl(A). i.e), f(A) is IP*C set in $\grave{Y}$. Hence f is IP*- closed map.

Theorem – 4.12. Let ($\grave{X}$,τ_{IT}) and ($\grave{Y}$,σ_{IT}) be an ITS in which every IP*C set is IC set. Then f : ($\grave{X}$,τ_{IT}) → ($\grave{Y}$,σ_{IT}) is a IP*- closed map iff IP*cl(f(A)) ⊆ f(IP*cl(A)) for every IS A in $\grave{X}$.

Proof: Let f be IP*- closed map and A be any IS of $\grave{X}$. Since, IP*cl(A) is IP*C set in $\grave{X}$. By hypothesis, IP*cl(A) is IC set in $\grave{X}$ then f(IP*cl(A)) is IP*C set in $\grave{Y}$. Therefore, IP*cl(f(IP*cl(A))) = f(IP*cl(A)). Hence, IP*cl(f(A)) ⊆ f(IP*cl(A)). Conversely, Let A be any IC set in $\grave{X}$ then A is IP*C set in $\grave{X}$. Therefore, A = IP*cl(A). By our assumption, IP*cl(f(A)) ⊆

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f(IP*cl(A)) = f(A). Also IP*cl(f(A)) ⊇ f(A). Therefore, f(A) is IP*C set in $\grave{Y}$. Hence f is IP*-closed map.

Theorem – 4.13. Let ($\grave{X}$,τ_{IT}) and ($\grave{Y}$,σ_{IT}) be an ITS and f : ($\grave{X}$,τ_{IT}) → ($\grave{Y}$,σ_{IT}) be a bi ection map then the following statements are equivalent.

a) f is IP*- continuous map.
b) f^{-1} is IP*- open map.
c) f^{-1} is IP* closed map.

Proof: (1) ⇒ (2), Let M be any IO set in $\grave{Y}$. Since, f is IP*- continuous. Therefore, f^{-1}(M) is IP*O set in $\grave{X}$. Hence, f^{-1} is IP*- open map.

(2) ⇒ (3), Let M be any IC set in $\grave{Y}$ then M^c is IO set in $\grave{Y}$. Since f^{-1} is IP*- open map then $f^{-1}(M^c)$ = $[f^{-1}(M)]^c$ is IP*O set in $\grave{X}$. Therefore, f^{-1}(M) is IP*C set in $\grave{X}$. Hence, f^{-1} is IP*- closed map.

(3) ⇒ (1), Let M be any IC set in $\grave{Y}$. Since f^{-1} is IP*- closed map then f^{-1}(M) is IP*C set in $\grave{X}$. Hence f is IP*- continuous map.

5. Conclusions

We discussed the IP*- Open maps, IP*- Closed maps and their contra versions in this paper. We intend to conduct research in the future on Per IP*- Open maps, Pre IP*- Closed maps, Super IP*- Open maps and so on.

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