# A Detailed Analysis on Fuzzy d-algebra and Intuitionistic Fuzzy d-algebra

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**Abstract**

The motive of this paper is to conduct the detailed analysis on fuzzy d-algebra and Intuitionistic fuzzy d-algebra. The intuitionistic fuzzy set theory is a valuable method for presenting and manipulating information. Also, it investigates the use of fuzzy d-ideals with its properties and relations. The most relevant properties of the fuzzy d sub-algebras have been analyzed with the sample illustrations. Based on this, it investigated that the degree of membership functions in the fuzzy set are not depends on the structure of d-algebra in the set $X$ Moreover, the intuitionistic fuzzy set with the degree of non-membership functions is highly. accurate for efficiently solving the problems related to the knowledge base and observations. In this paper th he notion of a mapping on intuitionistic fuzzy d -algebra set, several Proposition and definitions of intuitionistic fuzzy d -algebra are presented.

**Keywords: Fuzzy d-algebra, Intuitionistic Fuzzy Set, Intuitionistic Rules, and Intuitionistic Fuzzy d-subalgebra.**

## 1. Introduction

In earlier, there are two types of abstract algebras have been developed that includes the categories of BCK-algebra and BCI-algebra introduced by the authors of [1]. Consequently, the wide range of algebras are introduced by the authors of Hu and Li [2], where they depicted that the proper subclasses of the classes of BCI are defined by the classes of BCK algebras. Similarly, the proper subclasses of the classes of BCH algebras are defined by the classes of BCI algebras [3]. Based on this, some of the new classes of algebras have been introduced in works [4]. Typically, the fuzzy set is determined [5-9] based on the object classes with respect to the membership grade functions. Then, the fuzzy topology is designed with varying generalization properties of fuzzy sets. With the incorporation of degree of non-membership functions in the fuzzy set for forming the intuitionistic fuzzy sets [10-12] are seems highly accurate for uncertainty quantification. Also, it offers the opportunities for modeling the problems based on the conventional knowledge and observations. The Fuzzy BCK algebras are used to construct the elements of theory with respect to the topological structure of fuzzy sets. In addition to that, the notion of fuzzy d-ideals in fuzzy d-algebra [12, 13] are examined by the authors of Neggers and kim. Consequently, the concept of fuzzy algebras are extended with the following notions.

- Fuzzy d-subalgebra [14]
- Fuzzy d-ideals [15]
- Fuzzy B-algebras [16]
- Fuzzy d#-ideal [17]
- Fuzzy BCI algebras [18, 19]
- Fuzzy d*-ideal algebras [20]

  

## 2. Preliminaries of Fuzzy d-algebra

**Definition 1:** The fuzzy d-algebra is considered as the non-empty set $X$ with the constant of 0, where the binary operator * satisfies the following conditions:

- $x*x=0$
- $0*x=0$
- $x*y=0$ & $y*x=0$ which implies that $y=x \in x, y$ in $X$

**Definition 2:** Let, consider the set of $(X,*,0)$ is the d-algebra, where $a \in X$ . Here, it is represented that $a*X=\{a*x \mid x \in X\}$. The variable $X$ is termed as the edge if the $a*X=\{0,a\}$ for all $a \in X$ .

**Definition 3:** Here, the fuzzy set has been defined for the set of $X$ by the term of $\mu: X \rightarrow [0,1]$, in which $\mu(X)$ indicates the membership degree of $X$ used in the fuzzy set $A$. Any subset of $A$ represented in the $X$ can be estimated based on its characteristic function $x_A : X \rightarrow \{0,1\}$ is defined as follows:

$$x_A(x) = \begin{cases} 1, & if x \in A \\ 0, & if x \notin A \end{cases}$$ And this type of characteristic functions are defined as fuzzy sets in $X$ .

**Definition 4:** The crisp fuzzy sets in the element $X$ can be represented based on the characteristic functions of the subset of set of $X$ .

**Definition 5:** Let consider, $X$ is the d-algebra with $\Phi \neq A \subseteq X$ , where $A$ is considered as the d-sub algebra of $X$ , if it satisfy the condition of $xy \in A$ whenever $x, y \in A$. Also, if $\Phi \neq A \subseteq X$ where $A$ is termed as the BCK ideal of $X$ and it must satisfy the following conditions:

- $0 \in A$
- $xy \in A$ and $y \in A$ implies $x \in A$

**Definition 6:** In d-algebra of set $A$ in $(X;*,0)$, and $\Phi \neq A \subseteq X$ , where $A$ is considered as the d-ideal of $X$ and it must satisfy the following conditions:

- $xy \in A$ and $y \in A$ then $x \in A$
- $x \in A$ and $y \in X$ then $xy \in A$ that is $AX \subseteq A$

**Definition 7:** The fuzzy set $\mu$ in d-algebra $X$ is considered as the fuzzy d-sub algebra of $X$ that satisfies the $\mu(xy) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in X$. Then, it is termed as the fuzzy BCK ideal of $X$ and it must satisfy the below inequalities:

- $\mu(0) \geq \mu(x), \forall x \in X$
- $\mu(x) \geq \min\{\mu(xy), \mu(y)\}, \forall x, y \in X$

**Definition 8:** Let consider $\mu$ is the fuzzy set in d-algebra of $X$ , where $\mu$ is termed as the fuzzy d-ideal of $X$ that must satisfy the following conditions:

- $(Fd_1)\mu(x) \geq \min\{\mu(xy), \mu(y)\}$
- $(Fd_2)\mu(xy) \geq \mu(x), \forall x, y \in X$

**Definition 9:** Then, the union of fuzzy sets $\mu_i(i \in I)$ are described as follows:

$$\vee_{i\in I} \mu_i(X) = \sup\{\mu_i(X) : i \in I\}$$
$$\wedge_{i\in I} \mu_i(X) = \inf\{\mu_i(X) : i \in I\}$$

**Definition 10:** The fuzzy topology on the set in $X$ is defined as the collection of $\delta$ fuzzy sets in $X$ , which satisfies the following conditions:

- $0_X \in \delta$ and $1_X \in \delta$
- If the element $\mu$ and $v$ are belongs to the term $\delta$ , then $\mu \wedge v$
- If the element $\mu_i$ is belongs to the factor $\delta$ for each $i \in I$ , then $\vee_{i\in I} \mu_i$

**Definition 11:** The universe $X$ is represented as follows: $A = \{(x, \mu_A(x), v_A(x)), x \in X\}$, where $\mu_A(x) : X \to [0;1], v_A(x) : X \to [0;1]$ with the factor of $0 \leq \mu_A(x) + v_A(x) \leq 1, \forall x \in X$ . Here, the values of $\mu_A(x)$ and $v_A(x)$ are indicates the degree of membership and non-membership values of $x$ to $A$ correspondingly.

**Definition 12:** Let, consider the $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ is the index of $x$ in the set $A$ and is determined by the indeterminacy degree of $x \in X$ to the set $A$ and $\pi_A(x) \in [0,1]$ that is $\pi_A(x) : X \to [0,1]$ and $0 \leq \pi_A \leq 1$ for every element of $x \in X$ .

**3. Intuitionistic Fuzzy d-algebra**

**3.1 Preliminaries of Intuitionistic Rules**

**Definition 9:** The algebra with the set of $(X :*, 0)$ with type $(2,0)$ is termed as the BCK algebra, which must satisfies the following:

- $((x * y) * (x * z) * (z * y)) = 0$
- $((x * (x * y)) * y) = 0$
- $x * x = 0$
- $0 * x = 0$
- $x * y = 0$ and $y * x = 0 \Rightarrow x = y$ for all $x, y \in X$

Observation: The partial ordering of $\leq$on $X$ is represented based on the term of $x \leq y$ if satisfies the conditions of $x * y = 0$.

**Definition 13:** The non-empty set of $X$ with the constant value of $0$ and binary operator $*$ is termed as the d-algebra, which must satisfies the following conditions:

- $x * x = 0$
- $0 * x = 0$
- $x * y = 0$ and $y * x = 0 \Rightarrow x = y$ for all $x, y \in X$

Sample: Let consider, this example with the set of $X = \{0,1,2\}$,

| * | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2 | 0 | 2 |
| 2 | 2 | 0 | 1 |

Based on the typical calculation, it is determined that the set of $(X : 8, 0)$ is considered as the d-algebra.

**Definition 14:** Let, consider $X$ is d-algebra and $I$ is the subset of $X$, where $I$ is also denoted as the d-ideal of $X$ that must satisfies the below conditions:

- $0 \in I$
- $x * x \in I$ and $y \in I \Rightarrow x \in I$
- $x \in x, y \in I \Rightarrow x * y \in (i.e) I \times X \subseteq I$

**Definition 15:** Let consider, the set of $S$ is non-empty with the subset of d-algebra $X$ in which $S$ is represented as the d-sub algebra of $X$ if it satisfies the rule of,

$$x * y \in S \text{ For all } x, y \in S$$

**Definition 16:** Let consider, the intuitionistic fuzzy set $A$ with the non-empty set $X$ is represented by the term of,

$$A = \{(\mathrm{x}, \alpha_A(\mathrm{x}), \beta_A(\mathrm{x})) / \mathrm{x} \in \mathrm{X}\}$$

Where, the term $\alpha_A : x \to [0,1]$ and $\beta_A : x \to [0,1]$ are represented by the degree of membership and non-membership functions correspondingly, in which $0 \leq \alpha_A(x) + \beta_A(x) \leq 1$ for all $x \in X$. The intuitionistic fuzzy set is illustrated by, $A = \{(\mathrm{x}, \alpha_A(\mathrm{x}), \beta_A(\mathrm{x})) / \mathrm{x} \in X\}$, where the term $X$ is represented based on the ordered pair of set $(\alpha_A, \beta_A)$ in $I^X \times I^X$.

**Definition 17:** Similar to that, the fuzzy set $\mu$ in d-algebra $X$ is represented as the fuzzy d sub-algebra of $X$, which is required to satisfy the following condition:

$\mu(x * y) \geq \min\{\mu(\mathrm{x}), \mu(\mathrm{y})\}$ For all $x, y \in X$

### 3.2 Intuitionistic Fuzzy d-subalgebra

**Definition 18:** The intuitionistic fuzzy set $A = (\alpha_A, \beta_A)$ in $X$ is represented by the term if intuitionistic fuzzy d-subalgebra, which must satisfy the following constraints:

- $\alpha_A(x * y) \geq \min\{\alpha_A(\mathrm{x}), \alpha_A(\mathrm{y})\}$
- $\beta_A(x * y) \leq \max\{\beta_A(\mathrm{x}), \beta_A(\mathrm{y})\}$

Sample: Let consider the set of d-subalgebra with the following example, $X = \{0,1,2\}$

| * | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2 | 0 | 2 |
| 2 | 2 | 0 | 1 |

In this sample, the intuitionistic fuzzy set $A = (\alpha_A, \beta_A)$ in $X$ is illustrated as follows:

$$\alpha_A(0) = \alpha_A(1) = 0.7 > 0.3 = \alpha_A(2)$$

$$\beta_A(0) = \beta_A(1) = 0.2 > 0.5 = \beta_A(1)$$

Based on the general calculation, the set $A = (\alpha_A, \beta_A)$ is considered as the intuitionistic fuzzy d-sub algebra of $A = (\alpha_A, \beta_A)$ of $X$.

**Definition 19:** The intuitionistic fuzzy set $A$ is defined as the level of IFS, if it satisfies the following condition:

$$A_t = \{\mathrm{x} \in \mathrm{X} / \alpha_A(\mathrm{x}) \geq \mathrm{t} \text{ and } \alpha_A(\mathrm{x}) \geq \mathrm{t}\} \text{ for } 0 \leq t \leq 1$$

**Observations:**

- The upper and lower set of $\alpha_A$ is determined based on $\alpha_A^t = \{\mathrm{x} \in \mathrm{X} \mid \alpha_A(\mathrm{x}) \geq \mathrm{t}\}$ and $\alpha_{At} = \{\mathrm{x} \in \mathrm{X} \mid \alpha_A(\mathrm{x}) \leq \mathrm{t}\}$ correspondingly.
- Then, these two sets of $A_s$ and $A_t$ are said to be equal $(A_s = A_t)$ if the condition $\alpha_A^s = \alpha_A^t$ and $\beta_{A,s} = \beta_{a,t}$.

  

- Then, these two sets of $A_s$ and $A_t$ are $(A_s \subseteq A_t)$ if $\alpha_A^s \subseteq \alpha_A^t$ and $\beta_{A,s} \supseteq \beta_{a,t}((A_s \subseteq A_t)$ if $\alpha_A^s \subseteq \alpha_A^t$ and $\beta_{A,s} \subseteq \beta_{A,t})$.

**Proposition 1:** For every intuitionistic fuzzy d-sub algebra of $X$ :

- $\alpha_A(0) \geq \alpha_A(x)$ for all $x \in X$
- $\beta_A(0) \leq \beta_A(x)$ for all $x \in X$

**Proposition 2:** The intuitionistic fuzzy set $A$ of d-algebra $X$ is termed as fuzzy d-algebra, if and only if for each $0 \leq t \leq 1$, in which the set of $A_t$ may be empty of sub algebra of $X$ .

Let, consider that the set $A$ is an intuitionistic fuzzy d-algebra of $X$ and $A_t \neq 0$. Then, for each $x, y \in A_t$,

- $\alpha_A(x*y) \geq \min\{\alpha_A(\mathrm{x}), \alpha_A(\mathrm{y})\} = \mathrm{t}$ and
- $\beta_A(x*y) \geq \max\{\beta_A(\mathrm{x}), \beta_A(\mathrm{y})\} = \mathrm{t}$
- Therefore, $x*y = A_t$

On the other hand, consider that $t = \min\{\alpha_A(\mathrm{x}), \alpha_A(\mathrm{y})\}$ and $t = \max\{\beta_A(\mathrm{x}), \beta_A(\mathrm{y})\}$ for each $x, y \in X$ . If the set of $x, y \in A_t$ then $\alpha_A(\mathrm{x*y}) \geq \min = \{\alpha_A(\mathrm{x}), \alpha_A(\mathrm{y})\}$ and $\beta_A(\mathrm{x*y}) \leq \mathrm{t} = \max\{\beta_A(\mathrm{x}), \beta_A(\mathrm{y})\}$, Therefore, $A$ is considered as the intuitionistic fuzzy d-sub algebra of $X$ .

**Proposition 3:** Any intuitionistic d-sub algebra of $X$ sub algebra is considered as the sub d-algebra of any intuitionistic fuzzy d-sub algebra of $X$ .

**Definition 20:** Let consider, $A$ is the intuitionistic fuzzy d-sub algebra of $X$ which is defined as follows:

$$\alpha_A(x) = \beta_A(\mathrm{x}) = \begin{cases} t & if x \in A \\ o or 1 & if x \notin A \end{cases}$$

If $x, y \in A$, then $\alpha_A(x*y) \geq \min\{\alpha_A(\mathrm{x}), \alpha_A(\mathrm{y})\} = \mathrm{t}$

$$\beta_A(x*y) \leq \max(\beta_A(\mathrm{x}), \beta_A(\mathrm{y})) = \mathrm{t}$$

Therefore, $x*y \in A_t$

Similar to that if $x, y \notin A$ then $\alpha_A(x*y) \geq \min\{\alpha_A(\mathrm{x}), \alpha_A(\mathrm{y})\} = 0$

$$\beta_A(x*y) \leq \max\{\beta_A(\mathrm{x}), \beta_A(\mathrm{y})\} = 0$$

Therefore, $x*y \in A_t$

If, any of $x, y \in A$ then $\alpha_A(x*y) \geq \min\{\alpha_A(\mathrm{x}), \alpha_A(\mathrm{y})\} = \mathrm{t}$

$\beta_A(x*y) \le \max\{\beta_A(x), \beta_A(y)\} = t$

Therefore, $x*y \in A_t$

### 3.4 Intuitionistic Fuzzy d-ideals

**Definition 21:** Let consider, the intuitionistic fuzzy set $A=(\alpha_A, \beta_A)$ in $X$ is represented by the term of intuitionistic fuzzy d-ideals of the term $X$, which must satisfy the following condition:

- $\alpha_A(0) \ge \alpha_A(x) \beta_A(0) \le \beta_A(x)$
- $\alpha_A(x) \ge \min\{\alpha_A(x*y), \alpha_A(x)\}, \beta_A(x) \le \max\{\beta_A(x*y), \beta_A(x)\}$
- $\alpha_A(x*y) \ge \min\{\alpha_A(x), \alpha_A(y)\}, \beta_A(x*y) \le \max\{\beta_A(x), \beta_A(y)\}$

It is clearly stated that every intuitionistic fuzzy d-ideal of d-algebra is termed as the d-sub algebra of $X$.

**Example:** Let consider, the set $X=\{0,1,2\}$ is the d-algebra and is illustrated in the following sample,

| * | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2 | 0 | 2 |
| 2 | 2 | 0 | 1 |

Let consider the fuzzy set $A=\{\alpha_A, \beta_A\}$ in $X$ is illustrated as follows:

$\alpha_A(0) = \alpha_A(2) = 1$ $\quad$ $\alpha_A(1) = t$

$\beta_A(0) = \beta_A(2) = 1$ $\quad$ $\beta_A(1) = s$

Where, $0 \le t \le 1, 0 \le s \le 1$ and $t + s = 1$. Based on this calculation, it is observed that $A=(\alpha_A, \beta_A)$ is the intuitionistic fuzzy d-ideal of $X$.

**Theorem 1:** Let, consider $A=(\alpha_A, \beta_A)$ in the set of $X$ with the intuitionistic fuzzy d-ideal of $X$. If the condition $x*y \le z$,

- $\alpha_A(x) \ge \min\{\alpha_A(y), \alpha_A(z)\}, \beta_a(x) \le \max\{\beta_A(y), \beta_A(z)\}$
- $\alpha_A(0) \ge \alpha_A(z), \beta_A(0) \le \beta_A(z)$ for all $x, y, z \in X$

**Explanation:** Let take, $x, y, z \in X$, in which $x*y \le z$. If the set $(x*y \le z$ then $(x*y)*z) = 0$.

- $\alpha_A(x) \ge \min\{\alpha_A(y), \alpha_A(z)\}, \beta_A(x) \le \max\{\beta_A(y), \beta_A(z)\}$
- $\alpha_A((x*y)*z) \ge \min\{\alpha_A(x*y), \alpha_A(z)\}$
- $\beta_A((x*y)*z) \le \max\{\beta_A(x*y), \beta_A(z)\}$

**Theorem 2:** Let $A=(\alpha_A,\beta_A)$ in the set $X$ is an intuitionistic fuzzy d-ideal set of $X$.

If $x\le y$ then, the following conditions must be satisfied:

- $\alpha_A(0)\ge\alpha_A(x)\ge\alpha_A(y)$
- $\beta_A(0)\le\beta_A(x),\le\beta_A(y)$ for all $x,y\in X$

**Observations:**

The set $x,y\in X$ and $x\le y$ then $x*y=0$

$\alpha_A(x)\ge\min\{\alpha_A(\mathrm{x}*\mathrm{y}),\alpha_A(\mathrm{y})\}$

$\alpha_A(x)=\min\{\alpha_A(0),\alpha_A(\mathrm{y})\}$

$\alpha_A(x)=\alpha_A(y)$

Similar to that, $\alpha_A(x*y)\ge\min\{\alpha_A(\mathrm{x}),\alpha_A(\mathrm{y})\}$

$\alpha_A(0)\ge\min\{\alpha_A(\mathrm{x}),\alpha_A(\mathrm{y})\}$

$\alpha_A(0)\ge\alpha_A(\mathrm{x})\ge\alpha_A(\mathrm{y})\}$

**Definition 22:** Let consider, $A=(\alpha_A,\beta_A)$ and $B=(\alpha_B,\beta_B)$ are the two sets of intuitionistic fuzzy sets of d-algebra $X$, where the Cartesian product is estimated $A\times B: X\times X\rightarrow[0,1]$ as shown in below:

$(\alpha_A,\alpha_B)(\mathrm{x},\mathrm{y})=\min\{\alpha_A(x),\alpha_B(\mathrm{y})\}$ And $(\beta_A\times\beta_B)(\mathrm{x},\mathrm{y})=\max\{\beta_A(\mathrm{x}),\beta_B(\mathrm{y})\}$ for all $x,y\in X$.

**Theorem 3:** If $A=\{\alpha_A,\beta_A\}$ and $B=\{\alpha_B,\beta_B\}$ are considered as the intuitionistic fuzzy d-ideal d-algebra $X$, in which $A\times B$ is the intuitionistic fuzzy d-ideal of $X$.

**Explanation:** For any, $(\mathrm{x},\mathrm{y})\in\mathrm{X}\times\mathrm{X}$

- $(\alpha_A\times\alpha_B)(0,0)=\min\{\alpha_A(0),\alpha_B(0)\}$
  $\ge\min\{\alpha_A(\mathrm{x}),\alpha_B(\mathrm{y})\}$
  $=(\alpha_A\times\alpha_B)(\mathrm{x},\mathrm{y})$
- $(\beta_A\times\beta_B)(0,0)=\max\{\beta_A(0),\beta_B(0)\}$

  $\ge\max\{\beta_A(\mathrm{x}),\beta_B(\mathrm{y})\}$

  $=(\alpha_A\times\alpha_B)(\mathrm{x},\mathrm{y})$

Similar to that, Let consider the set $(\mathrm{x}_1,\mathrm{x}_2)$ and $(\mathrm{y}_1,\mathrm{y}_2)\in\mathrm{X}\times\mathrm{X}$

$(\alpha_A\times\alpha_B)(\mathrm{x}_1,\mathrm{x}_2)=\min\{\alpha_A(\mathrm{x}_1),\alpha_B(\mathrm{x}_2)\}$

$$\geq \min\{\min\{\alpha_A(x_1*y_1),\alpha_A(y_1)\}\min\{\alpha_B(x_2*y_2),\alpha_B(y_2)\}\}$$
$$=\min\{\min\{\alpha_A(x_1*y_1),\alpha_B(x_2,y_2)\},\min\{\alpha_A(y_1),\alpha_B(y_2)\}\}$$
$$=\min\{(\alpha_A\times\alpha_B)(x_1*y_1,x_2*y_2),(\alpha_A\times\alpha_B)(y_1\times y_2)\}$$
$$=\min\{(\alpha_A\times\alpha_B)((x_1,x_2)*(y_1*y_2)),(\alpha_A\times\alpha_B)(y_1\times y_2)\}$$
$$(\beta_A\times\beta_B)(x_1,x_2)=\max\{\beta_A(x_1),\beta_B(x_2)\}$$
$$\leq \max\{\beta_A(x_1*y_1),\beta_B(y_1)\}\max\{\beta_A(x_2*y_2),\beta_B(y_2)\}\}$$
$$=\max\{\max\{\beta_A(x_1*y_1),\beta_B(x_2,y_2)\},\max\{\beta_A(y_1),\beta_B(y_2)\}\}$$
$$=\max\{(\beta_A\times\beta_B)(x_1*y_1,x_2*y_2),(\beta_A\times\beta_B)(y_1\times y_2)\}$$
$$=\max\{(\beta_A\times\beta_B)((x_1,x_2)*(y_1*y_2)),(\beta_A\times\beta_B)(y_1\times y_2)\}$$

Let consider the set $(x_1,x_2)$ and $(y_1,y_2)\in X\times X$ then

$$(\alpha_A\times\alpha_B)(x_1,x_2)*(y_1,y_2)=(\alpha_A\times\alpha_B)((x_1*y_1,x_2*y_2))$$
$$=\min\{\alpha_A(x_1*y_1),\alpha_B(x_2*y_2)\}$$
$$\geq \min\{\min\{\alpha_A(x_1),\alpha_A(y_1)\}\min\{\alpha_B(x_2),\alpha_B(y_2)\}\}$$
$$=\min\{\min\{\alpha_A(x_1),\alpha_B(x_2)\}\min\{\alpha_A(y_1),\alpha_B(y_2)\}\}$$
$$=\min\{(\alpha_A\times\alpha_B)(x_1,x_2),(\alpha_A\times\alpha_B)(y_1,y_2)\}$$
$$(\beta_A\times\beta_B)(x_1,x_2)*(y_1,y_2)=(\beta_A\times\beta_B)((x_1*y_1,x_2*y_2))$$
$$=\max\{\beta_A(x_1*y_1),\beta_B(x_2*y_2)\}$$
$$\leq \max\{\max\{\beta_A(x_1),\beta_B(y_1)\},\max\{\beta_B(x_2),\beta_B(y_2)\}\}$$
$$=\max\{\max(\beta_A(x_1),\beta_B(x_2)\},\max(\beta_A(y_1),\beta_B(y_2)\}\}$$
$$=\max\{(\beta_A\times\beta_B)(x_1,x_2),(\beta_A\times\beta_B)(y_1,y_2)\}$$

Therefore, $A\times B$ is stated as the intuitionistic fuzzy d-ideal of $X$.

**Theorem 4:** If the set of $\{A_i, i\in\wedge\}$ is considered as an arbitrary group of intuitionistic d-ideal algebra, formerly $\mathrm{I}\,A_i$ is termed as the intuitionistic d-ideal algebra, when $\mathrm{I}\,A_i=\{<x,\wedge\alpha_{Ai}(x),\vee\beta_{Ai}(x)\mid x\in X\}$.

**Observation:** Meanwhile, $\alpha_A(x)\geq \min\{\alpha_A(xy),\alpha_A(y)\}$ and

$\beta_A(x)\leq \max\{\beta_A(xy),\beta_A(y)\}$ For all $x,y\in X$ and $i\in\wedge$.

$$\wedge\alpha_{Ai}(x)\geq\wedge\{\min\{\alpha_{Ai}(xy),\alpha_{Ai}(y)\}\}\geq\{\min\{\wedge\alpha_{Ai}(xy),\wedge\alpha_{Ai}(y)\}\}$$

$$\vee\beta_{Ai}(x)\leq\vee\{\max\{\beta_{Ai}(xy),\beta_{Ai}(y)\}\}\leq\{\max\{\vee\beta_{Ai}(xy),\vee\beta_{Ai}(y)\}\}$$

Subsequently, $\alpha_A(xy)\geq\alpha_A(x),\beta_A(xy)\leq\beta_A(x)$ for all $i\in\wedge$, and is get $\wedge\alpha_{A_i}(xy)\geq\wedge\alpha_{A_i}(x)$ and $\vee\beta_{A_i}(xy)\leq\vee\beta_{A_i}(x)$ for all $x,y\in X$ and for all $i\in\wedge$. Therefore, it is obtained as $\mathrm{I}\,A_i=\{<x,\wedge\alpha_{A_i}(x),\vee\beta_{A_i}(x)\mid x\in X\}$ is termed as an intuitionistic fuzzy d-ideal algebra.

## 4. Conclusion

As part of this research we investigated some results of intuitionistic fuzzy ideals of fuzzy d sub-algebras, namely intuitionistic fuzzy d-algebra and also examined some of their useful properties. In future based on our observations, these definitions and main results can be applied to several other algebraic systems, such as the KK-algebra, lattice algebras and Lie algebras. Conflicts of Interest.

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