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MAGIC LABELING FOR CERTAIN GRAPHS

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Abstract:

Graph theory is one of the few branches of mathematics that may be said to have a precise starting date. In 1736, Leonhard Euler solved a celebrated problem, known as the Konigsberg bridges problem. The question had been posed whether it was possible to walk over all the seven bridges spanning the river pregel in Konigsberg just once without retracing one's footsteps. Euler reduced the question to a graph theoretical problem, and found an ingenious solution. Euler's solution marked not only the introduction of the discipline of graph theory but also the first applications of the discipline to a specific problem. Since its inception, graph theory has been exploited for the solution of numerous practical problems, and today still retains an applied character. In the early days, very important strides were made in the development of graph theory by the investigation of some very concrete problems, e.g. Kirchhoff's study of

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electrical circuits, and Cayley's attempts to enumerate chemical isomers. Also many branches of mathematics such as group theory, matrix theory, numerical analysis, probability and topology have their interaction with graph theory. It has also become more and more clear in recent years that the two disciplines of graph theory and computer science have much in common and that each is capable of assisting significantly in the development of the other.

Introduction

A magic square is a set of integers arranged in the shape of a square with the total of the entries in the rows, columns, and diagonals always being the same. This kind of problem of building a magic square has been around for about 4000 years, with the first documented occurrence in China around 2200 BC. The well-known magic squares are linked to magic graphs. Attempts to apply this idea to graphs were attempted in the 1960s. The concept is to label the vertices and edges of a graph with positive integers in such a manner that the total of all the labels associated with the vertices or edges is a constant, and a graph with this characteristic is called magical.

Vertex magic, vertex antimagic and vertex bimagic labeling

In 2002, MacDougall et al. [31] introduced the notion of vertex magic total labeling. A one-to-one mapping λ from $V \cup E$ onto the integers $\{1, 2, ..., p + q\}$ is a vertex magic total labeling of a graph $G(V, E)$ if there is a constant k so that for every vertex u, weight of the vertex under λ, $wt_{\lambda(u)} = \lambda(u) + \sum_{UV \in E} \lambda(uv) = K$..

In [31] the authors have studied properties of vertex magic graphs and identified families of graphs having vertex magic total labeling and also classes of graphs which do not admit vertex magic total labeling. Vertex magic total labeling has also been extensively studied by many authors [11, 17, 33, 36]. MacDougall et al. [32] further introduced the concept of super vertex magic labeling. An vertex magic total labeling λ of a graph G is called super vertex magic labeling if $\lambda(V) = \{1, 2, ..., p\}$. Swaminathan and Jeyanthi [45] called a vertex magic total labeling λ of G as E-super vertex magic labeling if $\lambda(E) = \{1, 2, ..., q\}$. Recently Marimuthu and Balakrishnan [34] studied E-super vertex magic labeling extensively.

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The definition of (*a*, *d*) vertex antimagic total labeling was introduced by Baca et al. in [16]. A graph *G* is said to be (*a, d*) vertex antimagic total graph if there exist positive integers *a*, *d* and a bijection λ from $V \cup E$ on to the set of consecutive integers $\{1, 2, ..., p + q\}$ such that the induced mapping $g_\lambda : V \rightarrow W$ is also a bijection, where $W = \{wt_\lambda(x)/x \in V\} = \{a, a+d, a+2d, ..., a+(p-1)d\}$ is the set of weights of vertices in *G*. Such a labeling is called super if the smallest possible labels appear on the vertices. Yegnanarayanan [51] defined several variations of vertex magic labeling and vertex antimagic labeling namely, (1, 1) vertex magic labeling, (1, 0) vertex magic labeling, (0, 1) vertex magic labeling, (1, 1) − (*a*, *d*) vertex antimagic labeling, (1, 0) − (*a*, *d*) vertex antimagic labeling, (0, 1) − (*a*, *d*) vertex antimagic labeling and also investigated the existence of such labeling on a number of classes of graphs. Further (1, 1) vertex bimagic labeling was introduced by Baskar Babujee in [19].

MAGIC LABELLING

Sedlacek introduced magic labeling with impetus of magic squares. From 1960‟s several authors are working on various types of magic labeling on divergent regional anatomies of graphs. Majority of them studied various properties of the graphs. Magic labeling is exhibited in terms of Vertex magic total labeling, Edge magic total labeling and Total magic labeling.

The graphs that scrutinized are limited, modest and undirected. The standard notations to denote the graph are followed. An imprecise remission for graph theoretic notations is [2]. The graphs used in this thesis are paths, cycles, wheels, fan graphs and friendship graphs. Imprecise definitions of path, cycle, wheel, fan graph, friendship graph are as follows.

Path graph: A path is a populated graph $P_n(V,E)$of the form $V = \{v_1 v_2 \ldots v_n\}$and $E = \{(v_0,v_1), (v_1,v_2) \ldots (v_{n-1} v_n)\}$where $\forall i v_i$are well defined and *n*is the path length.

Cycle graph: A cycle graph$C_n(V,E)$ where$n \geq 3$, $V = \{v_1 v_2 \ldots v_n\}$and $E = \{(v_0 v_1, v_1, v_2 \ldots v_{n-1} v_n, v_n v_0,)\}$where $\forall i v_i$are distinct and *n*is the cycle size.

Wheel graph: A wheel graph $W_n(V,E)$is a cycle of size*n*with central hub and all vertices of cycle are adjacent to it.

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Fan graph: A Fan graph F_n $(V,E)$is a path of length *n*with a pivotal axis and all vertices of path adjacent to central hub.

Friendship graph: A Friendship graph $T_n(V,E)$consists of *n*triangles with one common vertex called as hub where *n*size of friendship graph and each triangle is a cycle of size 3.

Labeled graphs are becoming an increasingly useful family of Mathematical Models for a extensive scope of applications such as Conflict resolution in social psychology, electrical circuit theory and energy crisis, Coding Theory problems, incorporating the design of good Radar location codes, Synch set codes, Missile guidance codes and helix codes with unsurpassed autocorrelation properties and in determining ambivalence in X ray Crystallographic dissection, to Design Communication Network addressing Systems, in determining Optimal Circuit Layouts and Radio Astronomy., etc. Designated graphs are replicating pre eminent role substantially in the field of computer science. This appraisal showed its consequence in networking channels, data mining, cryptography, SQL query solving, etc. This colossal range of applications imputed us towards labeled graphs.

Hence the concentration is to design algorithms for various kinds of magic labeling. Labeling is the operation of allocating integers to graph elements under some constraint. If the constraint is applied on only vertex set V then it is called vertex magic total labeling, if the constraint is on the edge set E then it is called edge magic total labeling, if the constraint is applied on both vertices and edges it leads to total magic labeling. In other words categorizing of a graph is a map that takes graph elements such as vertices and edges to numbers usually non negative integers. The whole latest survey of graph labeling is done by Gallian[1]. Different Kinds of labeling are discussed in [3]. Kotzig and Rosa [4] worked on a magic valuation of a graph which is vertex magic labeling.

The survey Gallian collected everything they could find on graph labeling. This survey includes a detailed table of contents and index. It will give the discerning and amiable labeling on a variety of graphs. It gave the magic type labeling and antimagic type labeling. They gave miscellaneous for some other graphs. They have collected everything they could find on graph

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labeling and the survey includes a detailed table of contents and index. They give the modish and harmonious labeling on some graphs. And also show the variations of graceful and congenial labeling. And also they gave the magic type labeling and antimagic type labeling. They gave details of various types of labeling for a variety of graphs.

Various Techniques of Graph Labeling

In the previous chapter, we have mentioned the fundamental concepts and terminology of graphs. This chapter is aimed to discuss cordial labeling of graphs in detail. For occurrence the problems arise from coloring of the vertices of a graph remained unsolved for more than ten decades for its solution in 1976. The problem of catalogue of isomers in the hydrocarbon series C_nH_{2n+2} initiated by the first work of Cayley is as old as the map coloring problem. In recent times, new contexts have emerged wherein the labeling of the vertices or edges of a given graph by elements of certain subsets S of the set of real numbers R. This problem provided enough motivation to formulate more terse mathematical problems on graph labelings. The concept of β-valuation was introduced by Alexander Rosa [6] in 1967. Independent discovery of β-valuation termed as graceful labeling by S.W.Golomb [7] in 1972, who also pointed out the importance of studying graceful graphs in trying to settle another complex problem of decomposing the complete graph by isomorphic copies of a given tree of the same order.

The most popular Ringel-Kotzig-Rosa [6] conjecture and various attempts to settle it provided the reason for different ways for labeling of graph structures. This chapter is focused on graceful and graceful like labelings of graphs.

3.2 Labeling of Graph

If the vertices are assigned values subject to specified condition in a graph, then it is known as graph labeling. Most interesting graph labeling problems have following three

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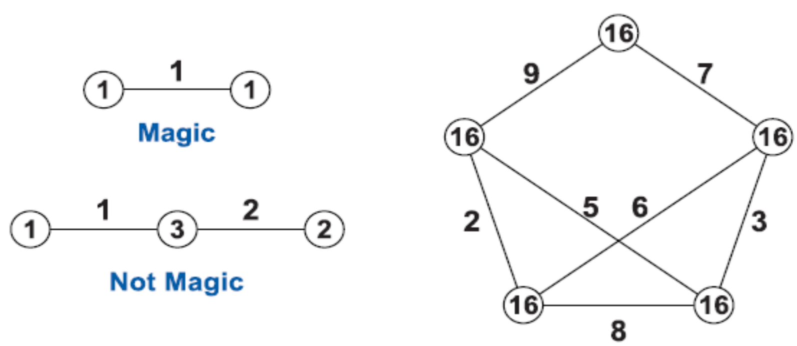
important rules. A set of numbers from which vertex labels are chosen.A rule that assigns a value to each edge.A condition that these values must satisfy. Now discussion about various graph labeling techniques will be carried out in sequential order as they were introduced.

3.3 Various Techniques of Graph Labeling

3.3.1 : Magic Labeling

Magic labeling was introduced by Sedl`acˇek [8] in 1963 motivated through the notion of magic squares in number theory. A function f is called magic labeling of a graph G if $f : V \cup E \longrightarrow \{1, 2, . . . , p + q\}$ is bijective and for any edge $e = (u, v)$, the value of $f(u) + f(v) + f(e)$ is constant. A graph which admits magic labeling is called magic graph.



figure–3.1

Some known results about magic labeling are listed.

Stewart [9] proved that

• $K_n$is magic for n = 2 and all n ≥ 5.

• $K_{n,n}$is magic for all n ≥ 3.

• Fans $F_n$are magic if and only if n ≥ 3 and n is odd.

• Wheels $W_n$are magic for all n ≥ 4.

For any magic labeling f of graph G, there is a constant such that for all edges $e = (u, v) \in G$, $f(u) + f(v) + f(e) = c(f)$. The magic strength $m(G)$ is defined as the minimum of $c(f)$, where the minimum is taken over all magic labeling of G.

The above definition and some facts listed below. They were obtained given by S. Avadyappanet. al. [10].

- $m(P_{2n}) = 5n + 1$, $m(P_{2n+1}) = 5n + 3$,
- $m(C_{2n}) = 5n + 4$, $m(C_{2n+1}) = 5n + 2$,
- $m(K_{1,n}) = 2n + 4$.

Hegde and Shetty [11] defined $M(G)$ analogous to $m(G)$ as follows:

$M(G) = max\{c(f)\}$, where maximum is taken over all magic labeling f of G.

For any graph G with p vertices and q edges following inequality holds:

$$p + q + 3 \leq m(G) \leq c(f) \leq M(G) \leq 2(p + q).$$

3.3.2 Graceful Labeling

A function f is called graceful labeling of a graph $G = (V, E)$ if $f : V \rightarrow \{0, 1, . . . , q\}$ is injective and the induce function $f^* : E \longrightarrow \{1, 2, . . . , q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge $e = (u, v) \in E$. A graph which admits graceful labeling is called graceful graph. Initially Rosa named above defined labeling as β−valuation. Golomb [7] renamed β−valuation as graceful labeling. We will discuss graceful labeling in detail in Chapter 4.

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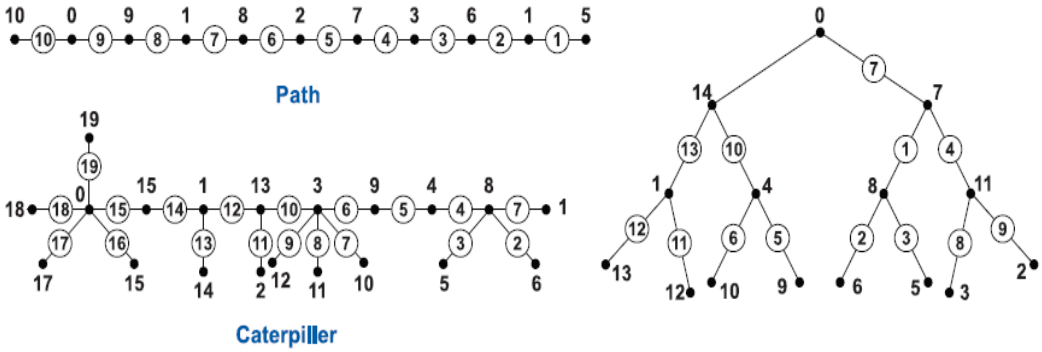

figure−3.2

3.3.3 Graceful Like Labeling

In 1967 Rosa [6] gave another result of graceful labeling

A function *f* is called graceful like labeling of a graph $G = (V, E)$ if $f : V \rightarrow \{0, 1, \ldots, q + 1\}$ is injective and the induce function $f^* : E \rightarrow \{1, 2, \ldots, q - 1, q + 1\}$ defined as

$f^*(e) = |f(u) - f(v)|$ is bijective for every edge $e = (u, v) \in E$.

Some known results of Graceful-like labeling are mentioned below.

• Frucht [12] has shown that $P_m \cup P_n$ admits graceful like labeling with edge labels $\{1, 2, \ldots, q - 1, q + 1\}$. $G \cup K_2$ (where G is graceful graph) admits graceful like labeling.

• Seoud and Elsahawi [13] have proved that all cycles C_n admit graceful like labeling.

• Barrientos [14] proved that cycle C_n is having graceful like labeling with edge labels $\{1, 2, \ldots, q - 1, q + 1\}$ if and only if $n \equiv 1$ or 2 (mod 4)

3.3.4 Harmonious Labeling

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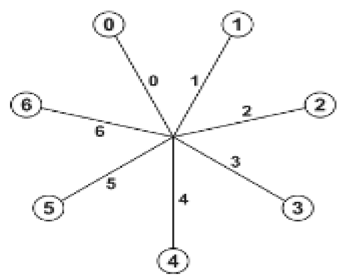
Graham and Sloane [15] introduced harmonious labeling in 1980. They have introduced this during their study of modular versions of additive bases problems stemming from error correcting codes.

A function f is called Harmonious labeling of a graph $G = (V, E)$ if $f : V \to \{0, 1, . . . , q - 1\}$ is injective and the induce function $f^* : E \to \{1, 2, . . . , q - 1\}$ defined as $f^*(e) = (u, v) = (f(u) + f(v)) \bmod q$ is bijective.

A graph which admits harmonious labeling is called harmonious graph. We will demonstrate harmonious labeling by means of following examples in figure 3.3.



figure−3.3

Graham and Sloane observed that if graph G is a tree then exactly two vertices are assigned same vertex label in harmonious labeling. Some known results about harmonious graph are listed below.

• Liu and Zhang [16] proved that every graph is a subgraph of a harmonious graph.

• Graham and Sloane [15] posed a conjecture Every tree is harmonious. In connection of above conjecture, Alderd and Mckay [17] proved that trees with 26 or less vertices are harmonious. They also proved that

– Caterpillars are harmonious.

– Cycles C_n are harmonious if and only if $n \equiv 1,3 \pmod 4$.

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– Wheels W_n are harmonious for all n.

– $C_m \times P_n$ is harmonious if n is odd.

– K_n is harmonious if and only if $n \leq 4$.

– $K_{m,n}$ is harmonious if and only if m or $n = 1$.

– Fans F_n are harmonious for all n.

• Liu [16] proved that all helms are harmonious.

• Jungreis and Reid [18] proved that grids $P_m \times P_n$ are harmonious if and only if (m, n) 6= (2, 2). In the same paper they proved that $C_m \times P_n$ is harmonious if $m = 4$ and $n \geq 3$.

• Gallian et al. [19] proved that $C_m \times P_n$ is harmonious if $n = 2$ and m 6= 4.

3.3.5 Elegant Labeling

Elegant labeling was introduced by Chang et al. [20] in 1981.

A function f is called elegant labeling of a graph G if $f : V \rightarrow \{0, 1, 2, . . . , q\}$ is injective and the induced function $f^{*}: E \rightarrow \{1, 2, . . . , q\}$ defined as $f^{*}(e = (u, v)) = (f(u) + f(v)) \bmod (q + 1)$ is bijective.

A graph which admits elegant labeling is known as elegant graph. We will note that as in harmonious labeling it is not necessary to make an exception for trees. Some known results for elegant labeling are listed below.

• Chang et al. [20] proved that C_n is elegant when $n \equiv 0, 3 \pmod 4$ and not elegant when $n \equiv 1 \pmod 4$ and P_n is elegant when $n \equiv 1, 2, 3 \pmod 4$.

• Cahit [21] proved that P_4 is the only path which is not elegant.

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• Balakrishnan et al. [22] proved that every simple graph is a subgraph of an elegant graph.

• Deb and Limaye [23] defined near elegant labeling by replacing co-domain of edge function $f^*$by $\{1, 2, . . . , q - 1\}$ and they proved that triangular snakes where the number of triangles is congruent to 3 (mod 4) are near elegant.

3.3.6 Concept of Prime Labeling

The concept of prime labeling was originated by Entringer and it was introduced in a paper by Tout et al. [24]

A graph G with p vertices and q edges is said to have a prime labeling if $f : V \rightarrow \{1, 2, . . . , p\}$ is bijective function and for every edge $e = (u, v)$ of G, gcdof $f(u)$ *and* $f(v)$ is 1.

• Around 1980 Entringer conjectured that All tree have a prime labeling. So far there has been little progress towards the proof of this conjecture.

• Some known classes of trees having prime labeling are paths, caterpillars, stars etc.

• Deretsky et al. [25] proved that

- All cycles have prime labeling.
- Disjoint union of C_{2k} and $C_n$have prime labeling.
- The complete graph $K_n$does not have a prime labeling for $n \geq 4$.

• Lee et al. [26] proved that $W_n$have prime labeling if and only if n is even.

• Seoud et al. [27] proved that all helms, fans, $K_{2,n}$, $K_{3,n}$ (where n 6= 3, 7) $P_{n+\bar{K}_2}$ (where n = 2 or n is odd) are having prime labeling. He also proved that $P_{n+\bar{K}_m}$does not have prime labeling if $m \geq 3$.

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• Seoud and Youssef [13] shown that $P_{n+\bar{K}_2}$ is having prime labeling if and only if n = 2 or n is odd.

In 1991 Deretsky et al. [25] introduced the notion of dual of prime labeling which is known as vertex prime labeling. According to them a graph with q edges has vertex prime labeling if its edges can be labeled with distinct integers {1, 2, . . . , q} such that for each vertex of degree at least two the greatest common divisor of the labels on its incident edges is 1. Some known results for vertex prime labeling are listed below.

• Deretsky et al. [25] proved that

– Forests, all connected graphs are having vertex prime labeling.

– $C_{2k} \cup C_n$, $C_{2n} \cup C_{2n} \cup C_{2k+1}$, $C_{2n} \cup C_{2n} \cup C_{2t} \cup C_k$ and $5C_{2m}$ are having vertex prime labeling.

– A graph with exactly two component one of them is not an odd cycle has a vertex prime labeling.

– 2 regular graph with at least two odd cycles does not have a vertex prime labeling.

– 2regular graph with at least two odd cycles does not have a vertex prime labeling

– He also conjectured that 2 regular graph has a vertex prime labeling if and only if it does not have two odd cycles.

3.3.7 k-Graceful Labeling

A natural generalization of graceful labeling is the notion of k-graceful labeling which was independently introduced by Slater [28] and by Maheo and Thuillier [29] in 1982. A function f is called k-graceful labeling of a graph $G = (V, E)$ if $f : V(G) \rightarrow \{0, 1, . . . , k + q - 1\}$ is injective and the induced function $f^* : E(G) \longrightarrow \{k, k + 1, k + 2, . . . , k + q - 1\}$ defined as

$f^*(e) = |f(u) - f(v)|$ is bijective for every edge $e = (u, v) \in E(G)$. A graph G is called k-graceful graph if it admits a k-graceful labeling.

Some known results for k-graceful graph are listed below.

• Liang and Liu [30] proved that $K_{m,n}$ is k-graceful, for all m, n ∈ N and for all k.

• Bu et al. [31] proved that $P_n \times P_2$ and $(P_n \times P_2)^S(P_n \times P_2)$ are k-graceful for all k.

• Acharya [32] proved that a k-graceful Eulerian graph with q edges satisfies one of the following:

(1) q ≡ 0 (mod 4), q ≡ 1 (mod 4) if k is even, (2) q ≡ 3 (mod 4) if k is odd.

3.3.8 Cordial Labeling

Cahit [21] introduced the concept of cordial labeling in 1987 as a weaker version of graceful and harmonious labeling. A function $f : V \rightarrow \{0, 1\}$ is called binary vertex labeling of a graph G and $f(v)$ is called label of the vertex v of G under f. For an edge e = (u, v), the induced function $f^*: E \rightarrow \{0, 1\}$ is given as $f^*(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0), e_f(1)$ be number of edges of G having labels 0 and 1 respectively under f^*. A binary vertex labeling f of a graph G is called cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and let $|e_f(0) - e_f(1)| \leq 1$. A graph which admits cordial labeling is called cordial graph.

In this chapter we have discussed various graph labeling techniques in detail. The discussion includes basic definitions and known results for some of the labeling technique. This chapter gives wider angle about various labeling techniques and it will provide ready reference for any researcher. The next chapter is devoted to the discussion on double path union and its α-labeling of some graphs.

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CONCLUSION

Graph labeling is becoming more interesting filed due to its broad range of applications. A vital role have played by labeled graphs in various fields of graph theory. Coding theory, missile guidance codes, design of good Radar type codes, astronomy, circuit design, X-ray crystallography, data base management are few names of such important fields. This chapter gives an overview of graph labeling as well as some information of important applications.

Here we would like to enhance the graph labeling applications in the filed of computer science. Graph labeling applications have been studied and here we explore the usage of this field in several area like communication networks, image processing, data mining, crypto systems and bird view has been proposed. Graph theory has been applied in investigation of electrical network is a collection of components and device interconnected electrical gazettes.

The network components are idealized physical devices and system, in order to represent several properties. Also they must obey the Kirchhoff's law of currents and voltage.

Bipartite Graph and Time Table Scheduling

Allocation of classes and subject to all the teachers in an institute is one of major issues, whenever constrain and complexity occur. A bipartite graph helps to solve such problem. Also it play an important role in this kind of problems. For m teachers and n subjects available periods p, the time table has to be prepared as follow. A bipartite graph G, we mean a set of teachers v_1, v_2, . . . ,$v_m$and another set of subjects u_1, u_2, . . . , u_n. These vertices have p_i periods. It is presumed that any one period, each teacher may engage almost one subject. Also each subject can be taught by maximum one teacher. For the first period, the time table for this single period correspond to a matching in the bipartite graph G and conversely, each matching correspond to a possible assignment of some teachers to subject taught during that period.

Hence, the solution for this will be obtained by partitioning the edges of the given graph into minimum number of matching. Also the edge have to be colored with minimum number of colors and this problem can be solved by the vertex coloring algorithm. The line graph of given graph has equal number of vertices and edges of the given graph. Also the vertices in the line graph are adjacent iff they are incident in the given graph. The line graph is a simple graph and its proper coloring gives a proper edges coloring of the given graph.

Application in Communication Network

For any kind of application, it depends on problem scenario a kind of graph is used for representing the problem. a suitable labeling is applied on that graph in order to solve the problem. Given a set of transmitters, each station is assigned a channel number(a positive integer) such that interference should be avoided. The smaller distance between two stations has stronger interference. Hence, the difference in channel assignment has to be greater.

Here each vertex represents a transmitter and any its pair connected to the neighbouring transmitters. Radio labeling is used to get effective network. For this we define some terminology as take $G = (V(G), E(G))$ a connected graph and $d(u, v)$ = distance between any two vertices of .

The Maximum distance between any pair of vertices is the diameter of G

and denote it by diam(G). A radio labeling on G, we mean an injective function $f : V(G) \longrightarrow N \cup \{0\}$ and define it such a way so that for any $u, v \in V(G)$, $|f(u) -$

$f(v)| \geq diam(G) - d(u, v) + 1$. The span of f is the difference of the largest and the smallest channel used, that is $max\{f(u) - f(v)\}$, *for every* $u, v \in V(G)$. The radio number of the given graph G is the maximum span of radio labeling of G and denote it by $\gamma_n(G)$. Given a set of transmitters, each station is defined as a channel such that the interference can be minimise or avoided.

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REFERENCES

[1] Anjali.N and Sunil Mathew, Energy of a fuzzy graph, An*nals of fuzzy Mathematics and Informatics*, Vol.6, No.3, (2013), 455-465.

[2] Arindam Dey, Vertex coloring of a fuzzy graph using Alpa cut, *IJMIE*, Vol.2, Issue 8, Aug. 2012.

[3] Anjalay Kishore and M.S.Sunitha, Chromatic number of fuzzy graphs, *Annals of fuzzy Mathematics and informatics,* (2013) 1–

[4] K.Arjunan and C.Subramani, Note on fuzzy graph, *International Journal of Emerging Technology and Advanced Engineer ing,* Vol.5, Issue 3, March 2015.

[5] R.Balakrishnan and K.Ranganathan, A text book of graph theor*y, Springer,* 2000.

[6] R.B.Bapat and S.Pati, Energy of a graph is never an odd integer, *Bull.Kerala Math. Assoc*. 1 (2004),129–132.

[7] Bhutani.K.R., On Automorphism of fuzzy graphs, *Pattern Recognition Lett9* (1989) 159–162.

[8] Chandrashekar Adiga and M.Smitha, On Maximum degree energy of a graph, *Int. J. Contemp. Math. Sciences,* Vol.4, (2009), No.8, 385–396.

[9] Chiranjib Mukherjee and Dr.Gyan Mukherjee, Role of Adjacency Matrix in Graph theory, *IOSR Journal of Computer Engineering,* Vol.16, Issue 2, Ver.III(Mar-Apr.2014) 58–63.

[10] Changiz Eslahchi and B.N.Onagh, Vertex strength of fuzzy graphs, *International journal of Mathematics and Mathematical Sciences,* (2006), 1–9.

[11] Douglas B. West, Introduction to Graph Theory, Second Edition, *PHI Learning Private Limited (*2009).

[12] K. M. Dharmalingam and R. Udaya Suriya, Chromatic excellence in fuzzy graphs, *Bulletin of the international mathematical virtual institute, V*o.7 (2017),305,315.

[13] M.Fathalian, R.A.Borzooei and M.Hamidi, Fuzzy magic labeling of simple graphs, *Journal of Applied Mathematics and Comput ing*, 60 (2019),369–385.

[14] Gallian.J.A, A Dynamic Survey of Graph Labeling, *The Electronic Journal of Combinatories* (2013).

[15] K.A.Germina and S. Hameed, Thomas Zaslavsky, On products and line graphs of signed graphs, their eigenvalues and energy, *Linear Algebra Appl.* 435 (2011), 2432–2450.

[16] Bundy, D., The Connectivity of Commuting Graphs, *J. Combin. Theory Se*r. A 113, Issue 6 (2006) 995–1007.

[17] S. Gnaana Bhragsam and S.K.Ayyaswamy, Neighbourly irregular graphs, *Indian J.Pure appl. Math* Vol.35, No.3, (2004),309–399.

[18] I. Gutman, The energy of a graph, *Ber. Math. Statist. Sekt. Forschung-szenturm Graz.*,103 (1978), 1–22.

[19] I. Gutman, O.E.Polansky, Mathematical concepts inorganic Chemistry, *Springer*-V*erlag*, Berlin, 1986.

[20] I. Gutman, The energy of a graph: Old and new results, In: A. Betten, A.Kohnert, R.Laue and A.Wasserman(Eds.), A*lgebraic Combinatorics and* A*pplications*, Springer, Berlin, (2001),196– 211.

[21] I. Gutman, S. Zare Firoozabadi, J. A. de la Pena, J. Rada, On the energy of regular graphs, MATCH *Commun. Math. Comput. Chem.*, 57(2007),435–442.

[22] Haray. F, Graph Theor*y, Narosa Publishing House*, New Delhi(1988).

[23] Hikoe Enomoto, Anna S. Llado, Tomoki Nakamigawa and Ger hard Ringel, Super EdgeMagic graphs, *SUT Journal of Mathe matics,* Vol. 34, No.2 (1988), 105-109.

[24] Huda Mutab Al Mutab, Fuzzy Graphs, *Journal of Advances in Mathematics,* Vol.17 (2019).

[25] Igor Shparlinski, On the energy of some circulant graphs, *Linear* A*lgebra and its Applications*, 414 (2006), 378-382.

[26] G.Indulal and A.Vijyakumar, Energies of some non-regular graphs, *Journal of Mathematica; Cemistry* ,vol.42, No.3, October 2007.

[27] Ivan Gutman, Jia-Yu Shao, The energy change of weighted graph, *Linear Algebra and its Application*s, 435(2011), 2425–2431.

[28] John N. Moderson, Premchand S.Nair, Fuzzy graphs and Fuzzy Hypergraphs, *Physica-Verlag Heidelberg*, 2000.

[29] Jack H.Koden and Vincent Moulton, Maximal Energy graphs, A*dvances in* A*pplied Mathematics*, 26, (2001)47–52.

[30] K.Kalpana and S.Lavanya, Connectedness Energy of fuzzy graph, *Journal of Computer and Mathematical Sciences,* Vol.9(5),485– 492, May 2018.

[31] Kinkar Ch.Das, Seyed Ahmad Mojalal, On Energy and Laplacian Energy of graphs, *Electronic Journal of Linear Algebra,* Vol.31, (2016), pp 167–186.

[32] S.Lavanya and R.Sattanathan Fuzzy total coloring of fuzzy graphs, *International Joural of Information Technology and Knowledge Management,* Vol.2, No.1,(2009), pp.37–39.

[33] J.N. Moderson, C. S. Peng, Operations on fuzzy graphs, *Information Sciences 7*9 (1994) 159-170.

[34] V. Mythili, M. Kaliyappan & S. Hariharan, A Review of fuzzy graph theory, International Journal of Pure and Applied Mathe matics, Vol.113,No.12 2017, pp 187-195.

[35] S.Lavanya and R.Sattanathan Fuzzy total coloring of fuzzy graphs, *International Joural of Information Technology and Knowledge Management,* Vol.2, No.1, (2009), pp.37-39.

[36] J.N. Moderson & P.S.Nair, *Information Scienc*es 90(1996) 39-49.

[37] D.Muthuramakrishnan and G.Jayaraman , *Total Coloring of split ting graph of path, cycle and Star graphs*,Int.J.Math.And Appl.,6 2018,659-664.

[38] Masthan Raju.U „Sharietf Basha.S,Application of Reverse vertex magic labeling of a graph, *International Journal of Innovative Technology and Exploring Engineering,* Vol.8, Issue 11, Sep.2019.

[39] S.Munoz, T.Ortuno, J.Ramirez and *Graphs, Omega* 32(2005),211-221.

[40] Nagoorgani .A., and Chandrasekaran V.T.,*Free nodes and busy nodes of a fuzzy graph, east Asian Math.J* 22(2006), No.2,163-172.

[41] Nagoorgani.A & Malarvizhi.J.Properties of *u*-complement of fuzzy graph, *International journal of Algorithms, Computing and mathematics,* Vol.2, No.3, and August 2009.

[42] A. Nagoorgani and K.Radha, The degree of a vertex in some fuzzy graphs, *International journal of algorithms,computing and mathematics,* vol.2 (2009) 107–116.

[43] A.Nagoorgani and S.R.Latha, On irregular fuzzy graphs, Applied Mathematical Sciences, Vol.6, No.11,(2012) 517–523.

[44] A.Nagoorgani and K.Radha, On regular graphs, *Journal of Physical Sciences,* Vol.12 (2008) 33–40.

[45] A.Nagoorgani and B.Fathima Kani, Fuzzy vertex order coloring, *International journal of pure and Applied Mathematics*, Vo.107, No.3 (2016) pp.601-614.

[46] N. Naga Maruthi Kumari, D. Venugopalam and M.Vijaya Kumar, *Complement of graph and fuzzy graph, International Journal of Mathematical Archive 4(*7), (2013) 14–18.

[47] A. Nagoorgani and S.R. Latha, Isomorhism on irregular fuzzy

graphs, *International Journal of Math. Sci of Engg.*, A*ppls.* Vol.6, No. III, (May 2012).

[48] A. Nagoorgani, Muhammad Akram and Rajalakshmi D, Novel properties of fuzzy labeling Graphs, *Hindawi Publishing Cooper ation,* Volume 2014, Article id 375135, 6 pages.

[49] A. Nagoorgani and Rajalakshmi. D, Properties of fuzzy labeling Graphs, A*pplied Mathematical Sciences,* Vol.6, 2012, No. 70, 3461-3466.

[50] A. Nagoorgani and Rajalakshmi. D, A Note on fuzzy labeling Graphs, *International Journal of fuzzy Mathematical Archive*, Vol.4, No.2, 2014, 88-95.