

STRUCTURES ON INTUITIONIST FUZZY R - IDEAL

¹G.Rethnarexlin, ^{2*}G.Subbiah and ³V.Nagarajan

1 Research scholar, Reg.No: 18123152092017, Department of Mathematics,
S.T.Hindu College, Nagercoil-629 002, Tamil Nadu, India.

2 *Associate Professor in Mathematics, Sri K.G.S. Arts College,
Srivaikuntam-628 619, Tamil Nadu, India.

3 Assistant Professor in Mathematics, S.T.Hindu College,
Nagercoil-629 002, Tamil Nadu, India.

* Corresponding author: E-mail Id: subbiahkgs@gmail.com

**Affiliated to Manonmaniam Sundaranar University, Abishekapatti,
Tirunelveli-627 012, Tamil Nadu, India.**

ABSTRACT :

In this paper, we introduce the notion of direct product of intuitionist fuzzy R – ideal of BCK – algebras and related properties are investigated. Characterizations of direct product intuitionist fuzzy R – ideals of BCK – algebras are given.

Key words: R-ideal, Fuzzy R-ideal, intuitionistic fuzzy ideal, intuitionistic Fuzzy R-ideal

1.Introduction:The notion of fuzzy sets in a set theory was introduced by Zadeh[27], and since then this concept has been applied to various algebraic structures. The idea of intuitionist fuzzy sets was first introduced by Atanassov [7,8], as a generalization of the notion of fuzzy set. Abdullah et al. [1 – 3,6], provided some interesting results on direct product of fuzzy ideals in different algebraic structures. The product of fuzzy subgroups was introduced in [20,26]. Imai and Iseki introduced two classes of algebras, BCK-algebras [12,13]. BCI-algebras are generalizations of BCK-algebras which were studied by many researchers [4,10,11,14,18,19]. A.Shehri[5], Jun et al. [15 – 17], Saeid et al. [21 – 23] and Satyanarayana et al. [24,25], applied the concept of fuzzy set to BCK-algebras, Zhan and Tan [28], introduced the concept of fuzzy R-ideals in BCK-algebras. In this paper, we introduce the notion of direct product of intuitionist fuzzy R-ideals in BCK-algebras and some related properties are investigated. Characterizations of direct product of

intuitionist fuzzy R-ideal of BCK-algebras are given. And also we introduce the notion of upper s-level cut of μ_{AXB} and lower t-level cut of λ_{AXB} . Also we proved, for an intuitionist fuzzy set of BCK-algebras $A \times B = \langle \mu_{AXB}, \lambda_{AXB} \rangle$ of $X_1 \times X_2$, then $A \times B = \langle \mu_{AXB}, \lambda_{AXB} \rangle$ is an intuitionist fuzzy R-ideals of BCK-algebras $X_1 \times X_2$ if and only if for any $s, t \in [0,1]$ upper and lower level sets are R-ideals of BCK-algebra $X_1 \times X_2$.

2. Preliminaries

Algebra $(X, *, 0)$ of type $(2,0)$ is called a BCI-algebra if satisfies the following conditions:

- (i) $\forall x, y, z \in X, ((x * y) * (x * z)) * (z * y) = 0,$
- (ii) $\forall x, y \in X, (x * (x * y)) * y = 0,$
- (iii) $\forall x \in X, x * x = 0,$
- (iv) $\forall x, y \in X, x * y = 0, y * x = 0 \implies x = y,$

We can define a partial order ' \leq ' on X by $x \leq y$ if and only if $x * y = 0$.

Any BCI-algebra X has the following properties:

- (T1) $\forall x \in X, x * 0 = x,$
- (T2) $\forall x, Y, Z \in X, (x * y) * z = (x * z) * y,$
- (T3) $\forall x, Y, Z \in X, x \leq y \implies x * z \leq y * z, z * y \leq z * x.$

Definition 2.1 A nonempty subset A of a BCI-algebra X is called an **ideal** of X if it satisfies:

- (II) $0 \in A, (I2) \forall x, y \in X, y \in A \implies x * y \in A$

Definition 2.2 A nonempty subset A of a BCI-algebra X is called a **a-ideal** of X if it satisfies:

- (II) and (I3) $\forall x, y \in X, (\forall z \in A) ((x * z) * (0 * y)) \in A \implies y * x \in A$

Definition 2.3 A nonempty subset I of a BCI-algebra X is called **R-ideal** of X, if

$$1. 0 \in I, 2. (x * z) * (z * y) \in I \text{ and } y \in I \implies x \in I$$

Definition 2.4 A fuzzy subset μ in a BCK- algebra X is called a fuzzy R- ideal of X , if

1. $\mu(0) \geq \mu(x)$
2. $\mu(x) \geq \min \{ \mu((x * z) * (y * z)), \mu(y) \}, \forall x, y, z \in X.$

Definition 2.5 Ideal I of a BCI-algebra $(X, *, 0)$ is called Closed if $0 * x \in I$, for all $x \in I$

Definition 2.6 Let A and B be two fuzzy ideal of BCI algebra X . The fuzzy set $A \cap B$

With membership function $\mu_{A \cap B}$ is defined by $\mu_{A \cap B}(x) = \min \{ \mu_A(x), \mu_B(x) \}, \forall x \in X$

Definition 2.7 Let A and B be two fuzzy ideal of BCI algebra X . The fuzzy set $A \cup B$.

With membership function $\mu_{A \cup B}$ is defined by $\mu_{A \cup B}(x) = \max \{ \mu_A(x), \mu_B(x) \}, \forall x \in X.$

Definition 2.8 Let A and B be two fuzzy ideal of BCI algebra X with membership function μ_A and μ_B respectively. A is contained in B if $\mu_A(x) \leq \mu_B(x), \forall x \in X.$

Definition 2.9 Let A be a fuzzy ideal of BCI algebra X . The fuzzy set A^m with membership function μ_A^m is defined by $\mu_A^m(x) = (\mu_A(x))^m, \forall x \in X.$

Definition 2.10 Let μ be a fuzzy set in X . The complement of μ is denoted by $\bar{\mu}$ and is defined as $\bar{\mu}(x) = 1 - \mu(x), \forall x \in X.$

Definition 2.11 Let $A = (X, \mu_A, \lambda_A)$ be an intuitionist fuzzy set in X . Then

- (i) $\bar{A} = (X, \mu_A, \mu_{\bar{A}})$ and
- (ii) $A = (X, \lambda_{\bar{A}}, \lambda_A)$.

Definition 2.12 An intuitionist fuzzy set A in non-empty set X is an object having the form

$A = \{ (x, \mu_A(x), \lambda_A(x)) : x \in X \}$, where the function $\mu_A : X \rightarrow [0,1]$ and $\lambda_A : X \rightarrow [0,1]$ denoted the degree of membership (namely $\mu_A(x)$) and the degree of non-Membership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$

Definition 2.13 An IFS $A = \langle X, \mu_A, \lambda_A \rangle$ in a BCI-algebra X is called an intuitionist fuzzy Sub-algebra of X if it satisfies: $\forall x, y \in X.$

1. $\mu_A(x * y) \geq \min \{ \mu_A(x), \mu_A(y) \}, \forall x, y \in X.$
2. $\lambda_A(x * y) \leq \max \{ \lambda_A(x), \lambda_A(y) \}$

Definition 2.14 An intuitionist fuzzy set $A = (X, \mu_A, \lambda_A)$ in X is called an intuitionist fuzzy ideal of X , if it satisfies the following axioms:

$$(IF1) \quad \mu_A(0) \geq \mu_A(x) \text{ and } \lambda_A(0) \leq \lambda_A(x),$$

$$(IF2) \quad \mu_A(x) \geq \min \{ \mu_A(x * y), \mu_A(y) \},$$

$$(IF3) \quad \lambda_A(x) \leq \max \{ \lambda_A(x * y), \lambda_A(y) \}, \forall x, y \in X.$$

Definition 2.15 An intuitionist fuzzy set $A = (X, \mu_A, \lambda_A)$ in X is called an intuitionist fuzzy ideal of X , if it satisfies (IF2), (IF3) and the following :

$$(IF4) \quad \mu_A(0 * x) \geq \mu_A(x) \text{ and } \lambda_A(0 * x) \leq \lambda_A(x), \forall x \in X$$

Definition 2.16 An IFS $A = \langle X, \mu_A, \lambda_A \rangle$ in X is called an intuitionist fuzzy R-ideal of X . If it satisfies (2.12) and $(\forall x, y, z \in X)$.

$$1. \mu_A(x) \geq \min \{ \mu_A((x * z) * (z * y)), \mu_A(y) \}$$

$$2. \lambda_A(x) \leq \max \{ \lambda_A((x * z) * (z * y)), \lambda_A(y) \}$$

3. Direct Product of Intuitionistic Fuzzy R-ideals

Definition 3.1 Let $A = \langle \mu_A, \lambda_A \rangle$ and $B = \langle \mu_B, \lambda_B \rangle$ be two intuitionist fuzzy sets in BCK-algebras X_1 and X_2 respectively. Then direct product of intuitionist fuzzy sets A and B is denoted by $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$, and defined as $\mu_{A \times B}(x, y) = \min \{ \mu_A(x), \mu_B(y) \}$ and $\lambda_{A \times B}(x, y) = \max \{ \lambda_A(x), \lambda_B(y) \}$, for all $(x, y) \in X_1 \times X_2$.

Definition 3.2 An IFS $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$, of $X_1 \times X_2$ is called an intuitionist fuzzy sub-algebra of $X_1 \times X_2$ if

$$(DIF1) \quad \mu_{A \times B}((x_1 y_1) * (x_2 y_2)) \geq \min \{ \mu_{A \times B}(x_1 y_1), \mu_{A \times B}(x_2 y_2) \}$$

$$(DIF2) \quad \lambda_{A \times B}((x_1 y_1) * (x_2 y_2)) \leq \max \{ \lambda_{A \times B}(x_1 y_1), \lambda_{A \times B}(x_2 y_2) \}$$

For all $(x_1 y_1), (x_2 y_2) \in X_1 \times X_2$

Definition 3.3 An IFS $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$, of $X_1 \times X_2$ is called an intuitionistic fuzzy R-ideal of $X_1 \times X_2$ if

$$(DIF3) \quad \mu_{A \times B}((0, 0)) \geq \mu_{A \times B}(x, y) \text{ and } \lambda_{A \times B}(0, 0) \leq \lambda_{A \times B}(x, y)$$

$$(DIF4) \mu_{A \times B}((x_1 y_1)) \geq \min\{ \mu_{A \times B}(((x_1 y_1) * (x_3 y_3)) * ((x_2 y_2) * (x_3 y_3))), \mu_{A \times B}((x_2 y_2))\}$$

$$(DIF5) \lambda_{A \times B}((x_1 y_1)) \leq \max\{ \lambda_{A \times B}, (((x_1 y_1) * (x_3 y_3)) * ((x_2 y_2) * (x_3 y_3))), \lambda_{A \times B}(x_2 y_2)\},$$

For all $(x_1 y_1), (x_2 y_2), (x_3 y_3) \in X_1 \times X_2$.

Definition 3.4 An IFS $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$, of $X_1 \times X_2$ is called an intuitionist fuzzy closed R-ideal of $X_1 \times X_2$ if it satisfies (DIF3), (DIF4), and (DIF5) and the following

$$(DIF6) \mu_{A \times B}((0,0) * (x, y)) \geq \mu_{A \times B}(x, y) \text{ and } \lambda_{A \times B}((0,0) * (x, y)) \leq \lambda_{A \times B}(x, y)$$

Theorem 3.5: Let $A = \langle \mu_A, \lambda_A \rangle$ and $B = \langle \mu_B, \lambda_B \rangle$ be two intuitionist fuzzy sub-algebra of BCK-algebras X_1 and X_2 respectively. Then $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionist fuzzy sub-algebra of BCK-algebras $X_1 \times X_2$.

Proof: For any $(x_1 y_1), (x_2 y_2) \in X_1 \times X_2$. Then

$$\mu_{A \times B}((x_1 y_1) * (x_2 y_2)) = \mu_{A \times B}(x_1 * x_2, y_1 * y_2)$$

$$= \min\{ \mu_A(x_1 * x_2), \mu_B(y_1 * y_2) \}$$

$$\geq \min\{ \min\{ \mu_A(x_1), \mu_A(x_2) \}, \min\{ \mu_B(y_1), \mu_B(y_2) \} \}$$

$$= \min\{ \min\{ \mu_A(x_1), \mu_A(y_1) \}, \min\{ \mu_B(x_2), \mu_B(y_2) \} \}$$

$$\geq \min\{ \mu_{A \times B}(x_1, y_1), \mu_{A \times B}(x_2, y_2) \}$$

$$\text{And } \lambda_{A \times B}((x_1 y_1) * (x_2 y_2)) = \lambda_{A \times B}(x_1 * x_2, y_1 * y_2)$$

$$= \max\{ \lambda_A(x_1 * x_2), \lambda_B(y_1 * y_2) \}$$

$$\leq \max\{ \max\{ \lambda_A(x_1), \lambda_A(x_2) \}, \max\{ \lambda_B(y_1), \lambda_B(y_2) \} \}$$

$$= \max\{ \max\{ \lambda_A(x_1), \lambda_B(y_1) \}, \max\{ \lambda_A(x_2), \lambda_B(y_2) \} \}$$

$$\leq \max\{ \lambda_{A \times B}(x_1, y_1), \lambda_{A \times B}(x_2, y_2) \}$$

Hence for all $(x_1, y_1), (x_2, y_2) \in X_1 \times X_2$, $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionist fuzzy sub-algebra of BCK-algebras $X_1 \times X_2$.

Theorem 3.6: Let $A = \langle \mu_A, \lambda_A \rangle$ and $B = \langle \mu_B, \lambda_B \rangle$ be two intuitionist fuzzy H-ideals of BCK-algebras X_1 and X_2 respectively. Then $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionist fuzzy R-ideal of BCK-algebra as $X_1 \times X_2$.

Proof: For any $(x, y) \in X_1 \times X_2$

$$\mu_{A \times B}(0,0) = \min \{ \mu_A(0), \mu_B(0) \} \geq \min \{ \mu_A(x), \mu_B(y) \} = \mu_{A \times B}(x, y).$$

$$\text{And } \lambda_{A \times B}(0,0) = \max \{ \lambda_A(0), \lambda_B(0) \} \leq \max \{ \lambda_A(x), \lambda_B(y) \} = \lambda_{A \times B}(x, y).$$

Now for any $(x_1 y_1), (x_2 y_2), (x_3 y_3) \in X_1 \times X_2$.

$$\begin{aligned} \mu_{A \times B}((x_1, y_1)) &= \min \{ \mu_A(x_1), \mu_B(y_1) \} \\ &\geq \min \{ \min \{ \mu_A((x_1 * x_3) * (x_2 * x_3)), \mu_A(x_2) \}, \min \{ \mu_B((y_1 * y_3) * (y_2 * y_3)), \mu_B(y_2) \} \} \\ &= \min \{ \min \{ \mu_A((x_1 * x_3) * (x_2 * x_3)), \mu_B((y_1 * y_3) * (y_2 * y_3)) \}, \min \{ \mu_A(x_2), \mu_B(y_2) \} \} \\ &= \min \{ \mu_{A \times B}((x_1 * x_3) * (x_2 * x_3)), ((y_1 * y_3) * (y_2 * y_3)), \mu_{A \times B}(x_2, y_2) \} \\ &\geq \min \{ \mu_{A \times B}(((x_1, y_1) * (x_3, y_3)) * ((x_2, y_2) * (x_3 y_3))), \mu_{A \times B}(x_2, y_2) \} \end{aligned}$$

$$\begin{aligned} \text{And } \lambda_{A \times B}((x_1, y_1)) &= \max \{ \lambda_A(x_1), \lambda_B(y_1) \} \\ &\leq \max \{ \max \{ \lambda_A((x_1 * x_3) * (x_2 * x_3)), \lambda_A(x_2) \}, \max \{ \lambda_B((y_1 * y_3) * (y_2 * y_3)), \lambda_B(y_2) \} \} \\ &= \max \{ \max \{ \lambda_A((x_1 * x_3) * (x_2 * x_3)), \lambda_B((y_1 * y_3) * (y_2 * y_3)) \}, \max \{ \lambda_A(x_2), \lambda_B(y_2) \} \} \\ &= \max \{ \lambda_{A \times B}((x_1 * x_3) * (x_2 * x_3)), ((y_1 * y_3) * (y_2 * y_3)), \lambda_{A \times B}(x_2, y_2) \} \\ &\leq \max \{ \lambda_{A \times B}(((x_1, y_1) * (x_3, y_3)) * ((x_2, y_2) * (x_3 y_3))), \lambda_{A \times B}(x_2, y_2) \} \end{aligned}$$

Hence for all $(x_1 y_1), (x_2 y_2), (x_3 y_3) \in X_1 \times X_2$, $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionist fuzzy R-ideal of BCK-algebra $X_1 \times X_2$.

Theorem 3.7: Let $A = \langle \mu_A, \lambda_A \rangle$ and $B = \langle \mu_B, \lambda_B \rangle$ be two intuitionist fuzzy closed R-ideals of BCK-algebras X_1 and X_2 respectively. Then $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionist fuzzy closed R-ideal of BCK-algebra $X_1 \times X_2$.

Proof: Let $A = \langle \mu_A, \lambda_A \rangle$ and $B = \langle \mu_B, \lambda_B \rangle$ be two intuitionist fuzzy closed R-ideals of BCK-algebras X_1 and X_2 respectively. Using Theorem 3.6, $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionist fuzzy closed R-ideal of BCK-algebra $X_1 \times X_2$, then

$$\mu_{A \times B}((0,0)^*(x, y)) = \mu_{A \times B}((0^*x, 0^*y)) = \min\{\mu_A(0^*x), \mu_B(0^*y)\}$$

$$\geq \min\{\mu_A(x), \mu_B(y)\} = \mu_{A \times B}(x, y).$$

$$\text{And } \lambda_{A \times B}((0,0)^*(x, y)) = \lambda_{A \times B}((0^*x, 0^*y)) = \max\{\lambda_A(0^*x), \lambda_B(0^*y)\}$$

$$\leq \max\{\lambda_A(x), \lambda_B(y)\} = \lambda_{A \times B}(x, y).$$

Hence $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionist fuzzy closed R-ideal of BCK-algebra $X_1 \times X_2$.

Lemma 3.8: If $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionist fuzzy closed R-ideal of BCK-algebra $X_1 \times X_2$. Then we have $(a,b) \leq (x, y) \Rightarrow \mu_{A \times B}(x, y) \leq \mu_{A \times B}(a,b)$ and

$$\lambda_{A \times B}(x, y) \geq \lambda_{A \times B}(a,b), \forall (a,b), (x, y) \in X_1 \times X_2$$

Proof:

Let $(a,b), (x, y) \in X_1 \times X_2$,

Such that $(a,b) \leq (x, y) \Rightarrow (a,b)^*(x, y) = (0,0)$.

$$\text{Consider } \mu_{A \times B}(x, y) = \mu_{A \times B}((x, y))$$

$$\geq \min\{\mu_{A \times B}(((x, y)^*(0,0))^*((0,0)^*((a,b))), \mu_{A \times B}(a,b)\}$$

$$= \min\{\mu_{A \times B}((x, y)^*((a,b))), \mu_{A \times B}(a,b)\} = \mu_{A \times B}(a,b).$$

$$\text{And } \lambda_{A \times B}(x, y) = \lambda_{A \times B}((x, y))$$

$$\leq \max\{\lambda_{A \times B}(((x, y)^*(0,0))^*((0,0)^*((a,b))), \lambda_{A \times B}(a,b)\}$$

$$= \max\{\lambda_{A \times B}((x, y)^*(a,b)), \lambda_{A \times B}(a,b)\} = \lambda_{A \times B}(a,b).$$

Theorem 3.9: Let $A = \langle \mu_A, \lambda_A \rangle$ and $B = \langle \mu_B, \lambda_B \rangle$ be two intuitionistic fuzzy closed R-ideals of BCK-algebras X_1 and X_2 respectively. Then $(A \times B) = \langle \mu_{A \times B}, \bar{\mu}_{A \times B} \rangle$ is an intuitionistic fuzzy R-ideal of BCK-algebra $X_1 \times X_2$ Where $\bar{\mu}_{A \times B} = 1 - \mu_{A \times B}$,

Proof:

Since by theorem 3.6 $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionistic fuzzy R-ideal of BCK-algebra $X_1 \times X_2$ Then $\mu_{A \times B}(0,0) \geq \mu_{A \times B}(x,y)$

$$1 - \mu_{A \times B}(0,0) \leq 1 - \mu_{A \times B}(x,y) \mu_{A \times B}(0,0) \leq \bar{\mu}_{A \times B}(x,y),$$

Now for any $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in X_1 \times X_2$ we have

$$\mu_{A \times B}((x_1, y_1)) \geq \min\{\mu_{A \times B}(((x_1, y_1) * (x_3, y_3)) * ((x_2, y_2) * (x_3, y_3))), \mu_{A \times B}(x_2, y_2)\}$$

$$\begin{aligned} 1 - \mu_{A \times B}((x_1, y_1)) &\leq 1 - \min\{\mu_{A \times B}(((x_1, y_1) * (x_3, y_3)) * ((x_2, y_2) * (x_3, y_3))), \mu_{A \times B}(x_2, y_2)\} \\ &= \max\{1 - \mu_{A \times B}(((x_1, y_1) * (x_3, y_3)) * ((x_2, y_2) * (x_3, y_3))), 1 - \mu_{A \times B}(x_2, y_2)\} \end{aligned}$$

$$\bar{\mu}_{A \times B}((x_1, y_1)) \leq \max\{((\bar{\mu}_{A \times B}(x_1, y_1) * (x_3, y_3)) * ((x_2, y_2) * (x_3, y_3))), \bar{\mu}_{A \times B}(x_2, y_2)\}$$

Hence $(A \times B) = \langle \mu_{A \times B}, \bar{\mu}_{A \times B} \rangle$ is an intuitionistic fuzzy R-ideal of BCK-algebra $X_1 \times X_2$

Theorem 3.10: Let $A = \langle \mu_A, \lambda_A \rangle$ and $B = \langle \mu_B, \lambda_B \rangle$ be two intuitionistic fuzzy R-ideals of BCK-algebras X_1 and X_2 respectively. Then $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionistic fuzzy R-ideals of BCK-algebras $X_1 \times X_2$.

Proof:

Since by theorem 3.6 $A \times B = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionistic fuzzy R-ideal of BCK-algebra $X_1 \times X_2$

$$\lambda_{A \times B}(0,0) \leq \lambda_{A \times B}(x,y)$$

$$1 - \lambda_{A \times B}(0,0) \geq 1 - \lambda_{A \times B}(x,y) \bar{\lambda}_{A \times B}(0,0) \geq \bar{\lambda}_{A \times B}(x,y),$$

Now for any $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in X_1 \times X_2$ we have

$$\lambda_{A \times B}((x_1, y_1)) \leq \max\{\lambda_{A \times B}(((x_1, y_1) * (x_3, y_3)) * ((x_2, y_2) * (x_3, y_3))), \lambda_{A \times B}(x_2, y_2)\}$$

$$1 - \lambda_{A \times B}((x_1, y_1)) \geq 1 - \max\{\lambda_{A \times B}(((x_1, y_1) * (x_3, y_3)) * ((x_2, y_2) * (x_3, y_3))), \lambda_{A \times B}(x_2, y_2)\}$$

$$\geq \min \{1 - \lambda_{A \times B}(((x_1, y_1) * (x_3, y_3)) * ((x_2, y_2) * (x_3, y_3))), 1 - \lambda_{A \times B}(x_2, y_2)\}$$

$$\bar{\lambda}_{A \times B}((x_1, y_1) *) \geq \min \{((\bar{\lambda}_{A \times B}(x_1, y_1) * (x_3, y_3)) * ((x_2, y_2) * (x_3, y_3))), \bar{\lambda}_{A \times B}(x_2, y_2)\}$$

Hence $A \times B = \langle \rangle$ is an intuitionistic fuzzy R-ideals of BCK-algebras $X_1 \times X_2$

Theorem 3.11: Let $A = \langle \mu_A, \lambda_A \rangle$ and $B = \langle \mu_B, \lambda_B \rangle$ be two intuitionistic fuzzy closed R-ideals of BCK-algebras X_1 and X_2 respectively. Then $(A \times B) = \langle \mu_{A \times B}, \lambda_{A \times B} \rangle$ is an intuitionistic fuzzy R-ideals of BCK-algebras $X_1 \times X_2$. If and only if $(A \times B) = \langle \mu_{A \times B}, \bar{\mu}_{A \times B} \rangle$ and are intuitionistic fuzzy R-ideals of BCK-algebras $X_1 \times X_2$

Proof:

The proof follows from theorem 3.9 and Theorem 3.10

References:

1. S.Abdulllah, M.Asalam, M.Imran, Direct product of intuitionistic fuzzy ideals in LA-Semigroups-II, Ann. FuzzyMath. Inform. 2(2011)151-160
2. S.Abdulllah, M.Asalam, T.khan, Direct product of finite fuzzy ideals in LA-Semigroups, Ann. FuzzyMath. Inform. (in press)
3. S.Abdulllah, M.Asalam, Direct product of finite intuitionistic fuzzy sets in BCK-algebras.(submitted)
4. J.Ahsan, E.Y. Deeba and A.B. Thaheem, On prime ideals of BCK-agebras, Math.Japon.36(1991)875-822.Salem Abdulla et al./Int J. of Algebra and Statistics 1(2012),8-16 16
5. N.O.Al-Shehri, Anti fuzzy implicative ideals in BCK-Algebras, Punjab Uni.J.Math.43(2011)85-91
6. S.Abdulllah, M.Asalam, N.Nasreen, Direct product of intuitionistic fuzzy ideals in LA-Semigroups, fuzzy sets, Rough Sets and Multivalued operations and Applications,3(1)(2011)1-9
7. K.T. atanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems, 20(1)(1986)87-96
8. K.T.Atanassov, New Operations defined over the intuitionistic fuzzy sets, Fuzzy Sets and system 61(2)(1994)137-142
9. K.T.Atanassov, Intuitionistic fuzzy sets, Springer, Heidelberg, 1999
10. S.Y. Huang, BCI-algebras, Science Press, China,2006
11. Y. Huang and Z. Chen, On ideals in BCK-algebras,Math.Japonica 50 211-266
12. K.Iseki, On BCI-algebras, Math.Sem.Notes,8(1980) 125-130