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Vague Normed Ideals

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Abstract: In this paper we introduce the inherent product of two vague normed ideals, and examine a number of associated properties of vague normed ideals .Also we illustrate the intrinsic create of two vague normed sets and demonstrate that the intrinsic invention can realistic to vague normed ideals.

Keywords: Vague normed ring, vague normed ideal, intrinsic product

Mathematics Subject Classification: 03E772, 03 F55

I. Introduction

The idea planned by Zadeh.L.A.[10] important a fuzzy subset A of a given space U characterizing the membership of an element x of X be in the right place to A by means of a association function $m_A(x)$ defined from X in to [0 1] has revolutionize the theory of Mathematical model Decision making etc.,in handling the imprecise real life situations mathematically.Zadeh's fuzzy set theory have been planned. Interval valued fuzzy sets, Intuitionistic fuzzy sets by Atanassov.K.T [1] ,Vague sets [3] are mathematically equivalent.

Any such set A of a given Universe X can be characterize by means of a pair of function (m_A , n_A) where m_A : U →[0 1] and n_A : U →[0 1] such that $0 \leq m_A(x) + n_A(x) \leq 1$ for all x in U.The set $m_A(x)$ is called the truth function or truth association function and the set $n_A(x)$ is called false function or non association function and $m_A(x)$ gives the evidence of how much x belongs to A, $n_A(x)$ gives the evidence of how much x does not belongs to A. The objectives of this paper is to contribute further to the study of vague algebra by introducing the concepts of the intrinsic product of two vague normed ideals, and investigate some associated properties of vague normed ideals. Also we describe the intrinsic product of two vague normed sets and show that the intrinsic product can applied to vague normed ideals.

2. Preliminaries:

Definition 2.1: [2]Let * :[0,1]X[0,1]→[0, 1] be a binary operation .Then *is t-norm if *satisfies the following conditions associativity,commutativity,monotonicity,monotoniccity and neutral element 1.We shortly use t-norm and write r*v instead of *(r,v) .

Example 2.2:[2] Continuous t-norm are r*v=rv and r*v=min{r,v}

Definition 2.3:[2] A t-norm T has the property ,for every x,y ∈[0,1] then T(x,y)≤min{x,y).

Definition 2.4 [4]A vague set A= (m_A, n_A),where m_A,, n_A are X *to* [0 1] is mappings X to [0 1]Such that $0 \leq m_A(x) + n_A(x) \leq 1$.

3. Properties of vague normed Ideals: In this section, we characterize several properties of vague normed ideals and elementary results are obtained

Definition 3.1: Let A and B be two vague sets of normed rings nr. The operations are defined as

$$(\boldsymbol{i}) \quad m_{(AOB)}(\mathrm{r}) = \begin{cases} o_{r=vz} \; (m_A(v) * m_B(z)) & if\ r = vz \\ 0 & otherwise \end{cases}$$

$$(ii) \quad n_{(AOB)}(\mathrm{r}) = \begin{cases} o_{r=vz} \; (m_A(v) * m_B(z)) & if\ r = vz \\ 0 & otherwise \end{cases}$$

· Therefore, the intrinsic product of A and B is considered to be vague normed sets $(AOB) = (m_{(AOB)}\ , n_{(AOB)}) = (m_A \mathrm{O}\, m_A\ , n_A \mathrm{o}\, n_B)$

Definition 3.2: Let nr be a normed ring .Then all vague set A=(x, $m_A(x)$, $n_A(x)$) of nr is an vague normed ring(VNR)of nr if it satisfies the following ,

1) $m_A(x-y) \geq m_A(x) * m_A(y)$

2) $m_A(xy) \geq m_A(x)*m_A(y)$

3) $n_A(x-y) \leq \max (n_A(x) \text{ o } n_A(y)$

4) $n_A(xy) \leq \max (n_A(x) \text{ o } n_A(y)$

Definition 3.3 Let nr be a normal ring .Then an vague set A=(x,$m_A(x)$,$n_A(x)$) of nr is an vague normed ideal (VNI) of nr if it satisfies the following conditions

1) $m_A(x-y) \geq m_A(x)*m_A(y)$

2) $m_A(xy) \geq m_A(x) \text{o } m_A(y)$

3) $n_A(x-y) \leq n_A(x) \text{ o } n_A(y)$

4) $n_A(xy) \leq n_A(x) * n_A(y)$.

Theorem 3.4: If A,B are any two vague ideals of a normed ring (**nr**). Then (A∩B) is vague normed ideals of nr.

The following example give the strength of the above Theorem.Let (**nr**)=Z the ring of integers under ordinary addition and multiplication of integer .Define the vague normed seta A=(m_A, n_A) and B=(m_B ,n_B) by r =5z, z is integer

A={(x_1 0.6 , 0.2) (x_2 0.3 , 0.2)} where r=5z, z is integer

B={(x_1 0.9 , 0.3) (x_2 0.2 , 0.6)} where r ∈ 5z

Theorem 3.5 Let A and B be an vague normed right ideal and an vague normed left ideal of a normed ring **(nr)** , respectively ,then AOB ⊆ (A∩B)

i.e (A*B) (r)≤(A∩B)(r) ≤ (A* B)(r) Where (A∩B)(r)={r, $m_{(A\cap B)}$(r), $n_{(A\cap B)}$(r); r ∈ nr}Where $m_{(A\cap B)}$(r)=min{ $m_{(A)}$(r), n_A(r)}

$n_{(A\cap B)}$(r)=max {n_A(r), n_B(r)}. Where r∈nr

Proof: Let A and B are vague normed right ideal of (nr**)**

Assume A is vague normed right ideal and B is vague normed left ideal

Let $m_{(A\wedge B)}(r) = {}_{r=vz}\overset{O}{}\{m_A(v) \text{o } m_B(z)\}$ and

let $n_{(A\wedge B)}(r)={*}_{r=vz}\{n_A(v)* n_B(z)\}$

since A is vague normed right ideal and B is a vague normed left ideal we hava $m_A(v) \leq m_A(vz)=m_A(r)$ and $m_B(v) \leq m_B(vz)=m_B(r)$

and $n_A(r)=n_A(vz) \geq n_A(v)$ and $n_B(r)=n_B(vz) \geq n_B(z)$

thus $m_{(A(+)B)}(r)=o_{r=vz}\{m_A(v)* m_B(z)\}$

$=\min(m_A(v),m_B(z)) \leq \min (m_A(r), m_B(r))$

$\leq m_{(A \wedge B)}(r)$ ---------(3.5.1)

$n_{(A \wedge B)}(r)=*_{r=vz}\{n_A(v)o\ n_B(z)\}$

$= \max(n_A(v),n_B(z)) \geq \max(n_A(r), n_B(r))$

$\geq n_{(A \wedge B)}(r)$ ---------(3.5.2)

From (3.5.1) and (3.5.2) the proof is concluded

Theorem .3.6: The union of two vague normed ideals of a ring (**nr)**, need not be always vague normed ideal.

Example.3.7: The following gives the strength of the above theorem.The following example give the strength of the above Theorem. Let (**nr)**=Z be the ring of integers under ordinary addition and multiplication of integers .Define the vague normed sets A=(m_A, n_A) and B=(m_B ,n_B) by r =5z, z is integer

A={(x_1 0.75 , 0.3) (x_2 0.4 , 0.2)} where r=5z, z is integer

B={(x_1 0.5 , 0.3) (x_2 0.35 , 0.5)} where r ∈ 5z.

$m_{(A \cup B)}(r)=\max\{ m_{(A)}(r), m_A(r)\}$ and $n_{(A \cup B)}(r)=\min\{n_A(r), n_B(r)\}$. Where r∈nr,then $m_{(A \cup B)}(r)=\min\{(3, 0.75,0.2),(-1,0.86,0.3),(7,0.35,0.4)\}$.

Let r=15 v=4 , $m_{(A \cup B)}(15)=0.75$ $m_{(A \cup B)}(4)=0.75$ and $m_{(A \cup B)}(15)=0.3$ $m_{(A \cup B)}(4)=0.3$.Hence $m_{(A \cup B)}(15-4)= m_{(A \cup B)}(11)=0.35$ not greater than

$n_{(A \cup B)}(15)* m_{(A \cup B)}(4)=\min\{0.96,0.75\}=0.36$ and $n_{(A \cup B)}(15-4)= n_{(A \cup B)}(11)=0.4$ not less than $m_{(A \cup B)}(15)* m_{(A \cup B)}(4)=\max\{0.2,0.3\}=0.3$.Thus,the union of two vague normed ideals of **(nr)** need not be an vague normed ideal. $m_{(A \cup B)}(15)* m_{(A \cup B)}(4)=\max\{0.2,0.3\}=0.3$.Thus,the union of two vague normed ideals of **(nr)** need not be an vague normed ideal.

Theorem 3.8: Let A=(x, m_A, n_A) be an vague normed ideal of a ring **(nr)** ,since we have r ∈**(nr)** :

(i)$m_A(o) \geq m_A(r)$ and $n_A(o) \leq n_A(r)$

ii) $m_A(-r)=m_A(r)$ and $n_A(-r)=n_A(r)$

iii) if $m_A(r-y)=m_A(o)$ then $m_A(r)=m_A(y)$

iv) if $n_A(r-y)=n_A(o)$ and $n_A(r)=n_A(y)$

Proof: (i) Since I is an vague normed ideal ,then $m_A(o)=m_A(r-r)$

$\geq m_A(r)*m_A(r)$

thefore $m_A(o)= =m_A(r)$

And $n_A(o)=n_A(r-r) \leq n_A(r)$ o $n_A(r)=n_A(r)$

thefore $n_A(o)= =n_A(r)$

(ii)$m_A(-r) \geq m_A(o-r) \geq m_A(o)*m_A(r)=m_A(r)$ and

$m_A(r)=m_A(o-(-r)) \geq m_A(o)*m_A(-r)=m_A(-r)$
Thefore $m_A(-r)= m_A(r)$
also
$n_A(-r)=n_A(o-r) \leq n_A(o)$o $n_A(r)=n_A(r)$ and
$n_A(r)=n_A(o-(-r)) \leq n_A(o)$o $n_A(-r)=mn_A(-r)$
Thefore $n_A(-r)= n_A(r)$.

iii) Since $m_A(r-y)=m_A(o)$ then $m_A(y)=m_A((r)-(r-y))$

$= m_A(r)*m_A(r-y)= m_A(r)*m_A(0) \geq m_A(r)$

Thefore $m_A(y) \geq m_A(r)$

Similarly $m_A(r) =m_A((r-y)-(-y))$

$\geq m_A(r-y)*m_A(-y)= m_A(0)*m_A(y) \geq m_A(y)$

Thefore $m_A(r) \geq m_A(y)$

Consequently,$m_A(r)= m_A(y)$

iv) Since $n_A(r-y)=n_A(o)$ then $n_A(y)=n_A((r)-(r-y))$

$= n_A(r)*n_A(r-y)= n_A(r)*n_A(0) \leq n_A(r)$

Thefore $n_A(y) \leq n_A(r)$

Similarly $n_A(r) =n_A((r-y)-(-y))$

$\geq n_A(r-y)*n_A(-y)= n_A(0)*n_A(y) \geq n_A(y)$

Thefore $n_A(r) \geq n_A(y)$.

Consequently, $m_A(r)= n_A(y)$

Theorem 3.9 Let $A=(x, m_A, n_A)$ be an vague normed ideal of a normed ring **(nr)** ,then $\delta_A=(m_A, {m_A}^C)$ is an vague normed ideal of **(nr)** .

Proof: Let r,y ∈ **(nr)**

$m_Ac(r-y)=1-n_A(r-y)$

$\geq 1-\max\{n_A(r), n_A(y)\}$

$\geq mIN\{1-m_A(r),1-m_A(y)\} \geq \max\{n_Ac(r), n_Ac(y)\}$

Therefore $n_Ac(r-y) \geq \min\{m_Ac(r),m_Ac(y)\}$

Hence $m_Ac(r-y) \leq n_Ac(r)o\, m_Ac(y)$-------------->(3.9.1)

Now take $n_Ac(ry)=1-n_A(ry)$

$\geq 1-\min\{n_A(r), n_A(y)\} \geq 1-\max\{1-m_A(r), 1-m_A(y)\}$

$\geq \min\{n_Ac(r), n_Ac(y)\}$

Therefore $m_Ac(ry) \geq m_Ac(r)*m_Ac(y)$--------------→(3.9.2).

From (3.12.1) and (3.12.2) ,$\delta_A=(m_A,m_Ac)$ is an vague normed ideal of **(nr)**

Thorem 3.10 If A is a vague normed ideal of the normed ring **(nr)** , then $\delta A= ({n_A}^C,n_A)$ is vague normed ideal of **(nr)**

Proof: (i) Let r, y ∈NR

$n_Ac(r-y)=1-n_A(r-y)$

$\geq 1\text{-max}\{n_A(r), n_A(y)\}$

$\geq \min\{1-n_A(r), 1-n_A(y)\}$

$\geq \min\{n_{Ac}(r), n_{Ac}(y)\}$

Therefore $n_{Ac}(r\text{-}y) \geq n_{Ac}(r)*n_{Ac}(y)$…..(3.10.1)

(ii) $n_{Ac}(ry)=1-n_A(ry) \geq 1-\min\{n_A(r), n_A(y)\}$

$\geq \max\{1-n_A(r), 1-n_A(y)\}$

$\geq \max\{n_A{}^C(r), n_A{}^C(y)\}$

$\geq n_A{}^C(r) o n_A{}^C(y)$

Thefore $n_{Ac}(ry) \geq n_A{}^C(r) o n_A{}^C(y)$………(3.10.2).

From (3.10.1) and (3.10.2) $A=(m_A{}^C, n_A)$ is a vague normed ideal of **(nr)**.

Thorem 3.11 An vague set $A=(m_A, n_A)$is an vague normed ideal of **(nr)** if the pair of fuzzy sets m_A, $n_A{}^C$ are vague normed ideals of **(nr)**.

Proof: Let (i) Let r, y ∈**(nr)**

$1-n_A(r\text{-}y)= n_A{}^C(r\text{-}y)$

$\geq \min\{n_A{}^C(r), n_A{}^C(y)\}$

$\geq \min\{1-n_A(r), 1-n_A(y)\}$

$\geq 1-\max\{n_A(r), n_A(y)\}= n_{Ac}(r) o\, n_{Ac}(y)$

Therefore $n_A(r\text{-}y) \geq n_{Ac}(r) o\, n_{Ac}(y)$…..(3.11.1)

(ii) Let $1-n_A(ry)= n_A{}^C(ry) \geq \max\{n_A{}^C(r), n_A{}^C(y)\}$

$\geq \max\{1-n_A(r), 1-n_A(y)\}$

$\leq 1-\min\{n_A(r), n_A(y)= n_A(r)*n_A(y)\}$

$\leq n_A(r)*n_A(y)\}$

Thefore $n_A(ry) \leq n_A(r)*n_A(y)\}$………(3.11.2).

Hence From (3.11.1) and (3.11.2) A= $A=(m_A, n_A)$ is a vague normed ideal of **(nr)**.

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Theorem, 3.12: Let A and B be two vague normal left (right) ideal of **(nr)**.Therefore,$(A_* \cap B_*) \subseteq (A \cap B)_*$

Proof: Let x ∈ $(A_* \cap B_*)$,then $m_A^{(x)} = m_A^{(0)}$ and $n_A^{(x)} = m_A^{(0)}$ and $m_B^{(x)} = m_B^{(0)}$ and $n_B^{(x)} = n_B^{(0)}$.

(i)$m_{(A\cap B)}{}^{(x)}$ =min{ $m_A^{(x)}$, $m_A^{(x)}$ }

=min{ $m_A^{(0)}$, $m_A^{(0)}$ }=$m_{(A\cap B)}(0)$=$m_{(A\cap B)}{}^{(0)}$.

(ii) $n_{(A\cap B)}{}^{(x)}$ =min{ $n_A^{(x)}$, $n_A^{(x)}$ }

=min{ $n_A^{(0)}$, $n_A^{(0)}$ }=$n_{(A\cap B)}(0)$=$n_{(A\cap B)}{}^{(0)}$

Therefore x ∈ $(A_* \cap B_*)$ implies x ∈ $(A \cap B)_*$

Thorem 3.13 Let g:R_1 to R_2 be an epimorphism maping of normed rings.If A=(m_A, n_A) is an vague normed ideal of of the normed ring ,then g(A) is also vague normed ideal of R_2.

Proof:- Suppose A=($m_A^{(x)}$, $n_A^{(x)}$) ,where x ∈ R_1 and

g(A)= Let A and B be two vague sets of normed rings nr. The operations are defined as

$$(\boldsymbol{i}) \quad m_{(AOB)}(x)= \begin{cases} o_{x=vz} \ (m_A(v) * m_B(z)) & if\ x = vz \\ 0 & otherwise \end{cases}$$

$$(\square\square) \quad \square_{(\square\square\square)}(x)= \begin{cases} \square_{\square=\square\square} \ (\square_{\square}(\square) * \square_{\square}(\square)) & \square\square\ \square = \square\square \\ 0 & \square\square h\square\square\square\square\square\square \end{cases}$$

.

Let $\square_1$, $\square_2$ ∈ nR_1, then there exist $\square_1$, $\square_2$ ∈ nR_2 such that g($\square_1$)= $\square_1$ and g($\square_2$)= $\square_2$.

(i) $\square_{\square(\square)}(\square_1$-$\square_2)$= $\square_{\square(\square_1-\square_2)=(\square_1-\square_2)}$ $\square_{(\square\square\square)}(\square_1 - \square_2) \geq$ g($\square_1$)= $\square_1$, g($\square_2$)= $\square_2$ $\square_{(\square\square\square)}(\square_1$-$\square_2)$

= $\square_{\square(\square_1=)}$ $\square(\square_1)$ * $\square_{(\square)}$ $\square(\square_2) \geq \square_{\square(\square)}(\square_1)$* $\square_{\square(\square)}(\square_2)$.

Thefore $\square_{\square(\square)}(\square_1$-$\square_2) \geq \square_{\square(\square)}(\square_1)$* $\square_{\square(\square)}(\square_2)$.

(ii) $\square_{\square(\square)}(\square_1\square_2)$= $\square_{\square(\square_1-\square_2)=(\square_1-\square_2)}$ $\square_{(\square\square\square)}(\square_1\square_2) \geq$ g($\square_1$)= $\square_1$, g($\square_2$)= $\square_2$ $\square_{(\square\square\square)}(\square_1 \square_2)$

$= \square_{\square(\square_1=)} \square (\square_1) * \square_{(\square)} \square (\square_2) \geq \square_{\square(\square)}(\square_1) * \square_{\square(\square)}(\square_2)$.

Therefore $\square_{\square(\square)}(\square_1 \square_2) \geq \square_{\square(\square)}(\square_1) * \square_{\square(\square)}(\square_2)$.

(iii) $\square_{\square(\square)}(\square_1 - \square_2) = \square_{\square(\square_1 - \square_2) = (\square_1 - \square_2)} \square_{(\square\square\square)}(\square_1 - \square_2) \geq$ g($\square_1$)= $\square_1$, g($\square_2$)= $\square_2$ $\square_{(\square\square\square)}(\square_1 - \square_2)$

$= \square_{\square(\square_1=)} \square (\square_1) * \square_{(\square)} \square (\square_2) \geq \square_{\square(\square)}(\square_1) * \square_{\square(\square)}(\square_2)$.

Thefore $\square_{\square(\square)}(\square_1 - \square_2) \geq \square(\square_1) * \square_{\square(\square)}(\square_2)$.

(iv) $\square_{\square(\square)}(\square_1 \square_2) = \square_{\square(\square_1 - \square_2) = (\square_1 - \square_2)} \square_{(\square\square\square)}(\square_1 \square_2) \geq$ g($\square_1$)= $\square_1$, g($\square_2$)= $\square_2$ $\square_{(\square\square\square)}(\square_1 \square_2)$

$= \square_{\square(\square_1=)} \square (\square_1) * \square_{(\square)} \square (\square_2) \geq \square_{\square(\square)}(\square_1) * \square_{\square(\square)}(\square_2)$.

Therefore $\square_{\square(\square)}(\square_1 \square_2) \geq \square_{\square(\square)}(\square_1) * \square_{\square(\square)}(\square_2)$.

Hence A is vague normed ideal of $\square_1$ then g(A) is also vague normed ideal of $\square_2$.

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