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Further Results on Pair Sum Graphs

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ABSTRACT

Let G be a (p, q) graph. An injective map $f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm p\}$ is called a pair sum labeling if the induced edge function, $f_e : E(G) \to Z - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-one and $f_e(E(G))$ is either of the form $\{\pm k_1, \pm k_2, \ldots, \pm k_{q/2}\}$ or $\{\pm k_1, \pm k_2, \ldots, \pm k_{(q-1)/2}\} \cup \{k_{(q+1)/2}\}$ according as q is even or odd. A graph with a pair sum labeling is called a pair sum graph. In this paper we investigate the pair sum labeling behavior of subdivision of some standard graphs.

Keywords: Path; Cycle; Ladder; Triangular Snake; Quadrilateral Snake

1. INTRODUCTION

The graphs considered here will be finite, undirected and simple. $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G. The cardinality of the vertex set of a graph G is denoted by p and the cardinality of its edge set is denoted by q. The corona $G_1 G_2$ of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 (with p_1 vertices) and p_1 copies of G_2

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and then joining the ith vertex of G_1 to all the vertices in the ith copy of G_2 . If $e = uv$ is an edge of G and w is a vertex not in G then e is said to be sub divided when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the subdivision graph of G and it is denoted by S(G). The graph $P_n \times P_2$ is called the ladder. A dragon is a graph formed by joining an end vertex of a path P_m to a vertex of the cycle C_n . It is denoted as $C_n @P_m$. The triangular snake T_n is obtained from the path P_n by replacing every edge of a path by a triangle C_3. The quadrilateral snake Q_n is obtained from the path P_n by every edge of the path is replaced by a cycle C_4 . The concept of pair sum labeling has been introduced in [1]. The pair sum labeling behavior of some standard graph like complete graph , cycle, path, bistar and some more standard graph are investigated in[1-3]. That all the trees of order ≤9 are pair sum have been proved in [4]. Terms not defined here are used in the sense of Harary[5]. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Here we investigate the pair sum labeling behavior of S(G), for some standard graphs G.

2. PAIR SUM LABELING

Definition 2.1 : Let G be a (p, q) graph. A injective map $g : V(G) \rightarrow \{\pm 1, \pm 2, \ldots, \pm p\}$ is said to be a pair sum labeling if the induced edge function $g_e : E(G) \rightarrow Z - \{0\}$ defined by $g_e(uv) = g(u) + g(v)$ is one-one and $g_e(E(G))$ is either of the form $\{\pm k_1, \pm k_2, \ldots, \pm k_{q/2}\}$ or $\{\pm k_1, \pm k_2, \ldots, \pm k_{(q-1)/2}\} \cup \{k_{(q+1)/2}\}$ according as q is even or odd. A graph with a pair sum labeling defined on it is called pair sum graph

3. ON STANDARD GRAPHS

Here we investigate pair sum labeling behavior of $C_n @P_m$ and $K_n^c + 2K_2$.

Theorem 3.1. If nis even, $C_n @P_m$ is a pair sum graph.

Proof: Let C_n be the cycle $x_1x_2x_3\ldots x_nx_1$ and lat P_m be the path $y_1y_2\ldots y_m$

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case 1. m=0 (mod 4)

Define g:V(C_n @P_m)□□□□□1, □2, , □ □ *n* □ *m* □□

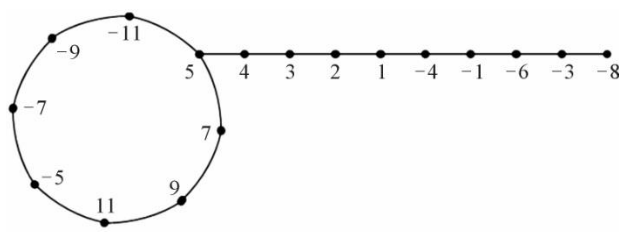
g□□ *y*□ (m)/ 2□ □*j+1* □ □ j , 1 □ *j* □□□*m*/2 □ .n

g□□ *y*□ (m)/ 2□ □□*j-1* □ □ -2j-2 , 1 □ *j* □□□*m*/4 □ .

g□□ *y*□ (m)/ 2□ □□*j* □ □□□ j +1, 1 □ *j* □□□*m*/2 □ .

g□□x_j□ □m/2+2j-1 , 1 □ *j* □□ n□□□ .

g□□$x_{n/2}$ □ □□ m/2-2j+1 , 1 □ *j* □□ n□□□



f_e □*E* □C_n @ P_m □□

□ □ □3, □ 5, □7, , □ *m*, □ □ *m* □1□□ □ □3,5, 7, , □ *m* □1□□
□ □ *m* □ 2, *m* □ 10, , □ *m* □ 2*n*□□

□ □ □ □ *m* □ 2 □ , □ □ *m* □ 10 □ , , □ □ *m* □ 2*n*□□ □ □(*n* □ 2), □ □ *n* □ 2 □□.

and

E□K_n^c □ 2 K_2 □ □ □*xy* , *zk* , xx_j , yx_j , zx_j , kx_j :1 □ *j* □ *n*□. Define

g : *V* □K_n^c □ 2 K_2 □ □ □□1, □2,..., □ □ *n* □ 4□□

by *g*□□x_j □ □ 2*j* □ 1, 1 □ *j* □ *n*/2

g□□$x_{n/2□j}$ □ □□□ □ 2*j* □ 1□, 1 □ *j* □ *n*/2

g □*x* □ □ *n* , *g* □*y* □ □ *n* □ 3

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g z n , g k n 3

Here ge E Kn^c 2K2

n 1, n 3, n 5, ,2n

n 1 , n 3 , n 5, , 2n

n 1, n 3, n 5, ,1

n 1 , n 3 , n 5, , 1 n 4, n 8, n 12, ,2n 2

n 4 , n 8 , n 12 , , 2n 2

n 2, n, n 2, ,2

n 2 , n, n 2, , 2

2 n 3, 2n 3 .

Therefore *f* is a pair sum labeling.

Illustration 2. A pair sum labeling of K_8^c $2K_2$ is shown in **Figure 2**.

4. ON SUBDIVISION GRAPH

Here we investigate the pair sum labeling behavior of S G for some standard graphs G.

Theorem 4.1. S L_n is a pair sum graph, where L_n is a ladder on n vertices.

Proof. Let

V S L_n x_i , y_i , z_i , a_j ,b_j :1 i n,1 j n 1

Let E S L_n $x_i z_i$, $z_i y_i$:1 i n

$x_i a_i$, $a_i x_{i 1}$, $y_i b_i$, $b_i y_{i 1}$:1 i n 1 .

Case 1: n is even.

When n = 2, the proof follows from the Theorem 2.3. For n > 2,

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Define *g* :*V* □ *S* □ L_n □□ □ □ □1, □2, □3, ..., □ □ 5*n* □ 2□□ by

g□□ *x n*/2 □ □□□ 1, *g*□□ *xn*/2□1 □ □□□3

g□□ $x_{n/2□i}$ □ □ 10*i* □ 3,1 □ *i* □ □*n* □ 2□/2

g□□ $x_{n/2 □ i □1}$ □ □□□ 10*i* □ 1,1 □ *i* □ □ *n* □ 2□/2

g□□ *zn*/2 □ □ 5, *g* □ *zn*/2□1 □ □□□5

g□□ $z_{n/2□i}$ □ □ 10*i* □ 1,1 □ *i* □ □*n* □ 2□/2

g□□ $z_{n/2 □1□i}$ □ □□□ □10*i* □ 1□ ,1 □ *i* □ □ *n* □ 2□/2

g□□ *yn*/2 □ □ 3, *g* □ *yn*/2□1 □ □ 1

g□□ $y_{n/2□i}$ □ □ 10*i* □ 1,1 □ *i* □ □*n* □ 2□/2

g□□ $y_{n/2 □1□i}$ □ □□□10*i* □ 3,1 □ *i* □ □ *n* □ 2□/2 *g* □ *an*/2 □ □□□2

g □ $a_{n/2□i}$ □ □ 10*i* □ 5,1 □ *i* □ □*n* □ 2□/2 *g* □ $a_{n/2□i}$ □ □□□10*i* □ 3,1 □ *i* □ □ *n* □ 2□/2 *g* □ bn/2 □ □ 2

g □ $b_{n/2□i}$ □ □ 10*i* □ 3,1 □ *i* □ □*n* □ 2□/2

g □ $b_{n/2 □ i □1}$ □ □□□10*i* □ 5,1 □ *i* □ □ *n* □ 2□/2.

When *n* = 4,

g_e □ *E* □*S* □ L_n □□□ □ □3,4,5,8,10,16,20,24,28□ □ □ □3, □4, □ 5, □8, □10, □ 16, □20, □24, □28□.

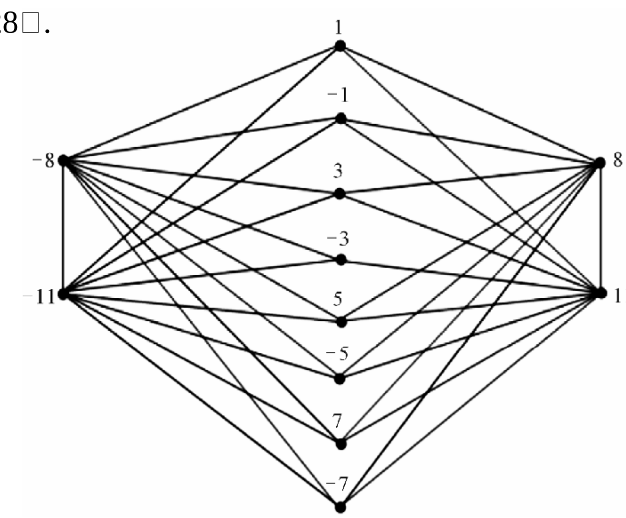

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Figure 2. A pair sum labeling of $K_8^c + 2K_2$.

For $n > 4$,

g_e □E □S □ L_n □□□

□g_e □E □ S □ L_4 □□□□□□ 26,36,40,44,48,38□,

□ □26, □36, □40, □44, □48, □38□,

□ 46,56,60,64,68,58□,

□ □46, □56, □60, □64, □68, □58□, ,

□10n □ 34,10n □ 24,10n □ 20,

10n □ 16,10n □ 12,10n □ 22□,

□ □10n □ 34, □ 10n □ 24, □10n □ 20,

□ 10n □ 16, □10n □ 12, □ 10n □ 22□□.

Therefore f is a pair sum labeling.

Case 2. n is odd.

Clearly S □ L_1 □ □ P_3 and hence S □ L_n □ is a pair sum graph by Theorem 2.2. For $n > 1$,

Define g :V □S □ L_n □□ □ □□1, □2, , □ □ 5n □ 2□□ by

g□□x□ n □1□/2 □ □ 6, g □x□ n□1□/2 □ □ 12

g □x□ n □ 3 □/2 □ □□□12, g □a□ n□1□/2 □ □□□9

g□□a□ n□1□/2 □ □ 3

g□□x□ n □ 3 □/2□j □ □ 10j □ 10, 1 □ j □ □ n □ 3 □/2

g□□x□ n □1□/2□j □ □□□ □10j □ 10 □ , 1 □ j □ □ n □ 3 □/2 g □y□ n □ 3 □/2□j □ □□□

□ 6 □ 10j □ ,1 □ j □ □ n □ 3 □/2

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g □y□ n □1□/2□i □ □ 6 □ 10i , 1 □ i □ □ n □ 3 □/2

g□ □z□ n □ 3 □/2□i □ □□□10i □ 2, 1 □ i □ □ n □ 3 □/2 g □z□ n □1□/2□i □ □ 10i □ 2,
1 □ i □ □ n □ 3 □/2

g □y□ n □1□/2 □ □ 2, g □y□ n□1□/2 □ □ 10

g □y□ n □ 3 □/2 □ □□□10, g □b□ n□1□/2 □ □□□6

g □b□ n □1□/2 □ □ 4, g □z□ n□1□/2 □ □□□4

g□ □z□ n □1□/2 □ □ 8, g □z□ n□1□/2 □ □□□8

g□ □a□ n □1□/2□i □ □□□ □10i □ 12 □ , 1 □ i □ □ n □ 3 □/2 g □a□ n □1□/2□i □ □ 10i
□ 12, 1 □ i □ □ n □ 3 □/2

g □b□ n □1□/2□i □ □□□ □10i □ 4 □ ,1 □ i □ □ n □ 3 □/2

g□ □b□ n □1□/2□i □ □ 10i □ 4, 1 □ i □ □ n □ 3 □/2.

Therefore

g_e □ E □S □ L_3 □□

□ □ 2, 3, 4, 6, 9,18, 20, □2, □ 3, □4, □6, □9, □18, □20□

and g_e □ E □S □ L_5 □□□ □ g_e □ E □S □ L_3 □□□

□□ 24,30,34,38,42,36, □24, □30, □34, □38, □42, □36□ when $n > 5$,

g_e □ E □S □ L_n □□□

□g_e □ E □ S □ L_5 □□□□□□ 40,50,54,58,62,52□,

□ □40, □50, □54, □58, □62, □52□,

□ 60,70,74,78,82,72□,

□ □60, □70, □74, □78, □82, □ 72□,□,

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□10*n* □ 30,10*n* □ 20,10*n* □16,

10*n* □ 12,10*n* □ 8,10*n* □18□,

□ □10*n* □ 30, □10*n* □ 20, □10*n* □16,

□ 10*n* □ 12, □10*n* □ 8, □10*n* □18□□.

Then *g* is a pair sum labeling.

Illustration 3. A pair sum labeling of *S* □L_7 □ is shown in **Figure 3**.

Theorem *S* □ C_n K_1 □ is a pair sum graph

Proof. Let

V □*S* □C_n K_1 □□ □ □x_j :1 □ *j* □ 2*n*□□ □ $z_j y_j$:1 □ *j* □ *n*□

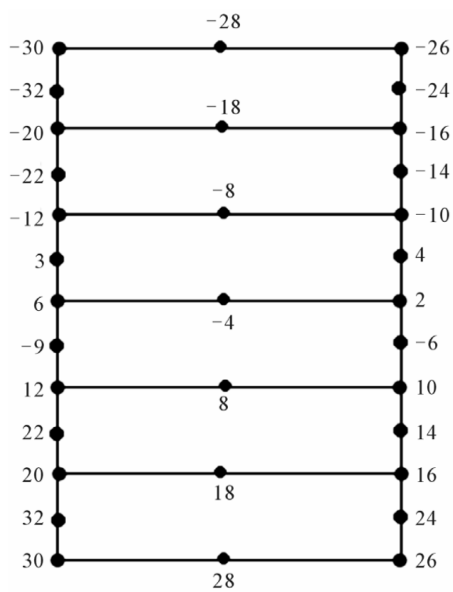

Figure 3. A pair sum labeling of *S* □L_7 □ .

Let *E* □*S* □C_n K_1 □□ □ □$x_j y_{j □1}$:1 □ *j* □ 2 *n* □1□

□ □□□□$x_{2j □1} x_j$:1 □ *j* □ *n*□ □ □$y_j z_j$:1 □ *j* □ *n*□.

Case 1. *n* is even.

Define $g : S$ □$C_n K_1$ □ □ □□1, □2, , □4n□ by

g□□x_j □ □ 2j □ 1, 1 □ j □ n

g□□$x_{n □ i}$ □ □□□ □ 2j □ 1□, 1 □ j □ n

g□□z_j □ □ 2 n □ 1 □ 2j , 1 □ j □ $n/2$

g □$z_{n/2□j}$ □ □□□2 n □ 1□ 2j,1 □ j □ $n/2$ g □y_j □ □ 3n □ 1 □ 2j , 1 □ j □ $n/2$

g □$z_{n/2□j}$ □ □□□3n □ 1□ 2j,1 □ j □ $n/2$ Here

g_e □ E □ □ □4,8,12, , □4 n □ 4□□

□ □ □4, □8, □12, , □ □ 4 n □ 4□□

□ □2n □ 2, 2 n □ 8, 2 n □ 14, , 5n □ 4□

□ □ □ □ 2 n □ 2 □ , □ □ 2 n □ 8 □ , □ □ 2 n □ 14 □ , , □ □ 5n □ 4□□

□ □5n □ 2,5n □ 6,5 n □ 10, , 7 n □ 2□

□ □ □ □ 5n □ 2 □ , □ □ 5 n □ 6 □ , □ □ 5n □ 10 □ , , □ □ 7 n □ 2 □□.

Then g is pair sum labeling.

Case 2. n is odd.

Define $g : V$ □ S □ $C_n K_1$ □□ □ □ □1, □2, ..., □4n□ by

g□□ x_j □ □ 4 n □ 2j □ 2, 1 □ j □ n

g□□ $x_{n/2□j}$ □ □□□4 n □ 2j □ 2,1 □ j □ n g □z_j □ □□□ n □ 1 □ j , 1 □ j
□□□□$n/2$□□

g□ $z_{□□ n/2 □□}$□j□ □ n □ j , 1 □ j □□□□$n/2$□□

g□□y_j □ □□□2 n □ 2 □ 2j , 1 □ j □□□□ $n/2$□□

g□ $v_{□□ n/2□□□j}$ □ □ 2 n □ 2 □ 2j , 1 □ j □□□□ $n/2$□□

Here

g_e □ E □S □ $C_n K_1$ □□□

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□ □8*n* □ 2,8 *n* □ 6, 4 *n* □ 10, 40 □ 6□

□ □ □ □ 8*n* □ 2 □ , □ □ 8 *n* □ 6 □ , , □ □ 4 *n* □ 10 □ , □ □ 4 *n* □ 6□□

□ □ 2 *n* □ 2, □2 *n* □ 2□

□ □3*n*, 3*n* □ 3,3*n* □ 6, , 3 □ *n* □1□/2□

□ □ □3*n*, □ □ 3*n* □ 3 □ , □ □ 3*n* □ 6 □ , , □3 □ *n* □1□/2□

□ □3*n* □ 1, 3*n* □ 4, ,3 □ *n* □ 7 □/2□

□ □ □ □ 3*n* □ 1□ , □ □ 3*n* □ 4 □ , , □3 □ *n* □ 7 □/2□.

Then *f* is pair sum labeling.

Illustration 4. A pair sum labeling of S □ C_7 K_1 □ is shown in **Figure 4**.

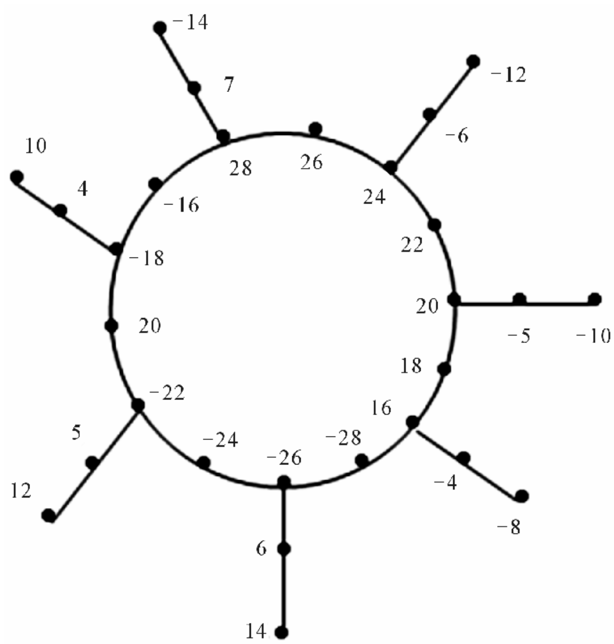

Figure 4. A pair sum labeling of S □C_7 K_1 □

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