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Research Paper

A Study on Topological Graphs in Certain Characterization and Other Graphs

[*1] AGILANDESWARI.A , MPhil Scholar,
Department of Mathematics,
Bharath Institute of Higher Education and Research,
Chennai,-73, India.
[*2] Dr.K.Arulmozhi, Associate Professor,
Department of Mathematics,
Bharath Institute of Higher Education and Research,
Chennai,-73, India.
mjsmpk@gmail.com arulmozhi.maths@bharathuniv.ac.in
Address for Correspondence
[*1] AGILANDESWARI.A , MPhil Scholar,
Department of Mathematics,
Bharath Institute of Higher Education and Research,
Chennai,-73, India.
[*2] Dr.K.Arulmozhi, Associate Professor,
Department of Mathematics,
Bharath Institute of Higher Education and Research,
Chennai,-73, India.
mjsmpk@gmail.com arulmozhi.maths@bharathuniv.ac.in

Abstract:

There are a few synthetic records that have been acquainted in hypothetical science with measure the properties of sub-atomic geography, for example, distance-based topological lists, degree-based topological files and counting-related topological files. Among the degree-based topological files, the molecule bond availability (ABC) record and mathematical number juggling (GA) file are the most significant, due to their synthetic importance. Certain physicochemical properties, for example, the limit, steadiness and strain energy, of substance compounds are related by these topological lists. In this paper, we concentrate on the atomic topological properties of some unique charts. The files (ABC), (ABC4), (GA) and (GA5) of these uncommon charts are processed.

Keywords: atom-bond connectivity (ABC) index; geometric–arithmetic (GA) index; edge partition.

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Introduction:

Diagram hypothesis, as applied in the investigation of sub-atomic constructions, addresses an interdisciplinary science called compound chart hypothesis or sub-atomic geography. By utilizing instruments taken from chart hypothesis, set hypothesis and measurements, we endeavor to distinguish primary elements engaged with structure–property movement connections. Atoms and displaying obscure constructions can be grouped by the topological portrayal of compound designs with wanted properties. Much exploration has been directed around here over the most recent couple of many years. The topological list is a numeric amount related with synthetic constitutions implying the connection of compound constructions with numerous physicochemical properties, substance reactivity or natural action. Topological records are planned on the grounds of the change of a sub-atomic chart into a number that portrays the geography of the sub-atomic diagram. We concentrate on the connection between the construction, properties, and movement of substance compounds in atomic displaying. Atoms and sub-atomic mixtures are frequently demonstrated by sub-atomic charts. A substance chart is a model used to portray a synthetic compound. A sub-atomic chart is a straightforward diagram whose vertices compare to the molecules and whose edges relate to the bonds. It very well may be portrayed in various ways, for example, by a drawing, a polynomial, an arrangement of numbers, a lattice or by an inferred number called a topological list. The topological record is a numeric amount related with a diagram, which portrays the geography of the chart and is invariant under a diagram automorphism. Some significant sorts of topological records of diagrams are degree-based topological lists, distance-based topological lists and counting-related topological files. The degree-based topological records, the molecule bond network (ABC) and mathematical number juggling (GA) files, are vital, with a huge job in substance chart hypothesis, especially in science. Unequivocally, a topological list Top(G) of a chart is a number with the end goal that, in case H is isomorphic to

G, then, at that point, Top(H) = Top(G). Obviously the quantity of edges and vertices of a chart are topological records [1–7]. We let G = (V, E) be a basic diagram, where V(G) signifies its vertex set and E(G) means its edge set. For any vertex u 2 V(G), we call the set NG(u) = fv 2 V(G)juv 2 E(G)g the open neighborhood of u; we signify by du the level of vertex u and by Su = åv2NG(u) d(v) the degree amount of the neighbors of u. The quantity of vertices and number of edges of the diagram G are indicated by jV(G)j and jE(G)j, individually. A basic chart of request n in which each pair of particular vertices is contiguous is known as a total diagram and is meant by Kn. The documentation in this paper is taken from the books [3,8–10].

$G, then, at\ that\ point, Top\ (H) = Top\ (G). Obviously\ the\ quantity\ of\ edges\ and\ vertices\ of\ a\ diagram\ are\ topological\ lists$ $[1-7]$. $We\ let\ G = (V, E) be\ a\ straightforward\ diagram, where\ V(G) signifies\ its\ vertex\ set\ and\ E(G) indicates\ itsedge\ set. For\ any\ vertex\ u \in V(G), we\ call\ the\ set\ N_G(u) = \{v \in V(G) \mid uv \in E(G)\} the\ open\ neighborhood\ of\ u;\ we\ indicate\ by\ d_u the\ level\ of\ vertex\ u\ and\ by\ S_z = \Sigma_{\nabla \in N_G(u)} d(D) the\ degree\ amount\ of\ the\ neighbors\ of\ u. The\ quantity\ of\ vertices\ and\ number\ of\ edges\ of\ the\ chart\ G\ are\ denobed\ by\ |V(G)| and\ |E(G)|, separately.$

$A\ straightforward\ chart\ of\ onder\ n\ in\ which\ each\ pair\ of\ particular\ vertices\ is\ adjoining\ is\ known\ as\ a\ total\ diagram\ and\ is\ meant\ by\ K_\pi. The\ documentation\ in\ this\ paper\ is\ taken\ from\ the\ books$ $[3,8-10]$

.In this paper, we study the molecular topological properties of some special graphs: Cayley trees, Γ_m^2; square lattices, $SL_{\pi;}$ a graph G_n; and a complete bipartite graph, $K_{m,\pi}$. Additionally, the indices $(ABC), (ABC_4), (GA)$ and (GA_5) of these special graphs, whose definitions are discussed in the materials and methods section, ane computed.

Definition 1. [11] The aldest degree-based topological index, the Randíc index, denoted by $R_{-\frac{1}{2}}(G)$, is difined as

$$R_{-\frac{1}{2}}(G) = \sum_{x=E(G)} \frac{1}{\sqrt{d_a d_v}}$$

Definition 2. [12] For any real numbo $a \in \mathbb{R}$, the general Randi'cindex, $R_a(G)$, is defined as

$$R_\Delta(G) = \sum_{x\nabla \in E(G)} (d_x d_v)^2$$

Definition 3. [13]. The degree-bused topological ABC index is defined as

$$ABC(G) = \sum_{a \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_x d_v}}$$

Definition 4. [2]. The degree-brsed topologion! GA index is defined as

$$GA(G) = \sum_{xD \in E(G)} \frac{2\sqrt{d_x d_v}}{d_x + d_v}$$

Recently, several authors have introduced new versions of the ABC and GA indices, which we derive in the two definitions below.

Definition 5. [14]. The faurth versian (ABC_4) of the, ABC index is defined as

$$ABC_4(G) = \sum_{xD\in E(G)} \sqrt{\frac{S_x + S_v - 2}{S_x S_v}}$$

Definition 6. [15]. The fifth versian (GAs) of the GA budex is defined as

$$GA_5(G) = \sum_{x\vee\in E\{G\}} \frac{2\sqrt{S_X S_V}}{S_x + S_v}$$

The idea of topological indioss came from Wiener [16] whik he was dealing with the limit of paraffin and was named the record way number. Afterward, the way number was renamed as the Wiener file [17]. Hayat et al. [1] concentrated on different degree-based topological files for certain ty pes of organizations, like silicates, hexagonal, honeycombs and oxides. Imran et al [7] concentrated on the atomic topological not set in stone the insightful shut equation of the ABC,ABC_4,ABC_5,GA,.

GA_4 and GA_5 records of Sierpinski organizations. M. Darafsheh [18] created various techniques to ascertain the Wiener list, Szeged record and Padmakar-Ivan file for different charts utilizing the gathering of automorphisms of G. He additionally tracked down the Wiener files of a couple of charts utilizing inductive, techniques. A. Ayache and A. Alameri [19] gave some topological files of m^k-diagrams, for example, the Wiener file, the hyper-Wiener list, the Wiener extremity, Zagreb lists, Schultz and changed Schultz records and the Wiener-type invariant Wei Gaoet al [20] acquired specific erraticism rendition topological files of the group of cycloalkanes C_π^(K_4). As of late, the ree has been broad investigation into ABC and GA lists, just as into their variations. For additional investigations of topological files of different charts and substance structures, see [21-26].

2. Materials and Methods

2.1. The Cayley Tree Γ^k

The Cayley tree Γ^k of order $k \geq 1$ is an infinite and symmetric regular tree, that is, a graph without cycles, from each vertex of which exactly $k + 1$ edges are issued. In this paper, we consider the Cay ley tree $\Gamma_\pi^2 = (V, E, i)$ of order 2 and with n levels from the root x_0, where V is the set of vertices of Γ_π^2, E is the set of edges of Γ_π^2, and i is the incidence function associating each edge $E \in E$ with its end vertices. If $i(e) = \{x, y\}$, then x and y are adjacent vertices, and we write $e =< x, y >$. For any $x, y \in V$, the distance $d(x, y)$ is defined as

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$$d(x,y) = \text{m} \ \{d \mid x = x_1, x_1, x_2, \ldots, x_{d-1}, x_d = y \in V\}$$

such that the pairs $< x_0, x_1 >< , x_1, x_2 >, \ldots, < x_{d-1}, x_d >$ are adjacent vertices [27]. For the root x_0 of the Cayley tree, we have

$$W_\pi = \{x \in V \mid d(x_0, x) = n\}$$
$$V_\pi = \{x \in V \mid d(x_0, x) \leq n\}$$
$$E_n = \{e =< x, y > \in E \mid x, y \in V_\pi\}$$

It is easy to compute the number of vertices reachable in step n or in level n starting from the root x_0, which is $|W_n| = 3 \times 2^{(\pi-1)}$. The number of vertices of Γ_n^2 is $|V_\pi| = 1 + 3 \times (2^\pi - 1)$, and the number of edges of Γ_π^2 is $|E_\pi| = 3 \times (2^\pi - 1)$, as is shown in Figure 1 below.

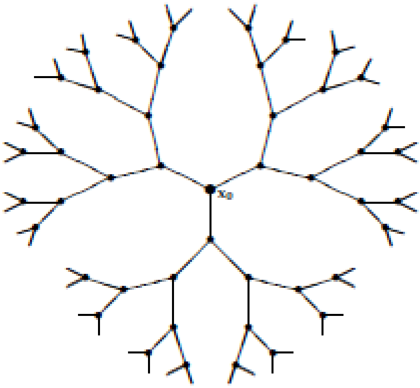

Figure 1 Cayley tree I_π^2 of order 2 with n levels where $n \geq 1$

2.2 The Square Lattice Graph SL_π

Lattice networks are widely used, for example, in distributed parallel computation, distributed control, wired circuits, and so forth. They are also known as grid or mesh networks. We choose a simple structure of lattice networks called a square lattioe because this allows for a theoretical analysis [28].

We consider a square graph $SL_\pi(V, E)$ of size $n \times n$ vertices, where V denotes the set of vertices of SL_π and E denotes the set of edges of SL_m, such that the number of vertices is $|V| = n^2$, and the number of edges is $|E| = 2n(n-1)$, as is shown in Figure 2 below.

Figure 1 Square lattice (SL_n) with $n \geq 1$.

2.3. The Special Graph $\mathbf{G_K}$

Other kinds of special graphs, denoted by $G_{\pi,}$ can be obtained from other subgraphs. The structures in Figure 3 show how to obtain a graph G_π.

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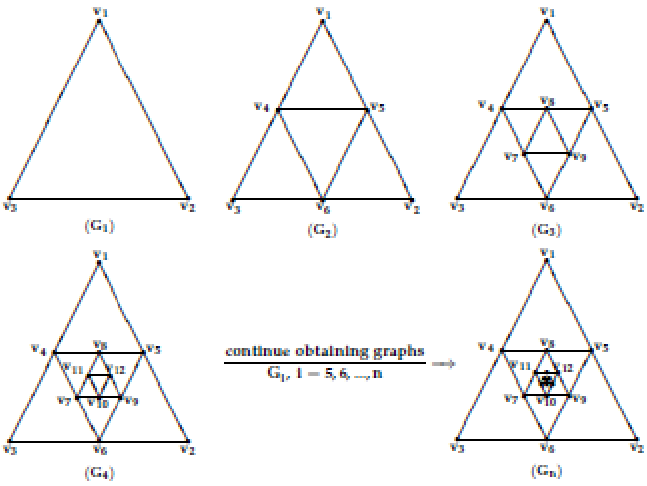

Figure 3 Structures of graphs $\mathbf{G_1, G_2, G_3, G_4 \ldots, G_n}$ of orders $\mathbf{3, 6, 9, 12, \ldots, 3n}$.

In Figure 3, we have obtained the graph sequence $G_1, G_2, G_3, G_4, \ldots, G_n$. We let G_1 be a complete graph of order $3 (G_1 \equiv K_3)$ and let $V(G_1) = \{v_1, v_2, v_3\}$; we have subdivided the three edges of G_1. The new vertices are denoted by $\{v_4, v_5, v_6\}$, and $\mathrm{G}_2[v_4, v_5, v_6] \equiv K_3$. Thus,

$$V(G_2) = V(G_1) \cup \{v_1, v_2, v_3\}$$
$$E(G_2) = \{v_1v_5, v_5v_2, v_2v_6, v_6v_3, v_3v_4, v_4v_1, v_4v_5, v_5v_6, v_6v_4\}$$
$$G_2 = G_1 - \{v_1v_2, v_2v_3, v_3v_1\} + \{v_1v_5, v_5v_2, v_2v_6, v_6v_3, v_3v_4, v_4v_1, v_4v_5, v_5v_6, v_6v_4\}$$

Now, for G_n, we have subdivided the edges $\{v_{3\pi-5}v_{3\pi-4}, v_{3n-4}v_{3n-3}, v_3 - 3^{v_{3\pi-5}}\}$. The new vertices are denoted by $\{v_{3\pi-2}, v_{3\pi-1}, v_{3\pi}\}$ and $G_\pi[v_{3\pi-2}, v_{3\pi-1}, v_{3\pi}] \equiv K_3$. Thus,

$$V(G_m) = V(G_{n-1}) \cup \{v_{3m-2}, v_{3m-1}, v_3\}$$
$$G_\pi = G_{m-1} - \{v_{3\pi-5}v_{3\pi-4}, v_{3\pi-4}v_{3\pi-3}, v_{3\pi-3}v_{3\pi-5}\} + \{v_{3\pi-5}v_{3\pi}, v_{3\pi}v_{3\pi-4}, v_{3\pi-4}v_{3\pi-2}, v_{3\pi-2}v_{3\pi-3}$$
$$v_{3\pi-3}v_{3\pi-1}, v_{3\pi-1}v_{3\pi-5}, v_{3\pi-2}v_{3\pi-1}, v_{3\pi-1}v_3, v_{3\pi}v_{3\pi-2}\}$$

It can be observed that the number of vertices of a graph G_n is $3n$ and that the number of edges is $6n - 3$ or, mathematically, $|V(G_m)| = 3n$, and $|E(G_m)| = 6n - 3$, respectively, where $n \geq 1$

2.4. The Camplede Biportite Grapli $K_{m\pi}$

A graph $K_{m,\pi}$ is a complete bipartite graph if its vertex set can be partitioned into two subsets X and Y, such that one of the two endpoints of each edge in X and the other in Y, as well as each vertex $v \in X$, is adjacent to all vertices of Y, as is shown in Figure 4 below. Clearly, if $|X| = m$ and $|Y| = n$, then $|V(K_{m,\pi})| = m + n$ and $|E(K_{mn})| = mn$. For more details, see [10].

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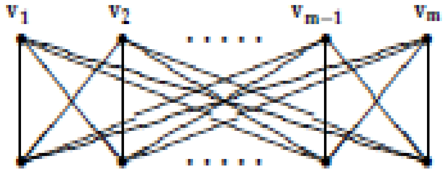

Figure 4 Complete bipartite graph K_{ml} π –

3. Results and Discussion

Prior to presenting our main results, the edge partitions of the Cayley tree, lattice square, G_n and comple te bipartite graph, on the basis of the degrees of end vertices of each edge and the degree sum of the neighbors of end vertices of eachedge, are discussed below.

Referring to Figure 1 , there are two types of edges in Γ_π^2 on the basis of the degrees of the end vertices of eachedge, as follows: the first type, for $e = uv \in E(\Gamma_\pi^2)$, is such that $d_x = 1$ and $d_v = 3$; the other type is for $E = uv \in \mathbb{E}(\Gamma_\pi^2)$ such $d_x = d_v = 3$. In the first type, there are $3 \times 2^{\pi-1}$ edges, and in the other type, because $|E(\Gamma_\pi^2)| = 3 \times (2^n - 1)$, there are $3 \times (2^n - 1) - 3 \times 2^{n-1} = 3 \times (2^n - 2^{n-1} - 1) = 3 \times (2^{n-1} - 1)$ edges, as is shown in Table 1. Similarly, from Figure 1, there are three types of edges in Γ_n^2 on the basis of the degree sum of vertioes lying at a unit distance from the end vertices of each edge, as follows: the first type, for $e = uv \in E(\Gamma_\pi^2)$, is such that $S_\alpha = 3$ and $S_v = 5$; the second type, for $\varepsilon = uv \in E(\Gamma_\pi^2)$, is such that $S_x = 5$ and $S_v = 9$; the third type, for $e = uv \in E(\Gamma_\pi^2)$, is such that $S_x = S_v = 9$. There are $3 \times 2^{\pi-1}, 3 \times 2^{\pi-2}$, and $3 \times (2^\pi - 1) - 3 \times 2^{\pi-1} - 3 \times 2^{\pi-2} = 3(2^\pi - 2^{n-1} - 2^{\pi-2} - 1) = 3 \times 2^{n-2}$ edges in the first, second and third ty pes of Γ_π^2, respectively, as is shown in Table 2 .

Table 1 Edge partition of Γ_π^2 on the basis of degrees of end vertics of eachedge.

$(d_{x,}d_v)$ Where uə ∈ E	Number of Edges
(1,3)	$3 \times 2^{\pi-1}$
(3,3)	$3 \times (2^{(\pi-1)} - 1)$

4. Conclusions

The Randi'c record has been firmly associated with numerous substance properties of particles and has been found to resemble the edge of boiling over and Kovats constants. The ABC file gives a decent model to the soundness of direct and extended alkanes just as for the strain energy of cycloalkanes. For certain physiochemical properties, the prescient force of the

GA file is to some degree better than the prescient force of the Randi'c availability file. In this paper, specific degree-based topological records, specifically, the ABC and GA lists, are contemplated. Still up in the air and registered the shut recipes of the ABC4 and GA5 records for these extraordinary charts and organizations. We have likewise registered their ABC and GA records. These outcomes are novel and critical commitments in chart hypothesis and organization science, and they give a decent premise to comprehend the geography of these diagrams and organizations. Later on, we are keen on contemplating and figuring topological records of a tree with breadth d.

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