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NON INTERSECTING PATH AND PLANE PARTITIONS AND GRAPH COLOURING

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Abstract:

In this article, basic ideas about the partitions of non intersecting paths are introduced. Different partitions are associated by means of intersecting and non intersecting paths by using permutations and combinations. We also see the trails of various numbers of n-tuples of non intersecting curves connecting two groups of n vertices. This can be

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Research Paper
Vol.11, Iss.9, Dec 2022

given by determinant methods. The aim of this paper will apply pfaflians to get non intersecting curves such that the end points of the curves are given in known domain.

1.1 NON INTERSECTING PATHS AND PFAFFIANS PATHS

The non intersection paths and pfaffians graphs Concerned the common solutions of with n-tuples are non-intersecting curve on acyclic digraph G. For enumerative purpose we give different definition of pfaffians, and group combinations of important properties of n-tuples of curves from the set of n corners to a well defined part of G. This can be done by various expressions depending on the problems pfaffian. The corresponding solutions given by Jozefia'k and Pragacz [JP], are also given [1-5].

The g-packing problem has been studied for certain families. A well known example is the case where F is the set consisting of the 2-cliques of a graph. In this case, the F-packing problem is the problem of finding a maximum matching in a graph. We are interested in a particular instance of the B-packing problem. Suppose G is a simple maximal planar graph; that is, G has no multiple edges and every face of G is a triangle. We denote by T(G) the set of all 2-cliques and facial triangles of G. Paper we derive a sufficient condition for G to have a perfect T(G)-packing. This result then allows us to construct special perfect path double covers of maximal planar graphs.A perfect path double couer (PPDC) of a graph G is a collection P of paths such that every edge of G lies in exactly two paths of P and every vertex of G occurs precisely twice as an endpoint of the paths of P (where a path of length zero is considered to have two identical endpoints) proves that every simple graph has a PPDC, thus settling a conjecture of Bondy. Li's proof also yields an algorithm for constructing PPDCs of simple graphs. Our result about perfect T(G) - packings of maximal planar graphs allows us to construct PPDCs of maximal planar graphs. These PPDCs have the additional property that when the graph has minimum degree at least four, the sequence of lengths of the paths of the PPDC is the same as the degree sequence of the graph. This leads to the question of which graphs have PPDCs with this property. We consider only simple graphs. For a graph G, we denote by I'(G) and E(G), respectively, the vertex set and the edge set of the graph, with | V(G) |= u(G) and | E(G)| =e(G). A planar graph is a graph which has an embedding in the plane, but we shall use this term to signify a planar graph together with a fixed planar embedding. If G is a planar graph, we let F(G) denote the set of faces of the graph, and set |F(G)|= f(G). If v is a vertex of G, then d(v) denotes the degree of v in G and δ(G) denotes the minimum degree of the vertices in G. A subgraph of G induced by a set S$\subseteq$ V(G) is the graph G[S] obtained from G by restricting the vertex set to S.

Let G be a maximal planar graph on at least four vertices. For vϵ V(G), we let N(v) denote the set of neighbours of u. Since G is maximal planar, there is a unique cycle C(v) with vertex set N(u), and this cycle is called the vertex cycle for v in G.

1.2 Multi-level Hypergraph Partitioning.

State-of-the-art hypergraph partitioners like hMetis and PaToH use a multi-level approach. This scheme divides the partitioning process into three phases [6-10]

1. Coarsening Phase
2. Initial Partitioning
3. Uncoarsening and Refinement Phase

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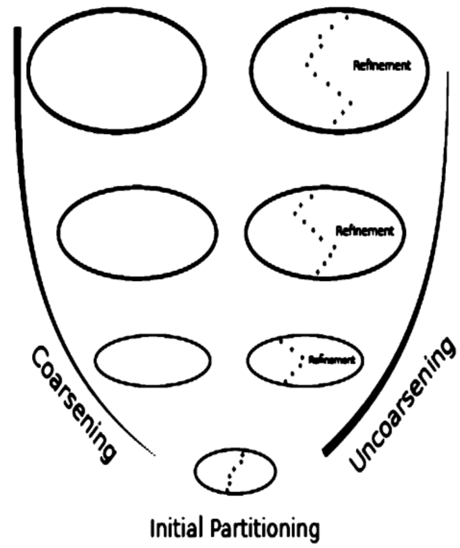

Figure 1.1 : Multi-Level Hypergraph Partitioning

The idea behind this division is to reduce the number of hypernodes and hyperedges in the coarsening phase significantly in order to decrease the cost of the initial partitioning. Coarsening is then reversed to obtain a 2- partition of the initial hypergraph [11-15].

The coar sening phase is itself divided into multiple steps, the so called levels, thus the name "multi-level". At each level groups of hypernodes are each merged into only one hypernode. Hypere dges, which only contain one hypernode or hyperedges that are parallel to another hyperedge after this merging are removed from the hypergraph. The hypernode which represents a merged group carries the weight of all hypernodes of the group [16-20].

As one hypernode at a coarse level may represent very many hypernodes of a finer level, we have a rather global view on the optimization problem at the coarse levels. The process of compiling and merging groups is repeated until the hypergraph is small enough for an efficient computation of an initial partition. In the second phase the initial partitioning takes place. The coarsened hypergraph is often partitioned by a heuristic as the loss of topological information in the coarsening phase can not be compensated by an optimal initial partition. For more information on initial partitioning we refer the reader to Bichot and Siarry.

The third phase consists of uncoarsening the hypergraph back to its initial state. At each level the 2-partition is refined locally by moving hypernodes between the blocks in order to decrease the hyperedge cut.

Theorem:

For $k \geq 2, \quad \varphi[L(F_{2,k})] = k.$

Proof:

Let $G = F_{2,k}$ be a fire cracker graph. By definition, $(2,k)$ fire cracker graph is obtained by concatenation of $2, k$ stars by linking a leaf from each. Consider the line graph of $F_{2,k}$. Let S be the vertex adjacent with both v and v'. Here the vertex v along with $v_1, v_2, .., v_{k-1}$, induces a clique of the order k also the vertex v' with $v_1', v_2' ....,$ v_{k-2}' induces another clique of order k. Thus in $L(F_{2,k})$, we find two copies of mutually disjoint complete subgraphs.

Consider a colour class $C = \{c_1, c_2, \ldots c_k\}$. Assign a proper colouring to these vertices as follows. Assign the colour c_1 to the vertex v and c_{i+1} to the vertices $v_1, v_2, ..., v_{k-2}$ for $i=1,2,3...k$ which produces a b-chromatic colouring. Next suppose if we assign any new colours to v' and v_i' for $i=1,2 \ldots k-1$ then it will not produce a b-chromatic colouring. Similarly if we assign any colour to the root vertex S, again it fails to produce the b-chromatic colouring.

Because here the vertex set v and v' are mutually disjoint to each other.

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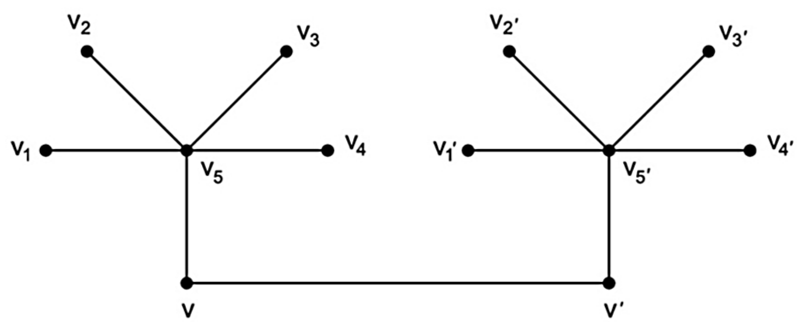
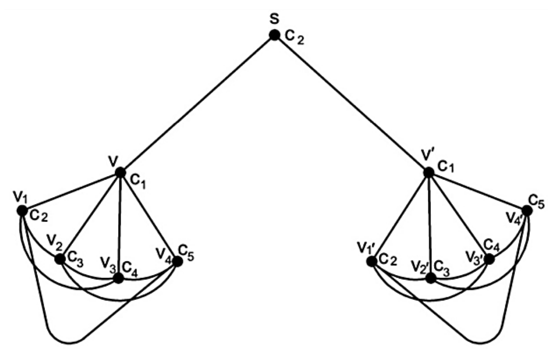
Research Paper

Vol.11, Iss.9, Dec 2022

Thus the only possibility is to assign the same colour which we already assigned for the vertices v and v_i for $i=1,2,3..k-1$ such as c_1 to v' and c_{i+1} to v_i' for $i=1,2,3...k-1$ and the colour c_2 to the root vertex. Now all the vertices vv_i and vv_i' realizes its own colour, which produce a b-chromatic colouring. Thus by the colouring procedure the above said colouring produces a maximum and b-chromatic colouring.





Theorem:

Let G be the square of the graph $C_{13} \times C_{13}$ Then G is b-colorable with13-colors and $b(G)$=13.

Proof:

Let the vertex set of G be $V=\{(i,j) : 1 \leq i \leq 13, 1 \leq j \leq 13\}$.

Since $b(G) = \Delta+1$ and $\Delta(G) = 12$, we have $b(G) = 13$. It remains to show that G is b-colorable with 13 colors.

Let us color the vertices of G as follows:

c((1, 1)) = 4,c((1, 2)) = 11,c((1, 3)) = 2,c((1, 4)) = 10,c((1, 5)) = 12,c((1, 6)) = 5, c((1,7)) = 13, c((1,8)) = 3, c((1,9)) = 8, c((1,10)) = 9, c((1,11)) = 1, c((1,12)) = 6, c((1,13)) = 7, c((2,1)) = 5,c((2, 2)) = 13,c((2, 3)) = 3,c((2, 4)) = 8,c((2, 5)) = 9,c((2, 6)) = 1, c((2,7)) = 6, c((2,8)) = 7, c((2,9)) = 4, c((2,10)) = 11, c((2, 11)) = 2,c((2,12)) = 10, c((2,13)) = 12,c((3, 1)) = 1,c((3, 2)) = 6,c((3, 3)) = 7,c((3, 4)) = 4,c((3, 5)) = 11,c((3, 6)) = 2, c((3,7)) = 10, c((3,8)) = 12, c((3,9)) = 5, c((3,10)) = 13, c((3,11)) = 3, c((3,12)) = 8, c((3,13)) = 9,c((4, 1)) = 2,c((4, 2)) = 10,c((4, 3)) = 12,c((4, 4)) = 5,c((4, 5)) = 13,c((4, 6)) = 3, c((4, 7)) = 8, c((4, 8)) = 9,c((4, 9)) = 1,c((4,10)) = 6,c((4,11)) = 7,c((4, 12)) = 4, c((4,13)) = 11,c ((5,1)) = 3,c ((5,2)) = 8,c ((5,3)) = 9,c ((5,4)) = 1,c ((5,5)) = 6 ,c ((5,6)) = 7, c((5,7)) = 4, c((5,8)) = 11, c((5,9)) = 2, c((5,10)) = 10, c((5,11)) = 12, c((5,12)) = 5, c((5,13)) = 13,c((6, 1)) = 7,c((6, 2)) = 4,c((6, 3)) = 11,c((6, 4)) = 2,c((6, 5)) = 10,c((6, 6)) = 12, c((6,7)) = 5, c((6,8)) = 13, c((6,9)) = 3,c((6, 10)) = 8,c((6,11)) = 9, c((6, 12)) = 1, c((6,13)) = 6, c((7,1)) = 12, c ((7,2)) = 5, c ((7,3)) = 13, c ((7,4)) = 3, c ((7,5)) = 8, c ((7,6)) = 9, c ((7, 7)) = 1, c((7, 8)) = 6,c((7, 9)) = 7,c((7,10)) = 4,c((7,11)) = 11,c((7,12)) = 2, c((7,13)) = 10,c ((8,1)) = 9, c ((8, 2)) = 1, c ((8,3)) = 6, c ((8, 4)) = 7, c ((8, 5)) = 4, c ((8, 6)) = 11, c ((8,7)) = 2, c((8,8)) = 10, c((8,9)) = 12,c((8,10)) = 5, c((8,11)) = 13,c((8,12)) = 3, c((8,13)) = 8,c ((9,1)) = 11, c ((9,2)) = 2,c ((9,3)) = 10,c ((9,4)) = 12,c ((9,5)) = 5, c ((9,6)) = 13, c ((9, 7)) = 3, c((9, 8)) = 8,c((9, 9)) = 9,c((9,10)) = 1, c((9,11)) = 6,c((9, 12)) = 7,c((9,13)) = 4,c((10,1)) = 13,c((10,2)) = 3,c((10,3)) = 8,c((10,4)) = 9,c((10,5)) = 1, c((10,6)) = 6, c((10,7)) = 7, c((10,8)) = 4,c((10,9)) = 11, c((10,10)) = 2,c((10,11)) = 10, c((10,12)) = 12,c((10,13)) = 5, c((11,1)) = 6,c((11,2)) = 7,c((11,3)) = 4,c((11,4)) = 11,c((11,5)) = 2, c((11,6)) = 10,c((11,7)) = 12, c((11,8)) = 5, c((11,9)) = 13, c((11,10)) = 3, c((11,11)) = 8, c((11,12)) = 9, c((11,13)) = 1, c((12,1)) = 10,c((12,2)) = 12,c((12,3)) = 5,c((12,4)) = 13, c((12,5)) = 3,c((12,6)) = 8,c((12,7)) = 9, c((12,8)) = 1, c((12,9)) = 6, c((12,10)) = 7, c((12,11)) = 4, c((12,12)) = 11, c((12,13)) = 2, c((13,1)) = 8,c((13,2)) = 9,c((13,3)) = 1,c((13,4)) = 6, c((13,5)) = 7, c((13, 6)) = 4,c((13, 7)) = 11, c((13, 8)) = 2,c((13, 9)) = 10, c((13, 10)) = 12, c((13,11)) = 5, c((13,12)) = 13, c((13,13)) = 3.

Note that in the above coloring all the vertices are colorful.

Conclusion:

In this article we have the concepts of Non-intersecting Paths and Pfaffians and Hyper graph. Also to make the colouring as *b*-chromatic one, we should assign only the same set of colours to $v_i'(i=1,2,3..n)$ which we already assigned for $u_i'(i=1,2,3...n)$.Now all the vertices u_i',v_i' and *w* realizes its own colour, which produces a *b*-chromatic colouring. Thus by the colouring procedure the above said colouring is maximum and *b*-chromatic.

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