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Graph theory in Pair sum tree

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Abstract

One of the importance classes of graphs is the trees. The importance of trees is evident from their application in various areas, especially theatrical computer science and molecular evolution. The edges of a tree are called branches. A graph with a pair sum labeling defined on it is called pair sum graph. Obtain from the path pm by appending n new pendent an edge to a vertex of the path pm adjacent to one vertex.

Keywords:
Edges, pendent, path, vertices, adjacent, pair sum tree, etc.,

Introduction

Graph Theory is concerned with various types of networks or really models of networks called graphs. These are not the graphs of analytic geometry, but what are often described as pointes connected by lines. The preferred terminology is vertex for a point and edge of a line. The lines need not be straight lines and in simply a visualization of a graph is not a geometric definition.

Theorem

Let G be the tree with V(G)= V(Bn,m□ □ □ zj:1≤j≤5} and

E□*G* □□ *E* □*Bn,m*□□□*z*1 *z*2, *z*2 *x* , *x z*3, *z*3*y* , *yz*4, *z*4 *z*5□ □*xy*□.Then *G* is a pair sum tree.

Proof

Define a function g: *V* □ *G* □□□□1,□2,…, □□*n*□*m*□ 7□□

By g□*x* □□□4,

g□*y* □□2,

g□*z*1□□□

6,

g□*z*2□□□1,

g□

*z*3□□1,g□

*z*4□□3,

g □ *z*5□□4.

Case 1): *n*□*m*.

g□*x*j□□ □6□ *j* , 1□ *j* □ *m*

andg□*yj*□□8□*j* , 1□*j* □ *m*.

Case 2): *n*□*m*.

Assign the label to *xj*, *yj*□1 □*j*□*n*□ as in case 1).

Define

g(yn+j)=8+n+j, 1≤j≤□ $\frac{m □ n}{2}$ □

and g □ *y*□(*m* -n)/ 2□□*j*□□ -*12*-n-j , 1□ *j* □ □ $\frac{m □ n}{2}$ □ .

Then *G* is a pair sum graph.

Theorem

Let *G* be the tree with *V*□*G* □□ *V* □*Bn,m*□□□*zj*:1 □ *j* □ 5□

and *E* □ *G* □□ *E* □*Bn*,*m*□□□*yz*1, *z*1 *z*2, *z*2 *z*3, *yz*4, *z*4 *z*5□. Then *G* is a pair sum graph.

Proof

Define a map

g:*V* □ *G*□□□□1,□2,… , □□*n*□*m*□ 7□□

by g□ *x* □□1,

g□ *y*

□□2, g□

*z*1□□3,

g□ *z*2□□ □4,

g □ *z*3□□

□1, g□

*z*4□□ □5,

g□ *z*5□□4.

Case 1): *n* □ *m*.

g□*xj*□ □ □7□ *j* , 1□ *j* □ *n*

and g□*yj*□ □4□ *j* , 1□ *j* □ *n*.

case 2): n<m

Assign the label to xj and, yj (1≤ j≤ m) as in case 1Define

$\overline{m □ n}$

g(yn+j)= 4+n+j, 1≤ j≤ □ 2 □

$\overline{n □ m}$

g □ *x*□(*n* □m)/ 2□□*j* □□ □□□n□j , 1□ *j* □ □ 2 □ .

case 3): n>m

Assign the label to xj, yj(1≤j≤m) as in case 1.

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n □ m

Define g(xm+j)= -7-m-j, 1≤ j≤ □ 2 □

n □ m

g □ x□(n □m)/ 2□□j □□ □□m+ j , 1□ j □ □ 2 □ .

Then *G* is a pair sum graph.

Theorem

The tree with V□ *G* □□ *V* □*Bn,m*□□□*zj*:1□ *j* □5□□nd

E □ *G*□□ *E* □*Bn,m*□□□ *z*1 *z*2, *z*2 *x*, *yz*3, *z*3 *z*4, *z*4 *z*5□□Then *G* is a pair sum graph.

Proof

Define a map *g* : *V* □*G* □□□□1,□2,…, □□*n*□*m*□

7□□by g□*x*□□ □□,g□*y* □□1,g□ *z*1□□ □6,

g□ *z*2□□ □1,g□ *z*3□□2,g□

z4□□3,g□ *z*5□□4.

Case 1): *n*□*m*.

g□*x*1□□ □5,

g□*xj*□1□□ □6□ *j* , 1□ *j*□ *n*

□1,g□ *y*1□□8 and

g□*xj*□1□□9□ *j* , 1□ *j*□ *n* □1

case 2): n<m

Assign the label to xj and, yj (1≤ j ≤ n) as in case 1

m □ n

g □ x□(n □m)/ 2□□j□□ -*10*-n-j , 1□ j □ □ 2 □ .

case 3): n>m

Assign the label to xj, yj(1≤j≤m) as in case 1.

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Define $g(x_{m+j})= -5-m-j,\ 1\leq j\leq \left\lceil \frac{n-m}{2} \right\rceil$ and

$g\left(x_{(n-m)/2+j}\right) = \ldots m+j,\ 1\leq j \leq \left\lceil \frac{n-m}{2} \right\rceil$

.Then G is a pair sum graph.

Theorem

Let G be the tree with $V(G) = V(B_{n,m}) \cup \{z_j : 1\leq j \leq 4\}$ and

$E(G) = E(B_{n,m}) \cup \{xz_1, z_1y, yz_2, z_2z_3, z_3z_4\} - \{xy\}$.Then G is a pair sum tree.

Proof

Define a function $g: V(G) \to \{\pm1, \pm2, \ldots, \pm(n+m+6)\}$

by $g(x) = -1$,

$g(y) = 1$,

$g(z_1) = -4$, $g(z_2) = 2$,

$g(z_3) = 3$, $g(z_4) = 4$.

Case 1): $n = m$

$g(x_j) = -6-j,\ 1\leq j \leq m$

and $g(y_j) = 6+j,\ 1\leq j \leq m$

case 2): n<m

Assign the label to xj and, yj (1≤j≤n) as in case 1 for 1≤j≤m

Define $g(y_{n+j})= -7-n-j,\ 1\leq j\leq \left\lceil \frac{m-n}{2} \right\rceil$ and g□ y□(n □m)/ 2□j□□6+n+j , 1□ j □ □ 2 □ .

case 3): n>m

Assign the label to xj, yj(1≤j≤m) as in case 1.

Define g(xm+j)= -5-m-j, 1≤ j≤ □ $\frac{n □ m}{2}$ □

g □ x□(n □m)/ 2□□j□□ □□m+ j , $\overline{1□ j}$ □ □2 □ .

Then G is a pair sum graph.

Theorem

LetGbe the tree with V □G □□ V □Bn,m□□□zj:1□j □4□ and

E □G□□ E □Bn,m□□□yz1, z1 z2, z2 z3, z3 z4□□ Then G is a pair sum tree.

Proof

Define a function

g: V □G □□□□1,□2,…, □□n□m□
6□□by g□x □□1,g□y □□2,g□
z1□□3,
g□ z2□□ □1,g□ z3□□ □2, g□ z4□□ □3.

Case 1): n□m

g□xj□□ □□□j , 1□ j □ n

and g□y1□j□□3□ j , 1□ j □ n □1.

case 2): n<m

Assign the label to xj and, yj (1≤ j≤ n) as in case 1 for 1≤ j ≤ m

Define g(yn+j)= 2+n+j, 1≤ j ≤ □ $\frac{m □ n}{2}$ □

m □ n

and g□ x□(n □m)/ 2□□j□□-7-n-j , 1□ j □□ 2 □ .

case 3): n>m

Assign the label to xj, yj(1≤j≤m) as in case 1.

Define g(xm+j)= -6-m-j, 1≤ j ≤□ 2 □

n □ m

g□ x□(n □m)/ 2□□j□□ □□m+ j , 1□ j □

□ 2 □

.Then *G* is a pair sum graph.

Theorem

The tree G with *V* □ *G* □□ *V* □*Bn,m*□□□*zj*:1□ *j* □4□

E □ *G* □□ *E* □*Bn,m*□□□*z*1 *z*2, *z*2 *x* , *x*z3, *z*3 *z*4, *z*4 *y*□ □*xy*□.Then *G* is a pair sum graph.

Proof

Define a map g: *V* □ *G* □□□□1,□2,..., □□*n*□*m*□ 6□□ by

g□ *x* □□1,

g□ *y* □□

□1,

g□*z*1□□3,

g□ *z*2□□2,

g □ *z*3□□ □4,g□ *z*4□□ □2.

Case 1): n=m

g□*xj*□ □ 4□ *j* , 1□ *j* □ *n*

and g□*yj*□ □ □5□ *j* , 1□ *j* □

n.**case 2): n<m**

Assign the label to xj and, yj (1≤j≤n) as in case 1

m □ n

Define g(yn+j) = 6+n+j, 1≤ j ≤□ 2 □and

$m \square n$

$g \square x \square (n \square m)/2 \square \square j \square \square$ -5-n-j , $\square$ and g $\square x \square (n \square m)/2 \square \square j \square \square \square \square \square$m- j ,
$1 \square j \square \square$ $1 \square j \square . \square$

case 3): n>m

Assign the label to xj, yj(1≤j≤m) as in case 1

Define g(xm+j)= 4+m+j, 1≤j≤□ 2

$\frac{n \square m}{2} \square$.

Then G is a pair sum graph.

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