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A STUDY ON GRACEFUL GRAPHS

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Abstract:

In this article we discuss about the topic under graph theory such as some basic definition, graceful graphs, Union of two graphs on classes of trees, pendant graceful graphs on edge labeling, Edge odd gracefulness graphs. Graph theory is the representation of an object in a set by graphically, which are associated by links and it is essentially concentrated in Mathematics as well as Computer Science. Graphs can be plotted by using vertices and edges. It is additionally used to examine the particles in physics and chemistry. In material physics we can study the atomic structures simulated by the structure of three dimensional. Also we have so many theorems in graph theory based on the coloring of graph, like edge coloring, vertex coloring. We can also study the path, walk, shortest path, Hamiltonian path etc. Nowadays Graph theory also used in Google map also to find the shortest distance between arrival and destination place.

Keywords: Graceful graphs, Pendant graceful graphs, Edge gracefulness graphs

1.1 Introduction:

Graph theory is a domain in mathematics that is both essential and efficient. In his explanation of the classic Konigsberg bridge problem, Euler established graph theory as a mathematical field. The study of the characteristics of diverse graphs is known as graph theory. Graphs may be used to simulate a variety of real-world scenarios, such as: Participants in the social network and their friendships; the country's cities and roads connecting them;

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communication networks such as the Internet and the World Wide Web; language structure, such as sentence structure; project management, project dependency management; Bioinformatics: protein-protein interactions, residual network communication, genetic control; mathematical relationships such as Fibonacci expansion using trees; decision trees and Besesia networks; electrical circuits: Kirchhoff rules are closely linked to the regional graph structure; and many others. Graph has evolved as a valuable mathematical field in its own right. As a result, there is a demand for a low-cost beginning work on the subject that is appropriate for both mathematicians taking graph theory courses and non-specialists who want to understand the subject as soon as possible. It is my goal that this work contributes to meeting this need.The only prerequisites for reading it are a basic understanding of elementary set theory and matrix theory, however more difficult activities will require knowledge of abstract algebra. A weighted graph is one in which the labels on edges are individuals from an arranged set. Graceful labeling, Edge-graceful labelling, and Lucky labelling are three exceptional instances in graph labelling.

1.1.Graph

A graph which is linear (or simply a graph) G= (V, E) comprises of a bunch of articles V= $\{v_1, v_2,...\}$ called vertices. One more set E= $\{e_1, e_2,...\}$ entire components are called edges, to such an extent that each edge e_k is related to an un ordered pair (v_i, v_j) of vertices.

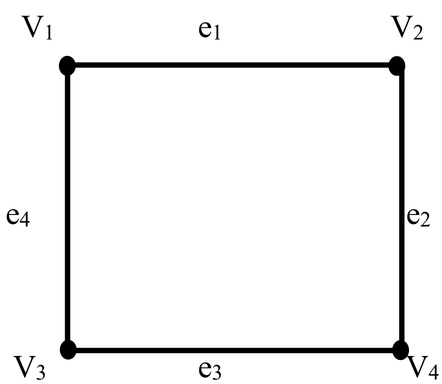
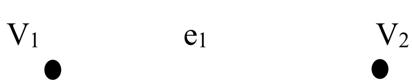


Figure 1.1

V= $\{v_1, v_2, v_3, v_4\}$, E= $\{e_1, e_2, e_3, e_4\}$

1.2.Parallel Edges

The graph contains more than one margin associated with the given vertices known as parallel edges.



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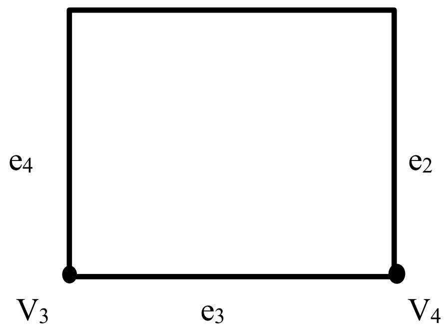


Figure 1.2

In the above figure, {e1, e2} are parallel edges.

1.3.Adjacent

Two non-parallel edges are said to be adjacent if they are on a common vertex.

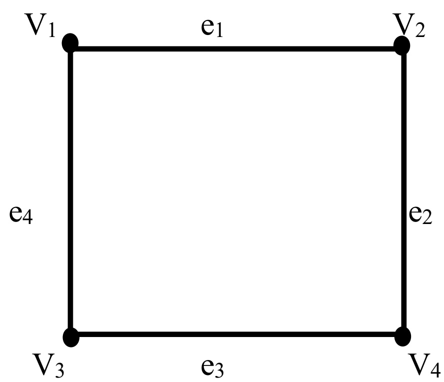


Figure 1.3

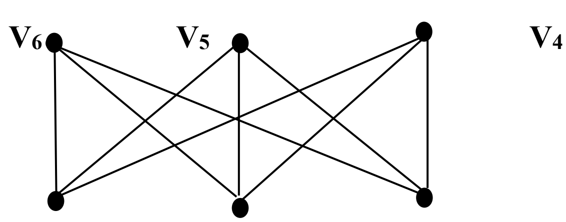
The point e_2 and e_4 are adjacent vertices

1.4.Degree

The number of event edges in vertex v_i with double-counted loops to itself (self-loops) is called the degree of vertex v_i. Indicated by d (v_i).

1.5. Bipartite Graph

The Graph G is said to be a bipartite graph when V is divided into two unrelated V_1 and V_2 headings in such a way that every line of G joints V_1 and V_2 area is called G-bipartition.



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V_1 V_2 V_3

Figure 1.4

2. Trees on graceful graphs

Lemma 2.1:

Let T be a tree with a graceful labeling and lt $u\epsilon V(T)$ be a vertex adjacent to u_1 and u_2. Consider $T = T - (V(Tu, u1) \cup V(Tu, u2))$ and V∈ $V(T), V \neq u$

(a) If u1 ≠ u2 and Ψ (u1) + Ψ (u2) = Ψ(u) + Ψ (v), then the tree obtained by a disjoint union of T^J, Tu,u_1 and Tu,u_2, and connecting v to u1 and u2 is graceful with the same graceful labeling Ψ.

(b) If u1 = u2 and 2Ψ (u1) = Ψ (u) + Ψ (v), then the tree obtained by a disjoint union of T^J and Tu,u_1, and connecting v to u1 is graceful with the same graceful labeling Ψ .

Proof:

It suffices to show that the edge labels of uu1 and uu2 are the same as of vu1 and vu2.

(a) $|\Psi(u_1) - \Psi(u)| = |\Psi(u) + \Psi(v) - \Psi(u2) - \Psi(u)| = |\Psi(v) - (u2)|$

$|\Psi(u_2) - \Psi(u)| = |\Psi(u) + \Psi(v) - \Psi(u1) - \Psi(u)| = |\Psi(v) - (u1)|$

(b) $|\Psi(u_1) - \Psi(u)| = \left|\frac{\Psi(u)+\Psi(v)}{2} - \Psi(u)\right| = \left|\frac{\Psi(u)-\Psi(v)}{2}\right|$

$|\Psi(u_1) - \Psi(v)| = \left|\frac{\Psi(u)+\Psi(v)}{2} - \Psi(v)\right| = \left|\frac{\Psi(v)-\Psi(u)}{2}\right|$

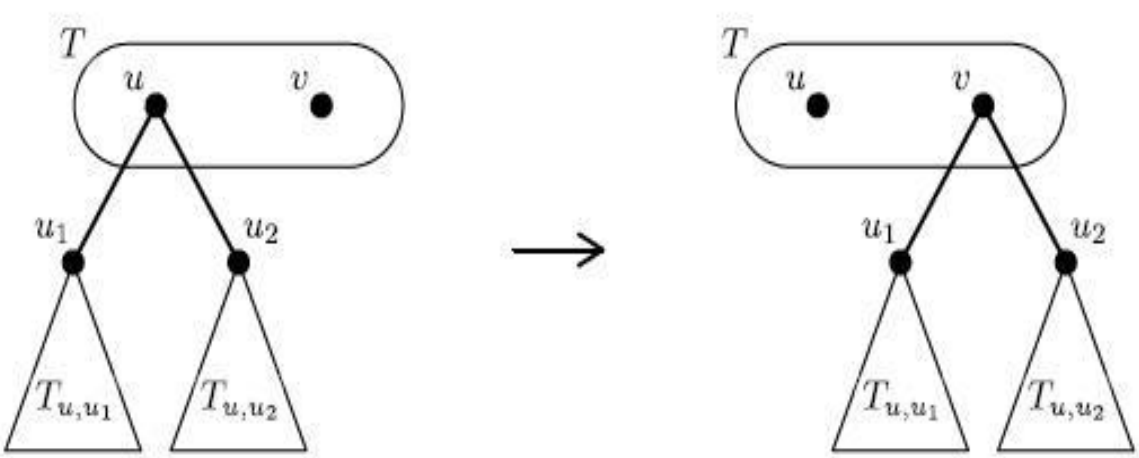

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Figure 2.1: Transfer of sub trees from *u* to *v*.

This operation is called a transfer and we mostly do transfers of leaves from one vertex to another. For the remaining of this section, for a graceful tree, we no longer distinguish the vertex label from the vertex itself since in a tree every number from $[0, n-1]$ must appear as a vertex label.

3. Even Graceful on $C_m \cup P_n$ and T_n CLASS OF TREES

Theorem 3.1

Every T_n tree is even graceful.

Proof:

Let's be a Tn tree with n + 1 vertices. By definition there is a P H (T) method corresponding to Tn.Using EPT, in the construction of the Hamiltonia PH (T) terminal Ed Ed {{d1, d2, d3 ... dr} ....., er}. Apparently Ep and Ed have the same number of edges. The number of T vertices and the number of vertices of the Hamiltonian path are equal to the edges of the Hamiltonia method by {E (T) −Ed} U EP. Now define the PH (T) vertices in sequence as v1, v2, v3, ..., vn + 1 from one vertex of PH (T) to another. Consider the vertex number f: V (P H (T)) → {0, 1, 2, 3… 2q-1} next

$$f(v_i) = \begin{cases} 2\left[\frac{i-1}{2}\right] & if\ i\ is\ odd, 1 \leq i \leq n+1 \\ 2q - 2\left[\frac{i-1}{2}\right] & i\ is\ even, 2 \leq i \leq n+1 \end{cases}$$

Where [ ] denote the integral part. Clearly f is an injective function.

Let $g_{f^*(uv)=|f(u)-f(v)|}$ be the induced mapping defined from the edge set of $P^H(T)$ into the set $\{2,4,6,\dots,2q-2\}$ whenever uv is an edge in $P^H(T)$.

Since $P^H(T)$ is a path, consider an arbitrary edge in $P^H(T)$ is of the form V_iV_{i+1}, $i = 1,2,\dots,2q$.

Case(1): when I is even, then

$$g_f{}^*(V_i, V_{i+1}) = |f(v_i) - f(v_{i+1})|$$

$$= \left|2q - 2\left[\frac{i-2}{2}\right] - 2\left[\frac{i+1-1}{2}\right]\right|$$

$= \left|2q - 2\left[\frac{i-2}{2}\right] - 2\left[\frac{i}{2}\right]\right|$

$= \left|2q - 2\left[\frac{i-2+i}{2}\right]\right|$

$= \left|2q - 2\left[\frac{2i-2}{2}\right]\right|$

$= \left|2q - 4\left[\frac{i-1}{2}\right]\right|$ ……………………………………(1)

Case (2): when n is odd, then

$g_f{}^*(V_i, V_{i+1}) \ = |f(v_i) - f(v_{i+1})|$

$= \left|2\left[\frac{i-2}{2}\right] - \left(2q - 2\left[\frac{i+1-2}{2}\right]\right)\right|$

$= \left|2\left[\frac{i-2}{2}\right] - 2q + 2\left[\frac{i-1}{2}\right]\right|$

$= \left|2q - 4\left[\frac{i-1}{2}\right]\right|$……………………………………(2)

From equation (1) and (2), we get for all I,

$g_f{}^*(V_i, V_{i+1}) \ = \left|2q - 4\left[\frac{i-1}{2}\right]\right|$……………………………………..(3)

It is clear that $g_f{}^*$ is injective and its range is $\{2,4,6, \dots ,2q\}$.

Then f is even graceful on $P^H(T)$.

In order to prove that f is also even graceful on T_n, it is enough to show that $g_f{}^*(d_s) = g_f{}^*(e_s)$.

We have $(V_{i+k}, V_{j-k}) = (V_{i+k,} V_{i+k+1})$, this implies $j - k = i + k + 1 \ j = i + 2k + 1$

One of i,j is odd and other is even.

Case (1): when i is even, $2 \le i \le n$.

$g_f{}^*(d_s) = g_f{}^*(V_i, V_j) = g_f{}^*(V_i, V_{i+2k+1})$

$= |f(v_i) - f(v_{i+2k+1})|$

$= \left|2q - 2\left[\frac{i-2}{2}\right] - 2\left[\frac{i+2k+1-1}{2}\right]\right|$

$= \left|2q - 2\left[\frac{i-2}{2}\right] - 2\left[\frac{i+2k}{2}\right]\right|$

$= |2q - (2i + 2k - 2)|$

$= |2q - 2(i + k - 1)|$

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$= |2(q-i-k+1)|$……………………….(4)

Case (2): when i is odd, $1 \leq i \leq n$

$g_f{}^*(d_s) = |f(v_i) - f(v_{i+2k+1})|$

$= \left|2\left[\frac{i-1}{2}\right] - \left(2q - 2\left[\frac{i+2k+1-2}{2}\right]\right)\right|$

$= \left|2\left[\frac{i-1}{2}\right] - 2q + 2\left[\frac{i+2k-1}{2}\right]\right|$

$= |2(i+k-1) - 2q|$

$= |2q - 2(i+k-1)|$

$= |2(q-i-k+1)|$……………………………….(5)

From (4) and (5) it follows that

$g_f{}^*(d_s) = |2(q-i-k+1)|, 1 \leq i \leq n$………………………….(6)

Now,

$g_f{}^*(e_s) = g_f{}^*(V_{i+k}, V_{j-k}) = g_f{}^*(V_{i+k}, V_{i+k+1})$

$= |f(v_{i+k}) - f(v_{i+k+1})|$

$= \left|2q - 2\left[\frac{i+k-2}{2}\right] - 2\left[\frac{i+k+1-1}{2}\right]\right|$

$= |2q - 2(i+k-1)|$

$= |2(q-i-k+1)|\,, 1 \leq i \leq n$…………..(7)

From (6) and (7)

$$g_f{}^*(e_s) = g_f{}^*(d_s)$$

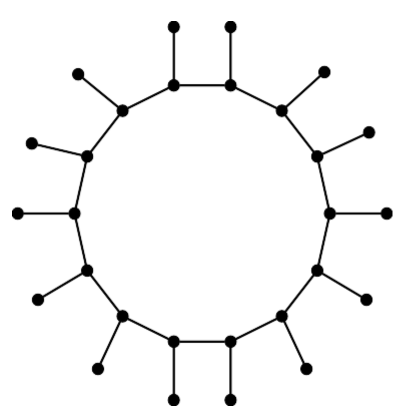
Then f is even graceful on T_n also. Hence the graph T_n tree is even graceful.

4. Pendant Graceful graphs on Edge Labeling

Definition:

Corona G 1⊙G 2 is a graph created by taking one duplicate of G 1, with p 1 vertices, and p 1 duplicates of G 2, also linking the vertex of G 1 by at the edge of all vertex duplicate G 2.

Example 4.1



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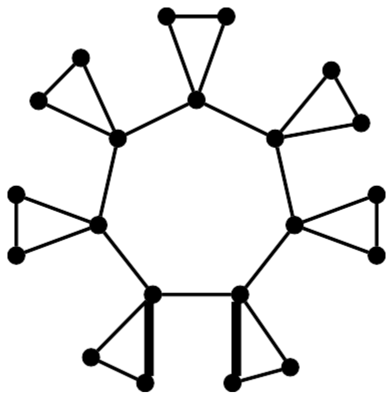

$C_7 \odot K_2$ $C_{14} \odot K_1$

Figure 4.1

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