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VAGUE WI-IDEALS OF LATTICE WAJSBERG ALGEBRAS

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Abstract:

In this paper, we introduce the concept of vague WI-ideal and vague lattice ideal of lattice wajsberg algebra. “We show that every vague WI-ideal of lattice wajsberg algebra is an vague lattice ideal of lattice wajsberg algebra”. Also,”we discuss its converse part. Further, we obtain “every vague lattice ideal is an vague WI-ideal in lattice H-wajsberjalgebra.Moreover” we discuss some characterizations of vague WI-ideal.

Keywords: “wajsberg algebra”; “Lattice wajsberg algebra”; WI-ideal; vague WI-ideal;

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1. **Introduction**

“The concept of Lattice was first defined by Dedekind in 1897 and then developed by Birkhoft.G, imposed an operation an open problem " ,”Is there a common abstraction which includes Boolean algebra, Boolean rings and lattice ordered group or L-group is an algebraic structure connecting lattic and group.” To answer this problem many common abstractions, namely dually residuated lattice ordered semigroups, commutative lattice ordered groups. lattice ordered rings, Also the concept proposed by Zadeh.L.A.[17] defining a fuzzy subset A of a given universe X characterizing the membership of an element x of X belonging to A by means of a membership function $\mu_A(x)$ defined from X in to [0 1] has revolutionized the theory of Mathematical modeling. Decision making etc.,in handling the imprecise real life situations mathematically.

The objective of this paper is to contribute further to the study of vague algebra , we introduce vague WI-ideal and vague lattice ideal of lattice of wajsbergalgebra.We have shown that every vague WI-ideal of lattice wajsberg algebra is an vague lattice ideal of lattice wajsbergalgebra.But ,the converse part is true only in lattice H-Wajsbergalgebras.Finially,we have shown that collection of WI-ideals of lattice wajsberj algebras is an vague WI-ideal of lattice wajsbergs and investigate some their properties with suitable examples.

2. Preliminaries

In this section, we recall some basic definitions and properties which are useful to develop the main results.

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Definition 2.1 [2] Let (A, →, *,1) be an algebra with a binary operation "→"and a quasi complement " * " is called a wajsberg algebra if and only if it satisfies the following axioms for all x,y,z ∈ A,

1. $1 \to x = x$
2. $(x \to y) \to ((y \to z) \to (x \to z)) = 1$
3. $(x \to y) \to y = (y \to x) \to x$
4. $(x^* \to y^*) \to (y \to x) = 1$.

Definition 2.2[2] The wajsberg algebra (A,→ **, *,** 1) satisfies the following properties for all x,y,z∈ A,

(i) $x \to x = x$

(ii) If $(x \to y) = y \to x = 1$ then $x = y$

(iii) $x \to 1 = 1$

(iv) $x \to (y \to x) = 1$

(v) If $x \to y = y \to z = 1$ then $x \to z = 1$

(vi) If $(x \to y) \to ((z \to x) \to (z \to y)) = 1$

(vii) $x \to (y \to z) = y \to (x \to z)$

(viii) $x \to 0 = x \to 1^* = x^*$

(ix) $(x^*)^* = x$

(x) $(x^* \to y^*) = y \to x$.

Definition 2.3 [2] The wajsberg algebra (A, →,*,1) is called a lattice wajsberg algebra if it satisfies the following properties for all x,y∈ A,

(1) a partial ordering "≤" on a lattice wajsberg algebra A, such that x≤y if and only if $x \to y = 1$
(2) (x□ y)=$(x \to y) \to y$
(3) (x□ y)= $((x^* \to y^*) \to y^*)^*$ Thus ,we have (A,□ ,□ , *, 0,1) is a lattice wajsberg algebra with lower bound 0 and upper bound 1.

Theorem 2.5 [2] Thewajsberg algebra (A, →,*,1) satisfies the following properties for all x,y, z ∈ A,

1. If $x \le y$ then $x \to z \ge y \to z$

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2. If $x \leq y$ then $z \to x \leq z \to y$

3. If $x \leq y \to z$ if and only if $y \leq x \to z$

4.$(x \square y)^* = (x^* \square y^*)$

5.$(x \square y)^* = (x^* \square y^*)$

6. $(x \square y) \to z = (x \to z) \square (y \to z$

7.$x \to (y \square z) = (x \to y) \square (x \to z)$

8.$(x \to y) \square (y \to x) = 1$

9.$x \to (y \square z) = (x \to y) \square (x \to z)$

10.$(x \square y) \to z = (x \to y) \square (x \to z)$

11.$(x \square y) \square z = (x \square z) \square (y \square z)$

12.$(x \square y) \to z = (x \to y) \to (x \to z)$ for all x,y,z in A.

Definition 2.6[2] A lattice wajsberg algebra $(A, \to, *, 1)$ is called a lattice H-Wajsbergalgebra,if it satisfies $x \square y \square ((x \square y) \to z) = 1$ for all $x, y.z \in A$. In a lattice H-wajsberg algebra A ,the following hold.

1.$x \to (x \to y) = (x \to y)$

2.$x \to (y \to z) = (x \to y) \to (x \to z)$ foa all x,y,z in A.

Definition 2.7[2] Let $(A_1, \to, *, 1)$ and $(A_2, \to, *, 1)$ be lattice wajsberg algebras, a maping

$f: A_1 \to A_2$ is called implication homomorphism if $f(x \to y) = f(x) \to f(y)$ holds,

Definition 2.8[2] Let $(A_1, \to, *, 1)$ and $(A_2, \to, *, 1)$ be lattice wajsberg algebras, $f: A_1 \to A_2$ is implication homomorphism from A_1 to A_2 satisfies the following properties.

1. $f(x \square y) = f(x) \square f(y)$ 2. $f(x \square y) = f(x) \square f(y)$ 3.$f(x^*) = [f(x)]^*$

Definition 2.9[2] Let L be a lattice. An ideal I of L is a non-empty set of Lis called a lattice ideal if it satisfies the following axioms for all $x, y \in I$

1.$x \in F$, $y \in F$ and $y \leq x$ I

2. $x, y \in I$ implies $x \square y \in F$.

Definition 2.10 [3] Let A be lattice wajsbergalgebra.Let I be a non-empty subset of A.Then I is called WI-ideal of a lattice wajsberg algebra A satisfies for all $x, y \in A$,

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1. $0 \in I$ 2. $(x \to y)^* \in I$ and $y \in I$ implies $x \in I$.

Definition 2.11[1] Let X be a set. A function $\mu: X \to [0,1]$ is called a fuzzy subset on X, for all $x \in X$ the value of $\mu(x)$ describes a degree of membership of x in μ.

Definition 2.12[10] Let μ be a fuzzy subset in set A. Then for $t \in [0,1]$,the st $\mu^t = \{ x \in A : \mu(x) \geq t \}$ is called a level subset of μ.

Definition 2.13 [16]Let$(A_1, \to, *, 1)$ be a lattice wajsberg algebra. A fuzzy subset μ of A is called a fuzzy WI-ideal of A if it satisfies the following properties for all x,y in A.

1. $\mu(0) \geq \mu(x)$
2. $\mu(x) \geq \min\{ \mu(y), \mu((x \to y)^*) \}$.

Definition 2.16 [4]: A vague set A in the universe of discourse X is a pair (t_A, f_A) where $t_A : X \to [0,1]$, $f_A: X \to [0,1]$ with $t_A(x)+f_A(x) \leq 1$ for all x in X. Here t_A is called the membership function and f_A is called non-membership function and also called true membership function, false membership function respectively.

3. Vague Wajsberg implicative ideal (Vage WI-Ideal)

In this section, we introduce the concept of an vague WI-ideal and an vague lattice ideal of a lattice wajsberg algebras. Also, we obtain some properties of an vague WI-ideal.

Definition 3.1 Letw $=(A, \to, *, 1)$ be a lattice wajsberg algebra. An vague set $A = (m_A, n_A)$ of w is called vague WI-ideal of A if it satisfies the properties.

1. $m_A{}^{(0) \geq} m_A{}^{(x)}$ and $n_A{}^{(0) \leq} n_A{}^{(x)}$

2. $m_A{}^{(x)} \geq \min\{m_A((x \to y)^*) , m_A{}^{(y)}\}$

3. $n_A{}^{(x)} \leq \max \{n_A((x \to y)^*) , n_A{}^{(x)} \}$ for all x, y ,z∈ A.

Example 3.2 Let A={0,a,b,c,d,r,s,t,1} be a set as a partial ordering .Define a quasi complement “ * ” and a binary operation ” → " on A .Then a vague set S ={(0,1,0), (a,0.6,0.4) (b,1,0),(c,0.6,0.4),(d 0.6,0.4), (r, 0.6,0.4), (s 0.6,0.4), (t, 0.6,0.4) (1,0.6,0.4} of A is a vague WI-ideal of A.

Sol:- Let A={0,a,b,c,d,r,s,t,1} be a set as a partial ordering .

Define a quasi complement “ * ” and a binary operation ” → " on A

Table 3.2.1 (Implication)

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→	0	A	B	C	D	1
0	1	1	1	1	1	1
A	D	1	A	C	C	1
b	c	1	1	c	c	1
C	B	A	B	1	A	1
D	A	1	A	1	1	1
1	0	A	B	C	D	1

Table:3.2 .2 (complement)

X	x^*
0	1
A	C
B	D
C	A
D	B
1	0

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Define '□ ' and '□ ' operations on A as follows:

a) (x□ y)=(x→y)→ y

b) (x□ y)=((x^* →y^*)→y^*))* for all x,y in A, Let a set A={0,a,b,c,d,r,s,t,1} be a set with figures 1 as a partial ordering and and (A, □ , □ , *, 0, 1) is a lattice wajsberg algebra, a vague set S ={(0,1,0), (a,0.6,0.4) (b,1,0),(c,0.6,0.4),(d 0.6,0.4), (r, 0.6,0.4), (s 0.6,0.4), (t, 0.6,0.4) (1,0.6,0.4} is a vague WI-ideal of A.

Sol:- (i) $n_A{}^{(0)} \geq n_A{}^{(x)}$ and $n_A{}^{(0)} \leq n_A{}^{(x)}$

(II) Put y=a

(a) $m_A{}^{(x)} \geq \min\{m_A((x \to y)^*)\ ,\ m_A{}^{(y)}\} = \min\{m_A(1)^*\ ,\ m_A{}^{(y)}\}$

$= \min\{m_A{}^{(0)}, m_A(1)^*\} = \min\{m_A{}^{(0)}, m_A{}^{(a)}\} = \min\{1,\ 0.6\} = 0.6$

and $m_A{}^{(1)}=1$

Therefore $m_A{}^{(0)} \geq \min\{m_A((0 \to a)^*), m_A{}^{(a)}\}$.

(b) Put y=b

$m_A{}^{(x)} \geq \min\{m_A((x \to b)^*)\ ,\ m_A{}^{(b)}\} = \min\{m_A(1)^*\ ,\ m_A{}^{(b)}\}$

$= \min\{m_A{}^{(0)}, m_A{}^{(b)}\} = \quad \min\{m_A{}^{(0)}, m_A{}^{(b)}\} = \min\{1,\ 1\} = 1$

and $m_A{}^{(1)}=1$

Therefore $m_A{}^{(0)} \geq \min\{m_A((0 \to b)^*), m_A{}^{(b)}\}$.

(c)Put y=c

$m_A{}^{(x)} \geq \min\{m_A((x \to c)^*)\ ,\ m_A{}^{(c)}\} = \min\{m_A(1)^*\ ,\ m_A{}^{(c)}\}$

$= \min\{m_A{}^{(0)}, m_A{}^{(c)}\} = \quad \min\{m_A{}^{(0)}, m_A{}^{(c)}\} = \min\{1,\ 0.6\} = 0.6$

and $m_A{}^{(1)}=1$

Therefore $m_A{}^{(0)} \geq \min\{m_A((0 \to c)^*), m_A{}^{(c)}\}$.

(d)Put y=d

$m_A{}^{(x)} \geq \min\{m_A((x \to d)^*)\ ,\ m_A{}^{(d)}\} = \min\{m_A(1)^*\ ,\ m_A{}^{(d)}\}$

$= \min\{m_A{}^{(0)}, m_A{}^{(c)}\} = \quad \min\{m_A{}^{(0)}, m_A{}^{(c)}\} = \min\{1,\ 0.6\} = 0.6$

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and $m_A{}^{(1)}=1$

Therefore $m_A{}^{(0)} \geq \min\{m_A((0 \to c)^*), m_A{}^{(d)}\}$.

(c)Put y=r

$m_A{}^{(x)} \geq \min\{m_A((x \to r)^*), m_A{}^{(r)}\} = \min\{m_A(1)^*, m_A{}^{(r)}\}$

$= \min\{m_A{}^{(0)}, m_A{}^{(c)}\} = \min\{m_A{}^{(0)}, m_A{}^{(c)}\} = \min\{1, 0.6\} = 0.6$

and $m_A{}^{(1)}=1$

Therefore $m_A{}^{(0)} \geq \min\{m_A((0 \to r)^*), m_A{}^{(r)}\}$.

(c)Put y=s

$m_A{}^{(x)} \geq \min\{m_A((x \to s)^*), m_A{}^{(s)}\} = \min\{m_A(1)^*, m_A{}^{(s)}\}$

$= \min\{m_A{}^{(0)}, m_A{}^{(c)}\} = \min\{m_A{}^{(0)}, m_A{}^{(c)}\} = \min\{1, 0.6\} = 0.6$

and $m_A{}^{(1)}=1$

Therefore $m_A{}^{(0)} \geq \min\{m_A((0 \to s)^*), m_A{}^{(s)}\}$.

(c)Put y=t

$m_A{}^{(x)} \geq \min\{m_A((x \to t)^*), m_A{}^{(t)}\} = \min\{m_A(1)^*, m_A{}^{(t)}\}$

$= \min\{m_A{}^{(0)}, m_A{}^{(c)}\} = \min\{m_A{}^{(0)}, m_A{}^{(t)}\} = \min\{1, 0.6\} = 0.6$

and $m_A{}^{(1)}=1$

Therefore $m_A{}^{(0)} \geq \min\{m_A((0 \to t)^*), m_A{}^{(x)}\}$.

3. $n_A{}^{(x)} \leq \max\{n_A((x \to y)^*), n_A{}^{(x)}\}$ for all x, y ,z∈ A.

(a) Put y=0

$n_A{}^{(0)} \leq \max\{n_A((0 \to a)^*), n_A{}^{(0)}\} = \max\{n_A((1)^*), n_A{}^{(0)}\}$

$= \max\{n_A{}^{(0)}, n_A{}^{(0)}\} = 0$

Therefore $n_A{}^{(0)} \leq \max\{n_A((0 \to a)^*), n_A{}^{(0)}\}$.

(b) Put y=a

$n_A{}^{(0)} \leq \max\{n_A((0 \to a)^*), n_A{}^{(a)}\} = \max\{n_A((1)^*), n_A{}^{(a)}\}$

$= \max\{n_A{}^{(0)}, n_A{}^{(a)}\} = \max\{0, 0.4\} = 0.4$ and $n_A{}^{(0)} = 0$

Therefore $n_A{}^{(0)} \leq \max\{n_A((0 \to a)^*), n_A{}^{(a)}\}$.

(c) Put y=b

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$n_A{}^{(0)} \le \max\{n_A((0 \to b)^*),\ n_A{}^{(b)}\} = \max\{n_A((1)^*),\ n_A{}^{(b)}\}$
$= \max\{n_A{}^{(0)},\ n_A{}^{(b)}\} = \max\{0, 0\} = 0$ and $n_A{}^{(0)} = 0$
Therefore $n_A{}^{(0)} \le \max\{n_A((0 \to a)^*),\ n_A{}^{(b)}\}$.

(d) Put y=c
$n_A{}^{(0)} \le \max\{n_A((0 \to c)^*),\ n_A{}^{(c)}\} = \max\{n_A((1)^*),\ n_A{}^{(c)}\}$
$= \max\{n_A{}^{(0)},\ n_A{}^{(c)}\} = \max\{0, 0.4\} = 0.4$ and $f_A{}^{(0)} = 0$
Therefore $n_A{}^{(0)} \le \max\{n_A((0 \to a)^*),\ n_A{}^{(c)}\}$.

(e) Put y=d
$n_A{}^{(0)} \le \max\{n_A((0 \to d)^*),\ n_A{}^{(d)}\} = \max\{n_A((1)^*),\ n_A{}^{(d)}\}$
$= \max\{n_A{}^{(0)},\ n_A{}^{(d)}\} = \max\{0, 0.6\} = 0.6$ and $n_A{}^{(0)} = 0$
Therefore $n_A{}^{(0)} \le \max\{n_A((0 \to a)^*),\ n_A{}^{(c)}\}$.

(f) Put y=r
$n_A{}^{(0)} \le \max\{n_A((0 \to r)^*),\ n_A{}^{(r)})\} = \max\{n_A((1)^*),\ n_A{}^{(r)}\}$
$= \max\{n_A{}^{(0)},\ n_A{}^{(r)}\} = \max\{0, 0.4\} = 0.4$ and $n_A{}^{(0)} = 0$
Therefore $n_A{}^{(0)} \le \max\{n_A((0 \to r)^*),\ n_A{}^{(r)}\}$.

(g) Put y=s
$n_A{}^{(0)} \le \max\{n_A((0 \to s)^*),\ n_A{}^{(s)}\} = \max\{n_A((1)^*),\ n_A{}^{(s)}\}$
$= \max\{n_A{}^{(0)},\ n_A{}^{(s)}\} = \max\{0, 0.4\} = 0.4$ and $n_A{}^{(0)} = 0$
Therefore $n_A{}^{(0)} \le \max\{n_A((0 \to s)^*),\ n_A{}^{(s)}\}$.

S is a vague WI-ideal of A.

Example 3.3 Let A={0,a,b,c,d,r,s,t,1} be a set as a partial ordering .Define a quasi complement " * " and a binary operation " → " on A .Then a vague set S ={(0,0.32,0.56), (a,1,0) (b,1,0),(c,0.32,0.56),(d 0.32,0.56), (r, 0.32,0.56), (s 0.32,0.56), (t, 0.32,0.56) (1,0.32,0.56} of A. Then S is a vague WI-ideal of A.

Sol: Let A={0,a,b,c,d,r,s,t,1} be a set as a partial ordering . By the figures1,2,3

Define a quasi complement" * " and a binary operation " → " on A .

Let S ={ (a,1,0) (b,1,0),(c,0.32,0.56),(d 0.32,0.56), (r, 0.32,0.56), (s 0.32,0.56), (t, 0.32,0.56) (1,0.32,0.56} be a vague set of A.

(i) $m_A{}^{(a)} \ge m_A{}^{(x)}$ and $n_A{}^{(a)} \le n_A{}^{(x)}$

(II) $m_A{}^{(x)} \ge \min\{m_A((x \to y)^*),\ m_A{}^{(y)}\}$

(a) Put x=a, y=b
$m_A{}^{(x)} \ge \min\{m_A((x \to y)^*),\ m_A{}^{(y)}\} = \min\{m_A((a \to b)^*),\ m_A{}^{(b)}\}$

$= \min\{m_A(1)^*, m_A{}^{(b)}\}$

$= \min\{m_A{}^{(0)}, 1\} = \min\{1, 1\} = 1$

and $m_A{}^{(a)} = 1$

Therefore $m_A{}^{(a)} \geq \min\{m_A((0 \to a)^*), m_A{}^{(a)}\}$.

(b) Put x=a, y=c

$m_A{}^{(x)} \geq \min\{m_A((x \to y)^*), m_A{}^{(y)}\} = \min\{m_A(t)^*, m_A{}^{(c)}\}$

$= \min\{m_A{}^{(a)}, 0.32\} = \min\{1, 0.32\} = 0.32$

and $m_A{}^{(a)} = 1$

Therefore $m_A{}^{(a)} \geq \min\{m_A((0 \to b)^*), m_A{}^{(b)}\}$.

(c) Put x=a, y=d

$m_A{}^{(x)} \geq \min\{m_A((x \to y)^*), m_A{}^{(y)}\} = \min\{m_A(1)^*, m_A{}^{(d)}\}$

$= \min\{m_A{}^{(0)}, 0.32\} = \min\{1, 0.32\} = 0.32$

and $m_A{}^{(a)} = 1$

Therefore $m_A{}^{(a)} \geq \min\{m_A((0 \to c)^*), m_A{}^{(d)}\}$.

(d) Put x=a, y=r

$m_A{}^{(x)} \geq \min\{m_A((x \to y)^*), m_A{}^{(y)}\} = \min\{m_A(1)^*, m_A{}^{(r)}\}$

$= \min\{m_A{}^{(0)}, 0.32\} = \min\{1, 0.32\} = 0.32$

and $m_A{}^{(a)} = 1$

Therefore $m_A{}^{(a)} \geq \min\{m_A((a \to r)^*), m_A{}^{(r)}\}$.

(e) Put x=a, y=s

$m_A{}^{(x)} \geq \min\{m_A((x \to y)^*), m_A{}^{(y)}\} = \min\{m_A(t)^*, m_A{}^{(r)}\}$

$= \min\{m_A{}^{(a)}, 0.32\} = \min\{1, 0.32\} = 0.32$

and $m_A{}^{(a)} = 1$

Therefore $m_A{}^{(a)} \geq \min\{m_A((a \to s)^*), m_A{}^{(s)}\}$.

(f) Put x=a, y=t

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$m_A{}^{(x)} \geq \min\{m_A((x \to y)^*)\ ,\ m_A{}^{(y)}\} = \min\{m_A(1)^*\ ,\ m_A{}^{(t}\}$

$= \min\{m_A{}^{(a)}, 0.32\} = \min\{1, 0.32\} = 0.32$

and $m_A{}^{(a)} = 1$

Therefore $m^{(a)} \geq \min\{m_A((a \to t)^*), m_A{}^{(t)}\}$.

3. $n_A{}^{(x)} \leq \max\{n_A((x \to y)^*)\ , n_A{}^{(x)}\}$ for all x, y ,z∈ A.

(a) Put x=a,y=b
$n_A{}^{(a)} \leq \max\{n_A((a \to b)^*)\ , n_A{}^{(b)}\} = \max\{n_A((1)^*)\ , n_A{}^{(b)}\}$
$= \max\{n_A{}^{(0)}\ , n_A{}^{(b)}\} = \max\{0,0\} = 0$
Therefore $n_A{}^{(a)} \leq \max\{n_A((0 \to a)^*)\ , n_A{}^{(0)}\}$.

(b) Put x=a,y=c
$n_A{}^{(a)} \leq \max\{n_A((a \to c)^*)\ , n_A{}^{(c)}\} = \max\{n_A((1)^*)\ , n_A{}^{(c)}\}$
$= \max\{n_A{}^{(0)}\ , n_A{}^{(c)}\} = \max\{0, 0.56\} = 0.56$
Therefore $n_A{}^{(0)} \leq \max\{n_A((0 \to a)^*)\ , n_A{}^{(0)}\}$.

(c) Put x=a,y=c
$n_A{}^{(a)} \leq \max\{n_A((a \to c)^*)\ , n_A{}^{(c)}\} = \max\{n_A((t)^*)\ , n_A{}^{(c)}\}$
$= \max\{n_A{}^{(a)}\ , n_A{}^{(c)}\} = \max\{0, 0.56\} = 0.56$
Therefore $n_A{}^{(0)} \leq \max\{n_A((0 \to a)^*)\ , n_A{}^{(0)}\}$.

(d) Put x=a,y=d
$n_A{}^{(a)} \leq \max\{n_A((a \to d)^*)\ , n_A{}^{(d)}\} = \max\{n_A((1)^*)\ , n_A{}^{(d)}\}$
$= \max\{n_A{}^{(0)}\ , n_A{}^{(d)}\} = \max\{0, 0.56\} = 0.56$
Therefore $n_A{}^{(a)} \leq \max\{n_A((a \to d)^*)\ , n_A{}^{(d)}\}$.

(e) Put x=a,y=r
$n_A{}^{(a)} \leq \max\{n_A((a \to r)^*)\ , n_A{}^{(r)}\} = \max\{n_A((1)^*)\ , n_A{}^{(r)}\}$
$= \max\{n_A{}^{(0)}\ , n_A{}^{(r)}\} = \max\{0, 0.56\} = 0.56$
Therefore $n_A{}^{(a)} \leq \max\{n_A((a \to r)^*)\ , n_A{}^{(r)}\}$.

(f) Put x=a,y=s
$n_A{}^{(a)} \leq \max\{n_A((a \to s)^*)\ , n_A{}^{(s)}\} = \max\{n_A((t)^*)\ , n_A{}^{(c)}\}$
$= \max\{n_A{}^{(a)}\ , n_A{}^{(c)}\} = \max\{0, 0.56\} = 0.56$
Therefore $n_A{}^{(a)} \leq \max\{n_A((a \to t)^*)\ , n_A{}^{(t)}\}$.

(g) Put x=a,y=t
$n_A{}^{(a)} \leq \max\{n_A((a \to t)^*)\ , n_A{}^{(t)}\} = \max\{n_A((1)^*)\ , n_A{}^{(t)}\}$
$= \max\{n_A{}^{(0)}\ , n_A{}^{(t)}\} = \max\{0, 0.56\} = 0.56$
Therefore $n_A{}^{(a)} \leq \max\{n_A((a \to t)^*)\ , n_A{}^{(t)}\}$.

Therefore S is a vague WI-ideal of A.

Example 3.4 Let A be a lattice wajsberg algebra defined on the above examples, define an vague set A=(m_A , n_A) be a vague set of a partial ordered set P as follows.

1. $m_A{}^{(0)} = m_A{}^{(c)} = 1$

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2. $m_A^{(x)}=m$, for any x in {a,b,c,d,r,s,t,1}

3. $m_A^{(0)}= m_A^{(c)}=0$

4. $n_A^{(x)}=n$ for any x in {a,b,c,d,r,s,t,1} where $m,n\in [0,1]$ and $m+n\leq 1$.Then A is an vague WI-ideal of A.

Theorem 3.5 Every vague WI-ideal A=(m_A , n_A) of a lattice wajsberg algebra A is an vague monotonic, that is ,if $x\leq y$, then $m_A^{(x)} \geq m_A^{(y)}$ and $n_A^{(x)}\leq n_A^{(y)}$.

Proof: Let $x,y\in A$ $x\leq y$.Then $(x\rightarrow y)^* =1^*=0$

$m_A^{(x)} \geq \min\{m_A((x\rightarrow y)^*)\ ,\ m_A^{(y)}\}=\min\{ t_A^{(0)} ,m_A^{(y)}\}=m_A^{(y)}$

Therefore $m_A^{(x)} \geq m_A^{(y)}$.

Now ,$n_A^{(x)} \leq \max\ \{n_A((x\rightarrow y)^*)\ ,n_A^{(x)}\}=\max\ \{n_A^{(0)} ,n_A^{(y)}\}$
$= n_A^{(y)}$

Therefore $n_A^{(x)} \leq n_A^{(y)}$.

Theorem 3.6 Let an vague set A=(m_A , n_A) be a vague WI-ideal of a lattice wajesberg algebra A.for any $x,y\in A$ which satisfies $x\leq y^* \rightarrow z$ then $m_A^{(x)} \geq \min\{m_A^{(x)},\ m_A^{(y)}\}$ and

$n_A^{(x)} \leq \max\{n_A^{(y)},\ n_A^{(z)}\}$.

Proof: Let $x,y,z\in A$ $x\leq y^* \rightarrow z$

Then, we have $1=x\rightarrow (y^* \rightarrow z)$

$=z^* \rightarrow (x\rightarrow y)=(x\rightarrow y)^* \rightarrow z$ (since wajsberg algebra)

and so , $((x\rightarrow y)^* \rightarrow z)^*=0$.

It follows from the definition of WI-ideal, that

$m_A^{(x)} \geq \min\{m_A((x\rightarrow y)^*)\ ,\ m_A^{(y)}\}$

$\geq \min\ \{ min\ \{m_A((x\rightarrow y)^* \rightarrow z)^*\ ,\ m_A^{(z)}\}, m_A^{(y)}\}$

$=\min\{ \min\{ m_A^{(0)} ,m_A^{(z)}\},m_A^{(y)}\}$

$=\min\{ m_A^{(y)},m_A^{(z)}\}$.

Therefore $m_A^{(x)}\geq \min\{ m_A^{(y)},m_A^{(z)}\}$.

$n_A^{(x)} \leq \max\ \{n_A((x\rightarrow y)^*)\ ,\ n_A^{(y)}\}$

$\leq \max\ \{ max\ \{n_A((x\rightarrow y)^* \rightarrow z)^*\ ,\ n_A^{(z)}\}, n_A^{(y)}\}$

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$=\max\{ \max\{ n_A{}^{(0)}, n_A{}^{(z)}\}, n_A{}^{(y)}\}$

$=\max\{ n_A{}^{(y)}, n_A{}^{(z)}\}.$

Therefore $n_A{}^{(x)} \leq \max\{ n_A{}^{(y)}, n_A{}^{(z)}\}.$

Definition 3.7 An vague set A=(m_A , n_A) of a lattice A is called an vague lattice ideal of A if it satisfies the following for all x,y in A,

1.A=(m_A , n_A) is a vague monotonic

2. $m_A{}^{(x\square y)} \geq \min\{m_A{}^{(x)}, m_A{}^{(y)}\}$
3. $n_A{}^{(x\square y)} \leq \max\{n_A{}^{(x)}, n_A{}^{(y)}\}$ for all x,y in A.

Remark 3.8. In the above definition 3.7 conditions of 1 and 2 can be equivalently replaced by 1.$m_A{}^{(x\square y)} \geq \min\{m_A{}^{(x)}, m_A{}^{(y)}\}$ and 2. $n_A{}^{(x\square y)} \leq \max\{n_A{}^{(x)}, n_A{}^{(y)}\}$ for all x,y in A.

Example 3.9. Let A be a lattice wajsberg algebra defined in the Let A={0,a,b,c,d,r,s,t,1} be a set as a partial ordering .Define a quasi complement " * " and a binary operation " → " on A .Then a vague set S ={(0,1,0), (a,m,n) (b,m,n),(c, m,n),(d ,1,0), (r, m,n), (s m,n), (t, m,n) (1,m,n} of A. Then S is a vague lattice ideal of A. Wher m,n∈[0, 1] and m+n≤1.Then

A=(t_A , f_A) is an vague lattice of A.

Theorem 3.10. Let A be lattice wajsberj algebra. Every vague WI-ideal of A is an vague lattice ideal of A.

Proof: Let A=(m_A , n_A) be an vague lattice ideal of A. clearly 1. $m_A{}^{(0)} \geq m_A{}^{(x)}$

and $n_A{}^{(0)} \leq n_A{}^{(x)}$

For any x,y in A.Now$x \leq x\square y$ for all x,y in A it follows that,

$m_A{}^{(x)} \geq m_A{}^{(x\square y)} \geq m_A{}^{(y\,\square\, x^*\square\, y)^*)} = m_A{}^{(y\,\square\, x^* \to y)^*)} \geq \min\{m_A((x \to y)^*),\ m_A{}^{(y)}\}$

Therefore $m_A{}^{(x)} \geq \min\{m_A((x \to y)^*),\ m_A{}^{(y)}\}.$

$n_A{}^{(x)} \leq n_A{}^{(x\square y)} \leq n_A{}^{(y\,\square\, x^*\square\, y)^*)} = n_A{}^{(y\,\square\, x^* \to y)^*)} \leq \max\{n((x \to y)^*),\ n_A{}^{(y)}\}$

Therefore $n_A{}^{(x)} \leq \max\{n_A((x \to y)^*),\ n_A{}^{(y)}\}.$

Thus A is an vague WI-ideal of A.

Theorem 3.11. The converse of the above theorem need not be true . Let A be lattice wajsberj algebra. Every vague lattice ideal of A is not a vague WI-ideal of A .

Let A be lattice wajsberj algebra defined in the example 3.3 A=(m_A , n_A) be a vague set of A is an vague WI-ideal of A.

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Example 3.12 Let A={0,a,b,c,d,r,s,t,1} be a set as a partial ordering .Define a quasi complement " * " and a binary operation " → " on A .Then a vague set S ={(0,1,0), (a,0.7,0) (b,1,0),(c,0.7,0.56),(d ,1,0), (r, 0.7,0.3), (s 0.7,0.3), (t, 0.7,0.3), (1,0.7,0.3)} of A is vague lattice ideal of A but S is not a vague WI-ideal of A.

Theorem 3.13 In a H-wajsberg algebra A, every vague lattice ideal of A is an vague WI-ideal of A.

Proof: Let A=(m_A , n_A) be an vague lattice ideal of A.

Clearly (i) $m_A{}^{(0)} \geq m_A{}^{(x)}$ and $n_A{}^{(0)} \leq n_A{}^{(x)}$ for any x,y in A .

Now , x ≤ x□ y for all x,y in A ,it follows that,

(i) $m_A{}^{(x)} \geq m_A{}^{(x\,\square\,y)} = m_A{}^{(y\,\square}\,(x^*\square y)^*$
$= m_A{}^{(y\,\square}\,(x\to y)^*\,) = \min\{\, m_A{}^{(y)}, m_A\,(x\to y)^*\,\}$.

$m_A{}^{(x)} \geq \min\{\, m_A{}^{(y)}, m_A\,(x\to y)^*\,\}$.

(ii) $n_A{}^{(x)} \leq n_A{}^{(x\,\square\,y)} = n_A{}^{(y\,\square}\,(x^*\square y)^* = n_A{}^{(y\,\square}\,(x\to y)^*\,)$

$=\max\{\, n_A{}^{(y)}, n_A(x\to y)^*\,\}$.

$n_A{}^{(x)} \leq \max\{\, n_A{}^{(y)}, n_A(x\to y)^*\,\}$.

Thus A=(m_A , n_A) be an vague WI-ideal of A.

Theorem 3.14 Let A be lattice wajsberg algebra. An vague A=(m_A , n_A) is an vague WI-ideal of A if and only if the fuzzy set m_A and ${n_A}^c$ are vague ideals of A, where ${n_A}^c(x)$=1-${n_A}^{(x)}$ for any x ∈A.

Proof: A=(m_A , n_A) is an vague WI-ideal of A. Clearly, $t_A$is a vague WI-ideal of A. For any x, y ∈ A, we have ${n_A}^c(0)=1-{n_A}^{(0)} \geq 1-{n_A}^{(x)}$

Therefore ${n_A}^c(0)={n_A}^c(x)$ and ${n_A}^c(x)=1-f_A(x) \geq 1-\max\{\, n_A((x\to y)^*\,), n_A{}^{(y)}\,\}$

$=\min\{1- n_A((x\to y)^*\,), 1-{n_A}^{(y)}\,\}$

$=\min\{\, {n_A}^c\,((x\to y)^*\,), {n_A}^{(y)}\,\}$.

Hence, we have n^c is vague WI-ideal of A.

Conversely, assume that m_A and ${n_A}^c$ are vague WI-ideals of A. For any x,y∈ A, we get

$m_A{}^{(0)} \geq m_A{}^{(x)}$ and $1-{n_A}^{(0)} = {n_A}^c(0) \geq {n_A}^c(x)$

$=1- n^{(x)}$

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Implies that $n_A{}^{(0)} \leq n_A{}^{(x)}$

$$m_A{}^{(x)} \geq \min\{m_A((x \to y)^*),\ m_A{}^{(y)}\}\ldots\ldots.(3.14.1)$$

and $1 - m_A{}^{(x)} = m_A{}^{c\ (x)} \geq \min\{ m_A{}^{c}\ ((x \to y)^*),\ m_A{}^{c(y)} \}$

$$= \min\{1 - m_A((x \to y)^*),\ 1 - m_A{}^{(y)}\}$$

$$= 1 - \max\{m_A((x \to y)^*),\ m_A{}^{(y)}\}$$

$$n_A{}^{(x)} \leq \max\{n_A((x \to y)^*),\ n_A{}^{(y)}\}\ldots\ldots\ldots(3.14.2)$$

Hence, from (3.14.1) and (3.14.2) we have A=(t_A, f_A) is an vague WI-ideal of A.

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