ISSN PRINT 2319 1775 Online 2320 7876

Research paper Volume 11, Iss 12, 2022

INTUITIONISTIC SEMI * CONNECTEDNESS AND COMPACTNESS ON INTUITIONISTIC TOPOLOGICAL SPACES

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Abstract

In this article certain kinds of intuitionistic semi * connectedness and intuitionistic semi * compactness are defined in intuitionistic topological space and their characteristics are investigated. Here we introduce intuitionistic semi * connectedness, intuitionistic semi * C_i- connectedness (i = 1,2,3,4,5), intuitionistic semi * compactness and obtain many properties.

2020 Mathematics Subject Classification: 54D05, 54E45.

Key Words: intuitionistic semi * connectedness, intuitionistic semi * C_i- connectedness, intuitionistic semi * compactness intuitionistic semi * open, intuitionistic semi * closed, IS*O, IS*C.

1 INTRODUCTION

Atanassov [6] is the person who first presented the idea of intuitionistic set. After that this concept is generalized to intuitionistic sets in [1], [2] and intuitionistic topological spaces in [3]. An idea of intuitionistic connectedness and intuitionistic compactness in intuitionistic topological space is given in [5]. In this article we establish the concepts of intuitionistic semi * connectedness, intuitionistic semi * C_i- connectedness, intuitionistic semi *

ISSN PRINT 2319 1775 Online 2320 7876
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compactness, intuitionistic semi * lindelof spaces. Also we encounter their basic properties and explore their relationship with already existing concepts.

2 PRIME NEEDS

Definition 2.1. Let X be a nonempty fixed set. An intuitionistic set (IS in short) $\tilde{A}$ is an object having the form $\tilde{A}_G = < X, A^{(1)}, A^{(2)} >$ where $A^{(1)}$ and $A^{(2)}$ are subsets of X such that $A^{(1)} \cap A^{(2)} = \emptyset$. The set $^{(1)}$ is called the set of member of $\tilde{A}_G$, while $A^{(2)}$ is called the set of non member of $\tilde{A}_G$.

Definition 2.2. An intuitionistic topology (IT in short) by subsets of a nonempty set X is a family τ of IS's satisfying the following axioms.

(a) $\widetilde{\emptyset}_I$, $\widetilde{X}_I \in \tau$

(b) $\widetilde{U}_G \cap \widetilde{V}_G \in \tau$ for every $\widetilde{U}_G$, $\widetilde{V}_G \in \tau$

(c) $\cup \widetilde{U}_{G_i} \in \tau$ for any arbitrary family $\{ \widetilde{U}_{G_i} : i \in J\} \subseteq \tau$.

The pair (X, τ) is called an intuitionistic topological space (ITS in short) and any IS $\widetilde{U}_G$ in τ is called an intuitionistic open set (IOS). The complement of an IOS $\widetilde{U}_G$ in τ is called an intuitionistic closed set (ICS)

Definition 2.3. Let (X, τ) be an ITS and $\widetilde{U}_G =< X, U^{(1)}, U^{(2)} >$ be an IS in X, $\widetilde{U}_G$ is said to be intuitionistic generalized closed set (briefly Ig – closed set) $Icl(\widetilde{U}_G) \subseteq \tilde{A}_G$ whenever $\widetilde{U}_G \subseteq \tilde{A}_G$ and $\tilde{A}_G$ is IO in X.

Definition 2.4. If $\widetilde{U}_G$ is an IS of an ITS (X, τ), then the intuitionistic generalized closure of $\widetilde{U}_G$ is is denoted by $Icl^*(\widetilde{U}_G)$ and is defined as

$$Icl^*(\widetilde{U}_G) = \{\tilde{E}_G : \tilde{E}_G \text{ is Ig – closed set and } \widetilde{U}_G \subseteq \tilde{E}_G \}.$$

Definition 2.5.

(i) intuitionistic semi * open sets if there is an intuitionistic open set $\widetilde{G}$ in X such that $\widetilde{U}_G \subseteq \tilde{A}_G \subseteq Icl^*(\widetilde{U}_G)$.

(ii) intuitionistic semi * closed set if X - $\tilde{A}_G$ is intuitionistic semi * open.

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Definition 2.6. The intuitionistic semi * interior of $\tilde{A}_G$ is defined as the union of all intuitionistic semi * open sets of X contained in $\tilde{A}_G$. It is denoted by IS*int($\tilde{A}_G$).

Definition 2.7. The semi * closure of an IS $\tilde{A}_G$ is defined as the intersection of all intuitionistic semi * closed sets in X that containing $\tilde{A}_G$. It is denoted by IS*cl($\tilde{A}_G$).

Theorem 2.8. Let (X, τ_I) be an ITS and $\tilde{A}$ be any ITS. Then

(i) $\tilde{A}_G$ is intuitionistic semi * regular if and only if IS $*$Fr($\tilde{A}_G$)= $\tilde{\emptyset}_I$.

(ii) IS $*$Fr($\tilde{A}_G$) = IS $*$cl($\tilde{A}_G$) ∩ IS $*$cl(X − $\tilde{A}_G$).

Definition 2.9. The function f : (X, τ_1) → (Y, τ_2) is said to be intuitionistic semi * continuous (summarizing IS*-Cts) if $f^{-1}(\tilde{A}_G)$ is IS*O in (X, τ_1)for every IOS $\tilde{A}_G$ in (Y, τ_2).

Definition 2.10. Two IS's $\tilde{E}$ and $\tilde{F}$ are said to be overlapping if $\tilde{E} \not\subseteq X - \tilde{F}$. Conversely $\tilde{E}$ and $\tilde{F}$ are said to be nonoverlapping, if $\tilde{E} \subseteq X - \tilde{F}$. Notice that $\tilde{E} \not\subseteq X - \tilde{F}$ if and only if $E^{(1)} \not\subseteq F^{(1)}$ or $E^{(1)} \not\supseteq F^{(2)}$.

3 INTUITIONISTIC SEMI * CONNECTED

Definition 3.1. An ITS (X, τ) is said to be an intuitionistic semi * connected if $\tilde{X}_I$ cannot be expressed as the union of two disjoint nonempty IS*O sets in X.

Theorem 3.2. Every intuitionistic semi * connected is intuitionistic connected.

Proof. Let X be an intuitionistic semi * connected. To prove X is an intuitionistic connected. Suppose X is not an intuitionistic connected. Then there exist a disjoint nonempty IOS $\tilde{U}_G$ and $\tilde{V}_G$ such that $\tilde{X}_I = \tilde{U}_G \cup \tilde{V}_G$. Since $\tilde{U}_G$ and $\tilde{V}_G$ are IOS, both $\tilde{U}_G$ and $\tilde{V}_G$ are IS*O. This is a contradiction to X is an intuitionistic semi * connected. Hence X is an intuitionistic connected.

Remark 3.3. The converse of the above theorem need not be true as shown in the succeeding example

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Example 3.4. Let X = {i, j, k} and τ = {$\widetilde{X}_I$, $\widetilde{\emptyset}_I$, < X, {j}, {i, k} >, < X, {i}, {j} >, < X, {i, j}, ∅ >}. Then IS*O(X, τ) = {$\widetilde{X}_I$, $\widetilde{\emptyset}_I$, < X, {j}, {i, k} >, < X, {i}, {j} >, < X, {i, j}, ∅ >, < X, {i, k}, {j} >}. Clearly X is an intuitionistic connected but not an intuitionistic semi * connected.

Theorem 3.5. Every intuitionistic semi connected is intuitionistic semi * connected.

Proof. Let X be an intuitionistic semi connected. To prove X is an intuitionistic semi * connected. Suppose X is not an intuitionistic semi * connected. Then there exist a disjoint nonempty IS*O sets $\widetilde{U}_G$ and $\widetilde{V}_G$ such that $\widetilde{X}_I = \widetilde{U}_G \cup \widetilde{V}_G$. Since $\widetilde{U}_G$ and $\widetilde{V}_G$ are IS*O, both $\widetilde{U}_G$ and $\widetilde{V}_G$ are ISO sets. This is a contradiction to X is an intuitionistic semi connected. Hence X is an intuitionistic semi *connected.

Remark 3.6. The converse of the above theorem need not be true as shown in the succeeding example.

Example 3.7. Let X = {i, j, k} and τ = {$\widetilde{X}_I$, $\widetilde{\emptyset}_I$, < X, {i}, {j, k} >, < X, {k}, {i, j} >, < X, {i, k}, {j} >}. Then IS*O(X, τ) = {$\widetilde{X}_I$, $\widetilde{\emptyset}_I$, < X, {i}, {j, k} >, < X, {k}, {i, j} >, < X, {i, k}, {j} >, < X, {i}, {k} >, < X, {k}, {i} >, < X, {i. k}, ∅>}. Then X is an intuitionistic semi * connected but not an intuitionistic semi connected.

Theorem 3.8. An ITS (X, τ) has the only intuitionistic semi * regular subsets are $\widetilde{\emptyset}_I$ and $\widetilde{X}_I$ itself then (X, τ) is an intuitionistic semi * connected.

Proof. Assume that $\widetilde{\emptyset}_I$ and $\widetilde{X}_I$ are the only intuitionistic semi * regular subsets of X. To prove X is an intuitionistic semi * connected. Suppose X is not an intuitionistic semi * connected. Then there exist a disjoint nonempty IS*O sets $\widetilde{U}_G$ and $\widetilde{V}_G$ such that $\widetilde{X}_I = \widetilde{U}_G \cup \widetilde{V}_G$. Therefore $\widetilde{U}_G = X - \widetilde{V}_G$ is IS*C. Hence $\widetilde{U}_G$ is an intuitionistic semi * regular which is contradiction to our assumption. Hence X is an intuitionistic semi * connected.

Theorem 3.9. An ITS is an intuitionistic semi * connected if and only if every nonempty proper subsets of X has nonempty intuitionistic semi * frontier.

Proof. Let X be an intuitionistic semi * connected and $\widetilde{A}$ be any nonempty IS of X. To prove IS*Fr($\widetilde{A}$) ≠ $\widetilde{\emptyset}_I$. Suppose IS*Fr($\widetilde{A}$) = $\widetilde{\emptyset}_I$. Then by theorem 2.8, $\widetilde{A}$ is an intuitionistic semi *

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regular. Now by theorem 3.8, $\widetilde{A}$ is not an intuitionistic semi * connected. This is a contradiction to our hypothesis. Therefore IS*Fr($\widetilde{A}$) ≠ $\widetilde{\emptyset}_I$. Conversely, assume that $\widetilde{A}$ is any nonempty IS of X such that IS*Fr($\widetilde{A}$) ≠ $\widetilde{\emptyset}_I$. To prove X is an intuitionistic semi * connected. Suppose X is not an intuitionistic semi * connected. Then there exist a nonempty IS*O sets $\widetilde{U}_G$ and $\widetilde{V}_G$ such that $\widetilde{X}_I = \widetilde{U}_G \cup \widetilde{V}_G$. Therefore $\widetilde{U}_G = X - \widetilde{V}_G$. Hence $\widetilde{U}_G$ is both IS*O and IS*C. Therefore by theorem 2.8, IS*Fr($\widetilde{A}$) = $\widetilde{\emptyset}_I$ which is a contradiction to our assumption. Thus X is an intuitionistic semi * connected.

Theorem 3.10. Let (X, τ_1) and (Y, τ_2) be the two ITS and f: $X \to Y$ be the surjection map, intuitionistic semi * continuous and X be an intuitionistic semi * connected. Then Y is an intuitionistic semi * connected.

Proof. Let f: $X \to Y$ be the surjection, intuitionistic semi * continuous and X be an intuitionistic semi * connected. Assume that Y is not an intuitionistic semi * connected thats lead us to there exist a disjoint nonempty IS*O sets $\widetilde{U}_G$ and $\widetilde{V}_G$ such that $\widetilde{Y}_I = \widetilde{U}_G \cup \widetilde{V}_G$. Since f is an IS*-Cts, $f^{-1}(\widetilde{U}_G)$ and $f^{-1}(\widetilde{U}_G)$ is IS*O in X. Since $\widetilde{U}_G \neq \widetilde{\emptyset}_I$ and $\widetilde{U}_G \neq \widetilde{\emptyset}_I$, $f^{-1}(\widetilde{U}_G) \neq \widetilde{\emptyset}_I$ and $f^{-1}(\widetilde{U}_G) \neq \widetilde{\emptyset}_I$. We have $\widetilde{Y}_I = \widetilde{U}_G \cup \widetilde{V}_G$ implies $f^{-1}(\widetilde{Y}_I) = f^{-1}(\widetilde{U}_G) \cup f^{-1}(\widetilde{V}_G)$. Therefore $\widetilde{X}_I = f^{-1}(\widetilde{U}_G) \cup f^{-1}(\widetilde{V}_G)$ and $f^{-1}(\widetilde{U}_G) \cap f^{-1}(\widetilde{V}_G) = f^{-1}(\widetilde{U}_G \cap$ $VG= f-1\emptyset I=\emptyset I$. Therefore (X, τ_1) is not an intuitionistic semi * connected. This is a contradiction to our hypothesis. Hence (Y, τ_2) is an intuitionistic semi * connected.

Theorem 3.11. Let (X, τ_1) and (Y, τ_2) be the two ITS and f: $X \to Y$ be an injection map IPS*O and IPS*C. If Y is an intuitionistic semi * connected, then X is an intuitionistic semi * connected.

Proof. Assume (X, τ_1) is not an intuitionistic semi * connected thats lead us to there exist a nonvoid IS*O sets $\widetilde{U}_G$ and $\widetilde{V}_G$ such that $\widetilde{Y}_I = \widetilde{U}_G \cup \widetilde{V}_G$ and $\widetilde{U}_G \cap \widetilde{V}_G = \widetilde{\emptyset}_I$. Then $\widetilde{U}_G = X - \widetilde{V}$.Therefore $\widetilde{U}_G$ is both IS*O and IS*C in X. We have f: $X \to Y$ is both IPS*O and IPS*C, $f^{-1}(\widetilde{U}_G)$ is both IS*O and IS*C in Y . Therefore by theorem 2.8, IS $*$ Fr($f^{-1}(\widetilde{U}_G)$) = $\widetilde{\emptyset}_I$. Thus by theorem 3.9, Y is not an intuitionistic semi * connected which is contradiction. Hence (X, τ_1) is an intuitionistic semi * connected.

Theorem 3.12. Let (X, τ_1) and (Y, τ_2) be the two ITS and f: $X \to Y$ is an IS*O and IS*C injection map and (Y, τ_2) is an intuitionistic semi * connected, then (X, τ_1) is an intuitionistic connected.

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Proof. Assume (X, τ_1) is not an intuitionistic connected thats lead us to there exist a nonempty IO sets $\widetilde{U}_G$ and $\widetilde{V}_G$ such that $\widetilde{Y}_I = \widetilde{U}_G \cup \widetilde{V}_G$ and $\widetilde{U}_G \cap \widetilde{V}_G = \widetilde{\emptyset}_I$. Then $\widetilde{U}_G = X - \widetilde{V}_G$. Therefore $\widetilde{U}_G$ is both IOS and ICS in X. Then $\widetilde{U}_G$ is both IS*O and IS*C. Since f is both IS*O and IS*C, $f(\widetilde{U}_G)$ is an intuitionistic semi * regular in Y. Therefore by theorem 2.8, IS $*$Fr(f(($\widetilde{U}_G$)) = $\widetilde{\emptyset}_I$. Thus by theorem 3.9, Y is not an intuitionistic semi * connected which is contradiction. Thus (X, τ_1) is an intuitionistic connected.

Definition 3.13. Let (X, τ) be an ITS and $\widetilde{U}_G$ be any IS of X. If there exist IS*O sets $\widetilde{A}$ and $\widetilde{B}$ in X satisfying the following properties, then $\widetilde{U}_G$ is called intuitionistic semi * C_i– disconnected.

(i) C_1 : $\widetilde{U}_G \subseteq \widetilde{A} \cup \widetilde{B}$, $\widetilde{A} \cap \widetilde{B} \subseteq X - \widetilde{U}_G$, $\widetilde{U}_G \cap \widetilde{A} \neq \widetilde{\emptyset}_I$, $\widetilde{U}_G \cap \widetilde{B} \neq \widetilde{\emptyset}_I$.
(ii) C_2: $\widetilde{U}_G \subseteq \widetilde{A} \cup \widetilde{B}$, $\widetilde{U}_G \cap \widetilde{A} \cap \widetilde{B} = \widetilde{\emptyset}$, $\widetilde{U}_G \cap \widetilde{A} \neq \widetilde{\emptyset}_I$, $\widetilde{U}_G \cap \widetilde{B} \neq \widetilde{\emptyset}_I$.
(iii) C_3: $\widetilde{U}_G \subseteq \widetilde{A} \cup \widetilde{B}$, $\widetilde{A} \cap \widetilde{B} \subseteq X - \widetilde{U}_G$, $\widetilde{A} \nsubseteq X - \widetilde{U}_G$, $\widetilde{B} \nsubseteq X - \widetilde{U}_G$.
(iv) C_4: $\widetilde{U}_G \subseteq \widetilde{A} \cup \widetilde{B}$, $\widetilde{U}_G \cap \widetilde{A} \cap \widetilde{B} = \widetilde{\emptyset}$, $\widetilde{A} \subseteq X - \widetilde{U}_G$, $\widetilde{B} \subseteq X - \widetilde{U}_G$.

Definition 3.14. Let (X, τ) be an ITS and $\widetilde{U}_G$ be any IS of X. If $\widetilde{U}_G$ is said to be an intuitionistic semi * C_i– connected, then $\widetilde{U}_G$ is not an intuitionistic semi * C_i– disconnected where i = 1,2,3,4.

Theorem 3.15. Let (X, τ) be an ITS and $\widetilde{U}_G$, $\widetilde{V}_G$ be any two IS of X. If $\widetilde{U}_G$, $\widetilde{V}_G$ are intuitionistic semi * C_1– connected and $\widetilde{U}_G \cap \widetilde{V}_G \neq \widetilde{\emptyset}_I$, then $\widetilde{U}_G \cup \widetilde{V}_G$ is also an intuitionistic semi * C_1– connected.

Proof. Let $\widetilde{U}_G$, $\widetilde{V}$ be intuitionistic semi * C_1– connected. Suppose $\widetilde{U}_G \cup \widetilde{V}_G$ is not an intuitionistic semi * C_1– connected. Then there exist an IS*O set $\widetilde{C}$ and $\widetilde{D}$ such that $\widetilde{U}_G \cup \widetilde{V}_G \subseteq \widetilde{C} \cup \widetilde{D}$, $\widetilde{C} \cup \widetilde{D} \subseteq X - (\widetilde{U}_G \cup \widetilde{V}_G)$, $(\widetilde{U}_G \cup \widetilde{V}_G) \cap \widetilde{C} \neq \widetilde{\emptyset}_I$ and $(\widetilde{U}_G \cup \widetilde{V}_G) \cap \widetilde{D} \neq \widetilde{\emptyset}_I$. Since $\widetilde{U}_G$ and $\widetilde{V}_G$ are intuitionistic semi * C_1– connected, $\widetilde{U}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$ or $\widetilde{U}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$ or $\widetilde{V}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$. Since $\widetilde{U}_G \cap \widetilde{V}_G \neq \widetilde{\emptyset}_I$, $\widetilde{p}_{IV} \in \widetilde{U}_G \cap \widetilde{V}_G$.

Case (i) Let $\widetilde{U}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$ and $\widetilde{V}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$. Then $(\widetilde{U}_G \cap \widetilde{C}) \cup (\widetilde{V}_G \cap \widetilde{C}) = \widetilde{\emptyset}_I \Rightarrow (\widetilde{U}_G \cup \widetilde{V}_G) \cap \widetilde{C} = \widetilde{\emptyset}_I$ which is a contradiction.

Case (ii) Let $\widetilde{U}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$. Then $(\widetilde{U}_G \cap \widetilde{D}) \cup (\widetilde{V}_G \cap \widetilde{D}) = \widetilde{\emptyset}_I \Rightarrow (\widetilde{U}_G \cup \widetilde{V}_G) \cap \widetilde{D} = \widetilde{\emptyset}_I$ which is a contradiction.

Case (iii) Let $\widetilde{U}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$ and $\widetilde{V}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$. Then $\widetilde{p}_{IV} \notin \widetilde{C}$ and $\widetilde{p}_{IV} \notin \widetilde{D}$. This is impossible because $\widetilde{p}_{IV} \in \widetilde{U}_G \cap \widetilde{V}_G \subseteq \widetilde{C} \cup \widetilde{D}$.

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Case (iv) Let $\widetilde{U}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$. This case is similar to case (iii).
Hence from the above four cases $\widetilde{U}_G \cup \widetilde{V}_G$ is an intuitionistic semi * C_1- connected.

Theorem 3.16. Let (X, τ) be an ITS and $\widetilde{U}_G$, $\widetilde{V}_G$ be any two IS of X. If $\widetilde{U}_G$, $\widetilde{V}_G$ are intuitionistic semi * C_2- connected and $\widetilde{U}_G \cap \widetilde{V}_G \neq \widetilde{\emptyset}_I$, then $\widetilde{U}_G \cup \widetilde{V}_G$ is also an intuitionistic semi * C_2- connected.

Proof. Let $\widetilde{U}_G$, $\widetilde{V}_G$ be intuitionistic semi * C_2- connected. Suppose $\widetilde{U}_G \cup \widetilde{V}_G$ is not an intuitionistic semi * C_2- connected. Then there exist an IS*O set $\widetilde{C}$ and $\widetilde{D}$ such that $\widetilde{U}_G \cup \widetilde{V}_G \subseteq \widetilde{C} \cup \widetilde{D}$,$(\widetilde{U}_G \cup \widetilde{V}_G) \cap \widetilde{C} \cap \widetilde{D} = \widetilde{\emptyset}_I$,$(\widetilde{U}_G \cup \widetilde{V}_G) \cap \widetilde{C} \neq \widetilde{\emptyset}_I$ and $(\widetilde{U}_G \cup \widetilde{V}_G) \cap \widetilde{D} \neq \widetilde{\emptyset}_I$. Since $\widetilde{U}_G$ and $\widetilde{V}_G$ are intuitionistic semi * C_2- connected, $\widetilde{U}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$ or $\widetilde{U}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$ or $\widetilde{V}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$. Since $\widetilde{U}_G \cap \widetilde{V}_G \neq \widetilde{\emptyset}_I$, $\tilde{p}_{IV} \in \widetilde{U}_G \cap \widetilde{V}_G$.

Case (i) Let $\widetilde{U}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$ and $\widetilde{V}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$. Then $(\widetilde{U}_G \cap \widetilde{C}) \cup (\widetilde{V}_G \cap \widetilde{C}) = \widetilde{\emptyset}_I \Rightarrow (\widetilde{U}_G \cup \widetilde{V}_G) \cap \widetilde{C} = \widetilde{\emptyset}_I$ which is a contradiction.

Case (ii) Let $\widetilde{U}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$. Then $(\widetilde{U}_G \cap \widetilde{D}) \cup (\widetilde{V}_G \cap \widetilde{D}) = \widetilde{\emptyset}_I \Rightarrow (\widetilde{U}_G \cup \widetilde{V}_G) \cap \widetilde{D} = \widetilde{\emptyset}_I$ which is a contradiction.

Case (iii) Let $\widetilde{U}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$ and $\widetilde{V}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$. Then $\tilde{p}_{IV} \notin \widetilde{C}$ and $\tilde{p}_{IV} \notin \widetilde{D}$. This is impossible because $\tilde{p}_{IV} \in \widetilde{U}_G \cap \widetilde{V}_G \subseteq \widetilde{C} \cup \widetilde{D}$.

Case (iv) Let $\widetilde{U}_G \cap \widetilde{D} = \widetilde{\emptyset}_I$ and $\widetilde{V}_G \cap \widetilde{C} = \widetilde{\emptyset}_I$. This case is similar to case (iii).
Hence from the above four cases $\widetilde{U}_G \cup \widetilde{V}_G$ is an intuitionistic semi * C_2- connected.

Theorem 3.17. Let (X, τ) be an ITS and $\widetilde{U}_G$, $\widetilde{V}_G$ be any two IS of X. If $\widetilde{U}_G$ and $\widetilde{V}_G$ are overlapping intuitionistic semi * C_3- connected, then $\widetilde{U}_G \cup \widetilde{V}_G$ is also an intuitionistic semi * C_3- connected.

Proof. Assume $\widetilde{U}_G \cup \widetilde{V}_G$ is not an intuitionistic semi * C_3- connected thats lead us to there exist an IS*O sets $\widetilde{E}$ and $\widetilde{F}$ such that $\widetilde{U}_G \cup \widetilde{V}_G \subseteq \widetilde{E} \cup \widetilde{F}$, $\widetilde{E} \cap \widetilde{F} \subseteq X - (\widetilde{U}_G \cup \widetilde{V}_G)$, $\widetilde{E} \not\subseteq X - (\widetilde{U}_G \cup \widetilde{V}_G)$, $\widetilde{F} \not\subseteq X - (\widetilde{U}_G \cup \widetilde{V}_G)$. Since $\widetilde{U}_G$ and $\widetilde{V}_G$ are intuitionistic semi * C_3- connected, $\widetilde{E} \subseteq X - \widetilde{U}_G$ or $\widetilde{F} \subseteq X - \widetilde{U}_G$ and $\widetilde{E} \subseteq X - \widetilde{V}_G$ or $\widetilde{F} \subseteq X - \widetilde{V}_G$. Also by hypothesis $\widetilde{U}_G$ and $\widetilde{V}_G$ are overlapping, there is a point p, $(\tilde{p}_I \in \widetilde{U}_G,\ \tilde{p}_{IV} \in \widetilde{V}_G)$ or there is a point q, $(\tilde{q}_I \in \widetilde{V}_G,\ \tilde{q}_{IV} \in \widetilde{U}_G)$.

ISSN PRINT 2319 1775 Online 2320 7876

Case (i) Let $\widetilde{E} \subseteq X - \widetilde{U}_G$ and $\widetilde{E} \subseteq X - \widetilde{V}_G$. Then $\widetilde{E} \subseteq (X - \widetilde{U}_G) \cap (X - \widetilde{V}_G) = X - (\widetilde{U}_G \cup \widetilde{V}_G)$ which is contradiction to $\widetilde{E} \nsubseteq X - (\widetilde{U}_G \cup \widetilde{V}_G)$.

Case (ii) Let $\tilde{F} \subseteq X - \widetilde{U}_G$ and $\tilde{F} \subseteq X - \widetilde{V}_G$. This is similar to case (i).

Case (iii) Let $\widetilde{E} \subseteq X - \widetilde{U}_G$ and $\tilde{F} \subseteq X - \widetilde{V}_G$. Suppose there is a point p, ($\tilde{p}_I \in \widetilde{U}_G$, $\tilde{p}_{IV} \in \widetilde{V}_G$). Since $\widetilde{E} \subseteq X - \widetilde{U}_G$ and $\tilde{F} \subseteq X - \widetilde{V}_G$, $\widetilde{U}_G \cup \widetilde{V}_G \subseteq \widetilde{E} \cup \tilde{F} \subseteq (X - \widetilde{U}_G) \cup (X - \widetilde{V}_G) = X - (\widetilde{U}_G \cap \widetilde{V}_G)$. Therefore $\widetilde{U}_G \cap \widetilde{V}_G \subseteq X - (\widetilde{U}_G \cup \widetilde{V}_G) = (X - \widetilde{U}_G) \cup (X - \widetilde{V}_G)$. We have $\tilde{p}_I \in \widetilde{U}_G$ and $\tilde{p}_{IV} \in \widetilde{V}_G \Rightarrow \tilde{p}_{IV} \in \widetilde{U}_G \Rightarrow \tilde{p}_{IV} \in \widetilde{U}_G \cap \widetilde{V}_G \subseteq (X - \widetilde{U}_G) \cap (X - \widetilde{V}_G) \Rightarrow \tilde{p}_{IV} \in X - \widetilde{U}_G$ and $\tilde{p}_{IV} \in X - \widetilde{V}_G$ which is a contradiction. Similarly if there is a point q, ($\tilde{q}_I \in \widetilde{V}_G$, $\tilde{q}_{IV} \in \widetilde{U}_G$), we get a contradiction.

Case (iv) Let $\widetilde{E} \subseteq X - \widetilde{V}_G$ and $\tilde{F} \subseteq X - \widetilde{U}_G$. This is similar to case (iii).
Therefore from the above four cases $\widetilde{U}_G \cup \widetilde{V}_G$ is an intuitionistic semi * C_3- connected.

Theorem 3.18. Let (X, τ) be an ITS and $\widetilde{U}_G$, $\widetilde{V}_G$ be any two IS of X. If $\widetilde{U}_G$ and $\widetilde{V}_G$ are overlapping intuitionistic semi * C_4- connected, then $\widetilde{U}_G \cup \widetilde{V}_G$ is also an intuitionistic semi * C_4- connected.

Proof. The proof is similar to previous theorem.

Definition 3.19. The ITS (X, τ) is said to be an intuitionistic semi * C_5- disconnected if there exists an IS*O and IS*C set $\tilde{E}_G$ such that $\tilde{\phi} \neq \tilde{E}_G \neq \tilde{X}$.

An ITS (X, τ) is called intuitionistic semi * C_5- connected (summarizing IS*-C_5 ctd) if X is not an intuitionistic semi * C_5- disconnected.

Theorem 3.20. Every IS*-C_5 ctd space implies intuitionistic connected.

Proof. Let (X, τ) be an IS*-C_5 ctd. Assume X is not an intuitionistic connected thats lead us to there exist a nonempty IOS $\widetilde{U}_G$ and $\widetilde{V}_G$ such that $\tilde{X} = \widetilde{U}_G \cup \widetilde{V}_G$ and $\widetilde{U}_G \cup \widetilde{V}_G = \tilde{\phi}$. Since $\widetilde{U}_G$ and $\widetilde{V}_G$ are IOS, both $\widetilde{U}_G$ and $\widetilde{V}_G$ are IS*O. We have $\widetilde{U}_G \cap \widetilde{V}_G = \tilde{\phi}$ and $\widetilde{U}_G \cup \widetilde{V}_G = \tilde{X}$. Therefore $U_G^{(1)} \cap V_G^{(1)} = \phi$, $U_G^{(2)} \cup V_G^{(2)} = X$, $U_G^{(1)} \cup V_G^{(1)} = X$ and $U_G^{(2)} \cap V_G^{(2)} = \phi$. Thus $\widetilde{U}_G =$

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X − $\tilde{V}_G$ and $\tilde{V}_G$ = X − $\tilde{U}_G$. Therefore $\tilde{U}_G$ and $\tilde{V}_G$ are intuitionistic semi * regular which is contradiction to our assumption. Hence (X, τ) is an intuitionistic connected.

Theorem 3.21. Every IS*-C_5 ctd space implies intuitionistic C_5-connected.

Proof. Assume (X, τ) is not an intuitionistic C_5− connected thats lead us to there exist an intuitionistic clopen set $\tilde{E}_G$ such that $\tilde{\phi} \neq \tilde{E}_G \neq \tilde{X}$. Since $\tilde{E}_G$ is an intuitionistic clopen, $\tilde{E}_G$ is both IS*O and IS*C set. Thus $\tilde{E}_G$ is not an IS*-C_5 ctd which is a contradiction to our assumption. Thus (X, τ) is an intuitionistic C_5−connected.

Theorem 3.22. Every intuitionistic semi C_5-connected space implies IS*-C_5ctd.

Proof. Assume (X, τ) is not an IS*-C_5 ctd thats lead us to there exist a nonempty proper IS $\tilde{E}_G$ of X such that $\tilde{E}_G$ is an intuitionistic semi * regular. Since $\tilde{E}_G$ is both IS*O and IS*C, $\tilde{E}_G$ is an ISO and ISC. Thus X is an intuitionistic semi C_5− disconnected which is a contradiction to our assumption. Hence (X, τ) is an IS*-C_5 ctd.

Theorem 3.23. Every IS*-C_5 ctd space implies IS*-ctd.

Proof. Assume (X, τ) is not an IS*-ctd thats lead us to there exist nonempty IS*O sets $\tilde{E}_G$ and $\tilde{F}_G$ in (X, τ) such that $E_G^{(1)} \cup F_G^{(1)}$ = X, $E_G^{(2)} \cap F_G^{(2)}$ = ϕ, $E_G^{(1)} \cap F_G^{(1)}$ = ϕ and $E_G^{(2)} \cup F_G^{(2)}$ = X. Therefore $\tilde{E}_G$ = (X− $\tilde{F}_G$). Hence $\tilde{E}_G$ is both IS*O and IS*C. Thus X is an IS*-C_5 disconnected. Hence X is an IS*-ctd.

4 INTUITIONISTIC SEMI * COMPACT SPACES

Definition 4.1. Let $\tilde{Đ}$ be a family of IS*O sets of X, and let (X, τ) be an ITS. Then the collection $\tilde{Đ}$ is called an intuitionistic semi * open cover (summarizing IS*-OC) of X if ⋃ $\tilde{Đ}$ = $\tilde{X}_I$.

Definition 4.2. An ITS (X, τ) is said to be an intuitionistic semi * compact (summarizing IS*-cpt) if every IS*-OC of X has a finite subcover.

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Theorem 4.3. Let (X, τ) be an ITS. Then the following results hold.

(i) Every IS*-cpt implies intuitionistic compact.

(ii) Every intuitionistic semi compact implies IS*-cpt.

Proof. (i) Let (X, τ) be an IS*-cpt and $\{\widetilde{U}_\alpha\}$ be an intuitionistic open cover for X. Then $\{\widetilde{U}_\alpha\}$ is an IS*-OC for X. Since X is an IS*-cpt, $\{\widetilde{U}_\alpha\}$ has a finite subcover. Hence X is an intuitionistic compact.

(ii) Let (X, τ) be an intuitionistic semi compact and $\{\widetilde{D}_\alpha\}$ be an IS*-OC for X. Then $\{\widetilde{D}_\alpha\}$ is an intuitionistic semi open cover for X. Since X is an intuitionistic semi compact, $\{\widetilde{D}_\alpha\}$ has a finite subcover. Hence (X, τ) is an IS*-cpt.

Theorem 4.4. Let (X, τ) be an ITS. Then (X, τ) is IS*-cpt if and only if every family of IS*C sets in X with void intersection has a finite subfamily with void intersection.

Proof. Let (X, τ) be an IS*-cpt and $\{\widetilde{U}_\alpha\}_{\alpha\in J}$ be a family of IS*C sets in X such that $\cap\{\widetilde{U}_\alpha\}_{\alpha\in J} = \widetilde{\emptyset}_I$. Then $\cup\ \{X - \widetilde{U}_\alpha\}_{\alpha\in J} = \widetilde{X}_I$ is an IS*-OC for X. Since X is an IS*-cpt, X has a finite subcover, namely $\{X - \widetilde{U}_{\alpha1}, X - \widetilde{U}_{\alpha2}, ..., X - \widetilde{U}_{\alpha n}\}$ for X. Therefore $\widetilde{X} = \cup_{i=1 \text{ to } n} \{X - \widetilde{U}_{\alpha i}\}$. Thus $\cap_{i=1 \text{ to } n} \{\widetilde{U}_{\alpha i}\} = \widetilde{\emptyset}_I$. Conversely, assume that every family of IS*C sets in (X, τ) with empty intersection has a finite subfamily with void intersection. Let $\{\widetilde{D}_\alpha\}_{\alpha\in J}$ be an IS*-OC for (X, τ) . Then $\cup\ \{\widetilde{D}_\alpha\}_{\alpha\in J} = \widetilde{X}_I$. Therefore $\{X - \widetilde{D}_\alpha\}_{\alpha\in J} = \widetilde{\emptyset}_I$. Since $X - \widetilde{D}_\alpha$ is IS*C set for each $\alpha \in J$, by hypothesis there is a finite subfamily has a empty intersection. That is $\cap_{i=1 \text{ to } n} (X - \widetilde{D}_\alpha) = \widetilde{\emptyset}_I$. Then $\cup_{i=1 \text{ to } n} \widetilde{D}_\alpha = \widetilde{X}_I$. Hence (X, τ) is an IS*-cpt.

Theorem 4.5. Let (X, τ_1) and (Y, τ_2) be any two ITS and $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ be an IS*O function. If (Y, τ_2) is an IS*-cpt, then (X, τ_1) is an IS*-cpt.

Proof. Let $\{\widetilde{F}_\alpha\}$ be an IS*-OC for (X, τ_1). Then $\{f(\widetilde{F}_\alpha)\}$ is an IS*-OC for (Y, τ_2). Since (Y, τ_2) is an IS*-cpt, $\{f(\widetilde{F}_\alpha)\}$ has an finite subcover, namely $\{f(\widetilde{F}_{\alpha1}), f(\widetilde{F}_{\alpha2}), ..., f(\widetilde{F}_{\alpha n})\}$. Therefore $\{\widetilde{F}_{\alpha1}, \widetilde{F}_{\alpha2}, ..., \widetilde{F}_{\alpha n}\}$ is a finite subcover for(X, τ_1). Hence (X, τ_1) is an IS*-cpt.

Theorem 4.6. Let (X, τ_1) and (Y, τ_2) be any two ITS and $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ be an IS*O function. If (Y, τ_2) is an IS*-cpt, then (X, τ_1) is an intuitionistic compact.

IJFANS International Journal of Food And Nutritional Sciences
Official Publication of International Association of Food and Nutrition Scientists

ISSN PRINT 2319 1775 Online 2320 7876
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Proof. Let $\{\widetilde{E}_\alpha\}$be an intuitionistic open cover for (X, τ_1). Since f is an IS*O and $\{\widetilde{E}_\alpha\}$ is an intuitionistic open cover for (Y, τ_2), {f ($\widetilde{E}_\alpha$)} is an IS*-OC for (Y, τ_2). Since (Y, τ_2) is an IS*-compact, {f ($\widetilde{E}_\alpha$)} has an finite subcover,namely {f($\widetilde{E}_{\alpha 1}$), f($\widetilde{E}_{\alpha 2}$), ..., f($\widetilde{E}_{\alpha n}$)}. Therefore {$\widetilde{E}_{\alpha 1}$, $\widetilde{E}_{\alpha 2}$, ..., $\widetilde{E}_{\alpha n}$} is a finite subcover for(X, τ_1). Hence (X, τ_1) is an intuitionistic compact.

Theorem 4.7. Let (X, τ_1) and (Y, τ_2) be any two ITS and f : (X, τ_1) → (Y, τ_2) be a surjection and IS*-Cts function. If (X, τ_1) is an IS*-cpt, then (Y, τ_2) is an intuitionistic compact.

Proof. Let $\{\tilde{F}_\alpha\}$ be an intuitionistic open cover for (Y, τ_2). Since f is an IS*-Cts, $\{f^{-1}(\tilde{F}_\alpha)\}$ is an IS*-OC for (X, τ_1). Since (X, τ_1) is an IS*-cpt, $\{f^{-1}(\tilde{F}_\alpha)\}$ has finite subcover, namely $\{f^{-1}(\tilde{F}_{\alpha 1}), f^{-1}(\tilde{F}_{\alpha 2}), ..., f^{-1}(\tilde{F}_{\alpha n})\}$. Therefore {$\tilde{F}_{\alpha 1}$, $\tilde{F}_{\alpha 2}$, ..., $\tilde{F}_{\alpha n}$ } is a finite subcover for (Y, τ_2). Hence (Y, τ_2) is an intuitionistic compact.

Definition 4.8. An ITS (X, τ) is said to be an intuitionistic semi * Lindelof (summarizing IS*-L) if every IS*-OC contains countable subcover.

Theorem 4.9. Let f : (X, τ_1) → (Y, τ_2) be an surjection, IS*-Cts and (X, τ_1) be an IS*-L. Then (Y, τ_2) is an intuitionistic lindelof.

Proof. Let (X, τ_1) be an IS*-L and $\{\tilde{F}_\alpha\}$ be an intuitionistic open cover for (Y, τ_2).Then $\{f^{-1}(\tilde{F}_\alpha)\}$ is an IS*-OC for (X, τ_1). Since (X, τ_1) is IS*-L, $\{f^{-1}(\tilde{F}_\alpha)\}$ contains a countable subcover say, $\{f^{-1}(\tilde{F}_{\alpha n})\}$. Then $\{\tilde{F}_{\alpha n}\}$ has a countable subcover for (Y, τ_2). Thus (Y, τ_2) is an intuitionistic lindelof.

Theorem 4.10. Let f : (X, τ_1) → (Y, τ_2) be an surjection, IS*-Irresolute and (X, τ_1) be an IS*-L. Then (Y, τ_2) is an IS*-L.

Proof. Let (X, τ_1) be an IS*-L and $\{\tilde{F}_\alpha\}$ be an IS*-OC for (Y, τ_2). Then $\{f^{-1}(\tilde{F}_\alpha)\}$ is an IS*-OC for (X, τ_1). Since (X, τ_1) is IS*-L, $\{f^{-1}(\tilde{F}_\alpha)\}$ contains a countable subcover say, $\{f^{-1}(\tilde{F}_{\alpha n})\}$. Then $\{\tilde{F}_{\alpha n}\}$is a countable subcover for (Y, τ_2). Thus (Y, τ_2) is an IS*-L.

Theorem 4.11. Let f : (X, τ_1) → (Y, τ_2) be an intuitionistic pre semi * open and (Y, τ_2) be an IS*-L. Then (X, τ_1) is an IS*-L.

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Proof. Let (Y, τ_2) be an IS $*-$L and $\{\widetilde{D}_\alpha\}$ be an IS*-OC for (X, τ_1). Then $\{f(\widetilde{D}_\alpha)\}$ is an IS*-OC for Y . Since (Y, τ_2) is IS*-L, $\{f(\widetilde{D}_\alpha)\}$ contains a countable subcover say, $\{f(\widetilde{D}_{\alpha n})\}$. Then $\{\widetilde{D}_{\alpha n}\}$ is a countable subcover for (X, τ_1). Thus (X, τ_1) is an IS*-L.

Theorem 4.12. Let $f : (X, \tau_1) \to (Y, \tau_2)$ be an IS*O function and (Y, τ_2) be an IS*-L. Then (X, τ_1) is an intuitionistic lindelof.

Proof. Let (Y, τ_2) be an IS $*-$L and $\{\widetilde{D}_\alpha\}$ be an intuitionistic open cover for (X, τ_1). Then $\{f(\widetilde{D}_\alpha)\}$ is an IS*-OC for (Y, τ_2). Since (Y, τ_2) is IS*-L, $\{f(\widetilde{D}_\alpha)\}$ contains a countable subcover say, $\{f(\widetilde{D}_{\alpha n})\}$. Then $\{f(\widetilde{D}_{\alpha n})\}$ is a countable subcover for (X, τ_1). Thus (X, τ_1) is an intuitionistic lindelof.

5 CONCLUSION

The different qualities of intuitionistic semi * connectedness and compactness are covered in this article. We will continue to investigate different concepts, such as maximal and minimal open sets, separation axioms in IS*O sets.

6 ACKNOWLEDGEMENT

My heartful thanks to my guide Dr. M. Navaneethakrishnan for his valuable guidance on writing this paper and thanks to the referees for their time and comments.

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