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PAIR SUM LABELLING OF UNION OF GRAPHS

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ABSTRACT

The view of pair sum labeling has been introduced in this work. Pair sums labeling behavior of complete graph, cycle, path, bistar etc. Here we study pair sum labeling of union of some standard graphs and we find the maximum size of a pair sum graph. Although its primary interest was the graceful labeling of trees in order to solve Ringel's conjecture, labeling of graphs gained over the years its own beauty and interest. All the graphs considered here are finite and undirected. The graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of edges then the labelling called pair sums a labeling.

KEYWORDS: graph, labelling, edges, pair sum labelling etc.,

INTRODUCTION

The pair sum Labeling of graphs has been a topic of research for 50 years and it still has many properties to be found. Labeling was introduced in the late 1960's. Many studies in graph labelling refer to Rosa's research in 1967. Graphs that have some sort of regularity of structure are said to be Pair sum labeling. This work gives a brief overview of the subject, presenting not only theoretical results from the literature, but also some computational

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results. Furthermore, we give some contributions to this problem. Pair sum labeling some rather simple graphs classes like cycle and wheel. Also show necessary conditions to the existence of a graceful labeling for a graph, and two methods of constructing graceful graphs.

A pair sum labeling is an assignment of integers to the vertices or the edges, or both, subject to certain conditions. If the domain is the set of vertices we speak about the labeling. If the domain is the set of edges, then the labeling is called the edge labeling. If the labels are assigned to the vertices and also the edge of a graph such a labeling is called pair sum labeling

Definition: The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with
$V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$

Definition: If P_n denotes a path on n vertices, the graph $L_n = P_2 \times P_n$ is called a ladder .

Definition: The graph $C_n \hat{O} K_{1,m}$ is obtained from C_n and $K_{1,m}$ by identifying any vertex of C_n and central vertex of $K_{1,m}$.

Theorem

If $m \leq 4$,then nK_m is a pair sum graph.

Proof

Obviously m=1,the result is true.

Case 1: m=2.

Assign the label j and j+1 to the vertices of j^{th} copy of K_2 for all odd j. For even values of j,label the vertices of the j^{th} copy of K_2 by$-j+1$and$-j$.

Case 2: m=3.

Subcase1 m is even.

Label the vertices of first n/2 copies by $3j - 2, 3j - 1, 3j (1 \leq j \leq n/2)$. Remaining n/2 copies are labeled by $-3j + 2, -3j + 1, -3j$.

Subcase 2 n is odd.

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Label the vertices of first $(n-1)$ copies as in Subcase (a). In the last copy label the vertices by $\frac{3(n-1)}{2}+1$, $\frac{-3(n-1)}{2}-2$, $\frac{3(m-1)}{2}+3$ respectively.

Case 3 $m = 4$

Subcase1: n is even

Label the vertices of first $\frac{n}{2}$ copies by $4j-3, 4j-2, 4j-1, 4j$ $(1 \le j \le n/2)$. Remaining $\frac{n}{2}$ Copies are labeled by $-4j+3, -4j+2, -4j+1, -4j$.

Subcase2: n is odd.

Label the vertices of first $(n-1)$ copies as in Sub case(a). In the last copy label the vertices by $-2n, 2n+1, 2n+2$ and $-2n-3$ respectively.

Theorem

Ladder L_m admids pair sum labeling

Proof

Let $V(L_m) = \{x_j, y_j : 1 \le j \le m\}$ and

$E(L_m) = \{x_j y_j : 1 \le j \le m\} \cup \{x_j x_{j+1}, y_j y_{j+1} : 1 \le j \le m-1\}$.

When m is odd.

Let $m=2n+1$.consider $g: V(L_m) \to \{\pm1, \pm2, \ldots, \pm(4n+2)\}$ by

$$g(x_j) = -4(n+1)+2j, \quad 1 \le j \le n,$$

$g(x_{n+1}) = -(2n+1)$,

$g(x_{n+1+j}) = 2n+2j+2, \quad 1 \le j \le n$

$g(y_j) = -4n-3+2j, \quad 1 \le j \le n,$

$g(y_{n+1}) = 2n+2$

$g(y_{n+1+j}) = 2n+2j+1, \quad 1 \le j \le n.$

When m is even

Let $m=2n$. consider $g: V(L_m) \to \{\pm1, \pm2, \ldots, \pm(4n+2)\}$by

$g(x_{n+1-j}) = -2j, \quad 1 \le j \le n,$

$g(x_{n+j}) = 2j-1, \quad 1 \le j \le n,$

$g(x_{n+j}) = 2j, \quad 1 \le j \le n,$

$g(x_{n+1-j}) = -(2j-1), 1 \le j \le n.$

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Then L_m admits a pair sum labeling.

Theorem

Any triangular snake T_m is a pair sum graph.

Proof

Let $V(T_m)=\{x_i,y_j:1 \le i \le m+1, 1 \le j \le m\}$,

$E(T_m)=\{x_i x_{i+1}, x_i y_j, y_i y_{j+1}:1 \le i \le m, 1 \le j \le m-1\}$.

The proof consider three cases

Case1: $m= 4n-1$

Define

$$g(x_j)=2j-1, \quad 1 \le j \le 2n,$$

$g(x_{2n+j})=-2j+1,\ 1 \le j \le 2n,$

$g(y_j)=2j, \quad 1 \le j \le 2n-1,$

$g(y_{2n})=-8n+3,$

$g(y_{2n+j})=-2j, \quad 1 \le j \le 2n-1.$

Case2: $m=4n+1$

Define

$$g(x_i)=-8n-3+2(j-1), \quad 1 \le j \le 2n+1,$$

$$g(x_{2n+1+j})=8n+3-2(j-1), \quad 1 \le j \le 2n+1,$$

$$g(y_j)=-2+2(j-1), \quad 1 \le j \le 2n$$

$$g(y_{2n+1})=3,$$

$$g(y_{2n+j+1})=8n+2-2(j-1), \quad 1 \le j \le 2n.$$

Case3 $m=2n$

Define

$g(x_{n+1})=1,$

$g(x_{n+1+j})=2j, \quad 1 \le j \le n,$

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$g(x_{n+1-j})=-2j,\quad 1\leq j\leq n$

$g(y_n)=3,$

$g(y_{n+1})=-5,$

$g(y_{n+1+j})=5+2j,\quad 1\leq j\leq n-1,$

$g(y_{n-j})=-(5+2j),\quad 1\leq j\leq n-1.$

Clearly T_m is a pair sum labeling.

Theorem

The crown $C_m \odot K_1$ is a pair sum graph.

Proof

Let C_m be the cycle given by $x_1x_2,\ldots,x_mx_1$ and let $y_1,y_2,\ldots,y_m$ be the pendent vertices adjacent to $x_1,x_2,\ldots,x_m$respectively.

Case1: m is even.

Subcase(a): m=4n.

Define

$g(x_j)=2j-1,\quad 1\leq j\leq 2n$

$g(x_{2n+j})= -2j+1,\quad 1\leq j\leq 2n,$

$g(y_j)=4n+(2j-1),\quad 1\leq j\leq 2n,$

$g(y_{2n+j})=-(4n+2j-1),\quad 1\leq j\leq 2$

Theorem

One point union of a cycle C_3 and a path P_2 is not a pair sum graph.

Proof

Let $g(v_1)= y_1,g(v_2)=y_2,g(v_3)= y_3$ and $g(v_4)= y_4$.

By theorem2.1 $y_1+2y_2+2y_3+3y_4= 0$

=> $y_1+y_4= -2[y_2+y_3+y_4]$ (1)

=> $2|(y_1+y_4)$

=> $y_1+y_4= 2k \rightarrow (2)$.

Case(i): k = 1

=> $y_1+y_4=2 \rightarrow (3)$

=> $-\mathbf{y}_1+y_2+y_3= -3 \rightarrow (4)$

Let (l,m,n) be the ordered 3-tuples which satisfy the equation(4).Using theorem2.1 and definition3.1, we have the following ordered 3-tuples.

(i)(1,2,-4), (ii)(1,-4,2), (iii) (2,3,-4), (iv) (2,-4,3), (v)(-2,-1,-4), (vi)(-2,-4,-1)

(vii)(3,1,-1),(viii)(3,-1,1),(ix)(3,2,-2),(x)(3,-2,2),(xi)(3,4,-4),(xii)(3,-4,4),

(xiii)(-3, -2,-4),(xiv)(-3,-4,-2),(xv)(4, -1,2),(xvi)(4,2,-1),(xvii)(4,-2,3),(xviii)(4,3,-2).

Subcase (y_1,y_2, y_3)∈{(3,2, -2), (3,-2,2),(3,4,-4),(3,-4,4)}

By theorem, this is not possible.

Subcase (y_1, y_2, y_3)∈{(-3,-2,-4),(-3,-1,-2)} By(3),y_4= 5, this is not possible.

Subcase (y_1, y_2, y_3)∈{(4,-1, 2),(4,2,-1)}

By(3), y_4= -2 and in this case, the label of v_4v_3 is zero. This is a contradiction to definition

Subcase(xii): (y_1, y_2, y_3)∈{(4,-2, 3),(4,3,-2)}

By(3), y_4= -2.Then two edges receive 2 as an edge label.

In all the above subcases, the given graph is not a pair sum graph.

Case(ii):k = 2

By(2),$y_1+y_4= 4 \rightarrow (5)$

=>$y_1+y_2+y_3=-6 \rightarrow (6)$

In this case, we have the following ordered 3-tuples.

(i)(1, -2,-3)(ii)(1,-3,-2)(iii) (1,-4,-1)(iv)(1,-1,-4)(v)(-1,-3,-4)(vi)(-1,-4,-3) (vii)(2, -1,-3)(viii)(2,-3,-1)(ix)(3, -1,-2)(x)(3,-2,-1)(xi)(3,1,-4)(xii)(3,-4,1) (xiii)(4,1, -3)(xiv)(4,-3,1).

Subcase (y_1, y_2, y_3)∈{(1,-2,-3), (1,-3,-2)}

By(5), y_4= 3.Inthiscasetheedgelabel $v_4v_3$zerowhichisa contradiction.

Subcase (y_1, y_2, y_3)∈{(1,-4,-1), (1,-1,-4)}

By(5), y_4= 3, which is a contradiction.

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Subcase $(y_1, y_2, y_3)\in\{(-1,-3,-4), (-1,-4,-3)\}$

By(5), $y_4= 5$, which is not possible.

Subcase $(y_1,y_2,y_3)\in\{(2,-1,-3), (2,-3,-1)\}$

By(5),$x_4= 2$, which is not possible.

Subcase $(y_1, y_2, y_3)\in\{(3,-1,-2), (3,-2,-1)\}$

By(5), $y_4= 1$and this implies that zero appears as an edge label,a contradiction.

Subcase $(y_1, y_2, y_3)\in\{(3,1,-4), (3,-4,1)\}$

By(5), $y_4= 1$, a contradiction.

Subcase $(y_1, y_2, y_3)\in\{(4, 1, -3),(4,-3, 1)\}$

By(5), $y_4= 0$, a contradiction.

In all the above subcases, the given graph is not a pair sum graph.

Case(iii) k = 3

By(2), $y_1+y_4= 6\rightarrow(7)$

$\Rightarrow y+y_2+y_3= -9 \rightarrow(8)$

Here we have the following ordered 3-tuples.

(i)(2,-4,-3)(ii)(2,-3,-4)(iii)(3,-4,-2)(iv)(3,-2,-4)(v)(4,-2,-3)(vi)(-4,-3,-2) (vii)(4, -4,-1)(viii)(4,-1,-4)

Case(v) k >3.

Then $y_1+y_4\geq 8$,which is not possible.

Theorem

$C_3\hat{O}\, K_{1,n}$ is a pair sum graph if, $n > 1$

Proof

Case(i) n= 1

By theorem, $C_3\hat{O}\, K_{1,\,n}$ is not a pair sum graph.

Case(ii) n> 1

Let C_3 be the cycle $x_1x_2x_3x_1$ .Let V $(K_1, n)= \{y,y_j:1\leq j\leq m\}$and

$E\,(K_{1},n)= \{yy_j:1 \leq j\leq m\}$.

Without loss of generality we assume y is identified to x_1.

Define g(x)=1,

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$g(x_2)= -2$,

$g(x_3)=3$,

$g(y_j)=-4-j$, $1 \leq j \leq \frac{n}{2}$ if n is even and

$1 \leq j \leq \frac{n+1}{2}$ if n is odd,

$g(y_{(n/2)+j}) = 3+ j$ $1\leq j \leq \frac{n}{2}$ if n is even and

$1 \leq j \leq \frac{n-1}{2}$ if n is odd.

Clearly g is a pair sum labeling of $C_3\hat{O}\, K_{1, n}$.

Theorem

$C_4\hat{O}\, K_{1, n}$ is a pair sum graph for all n.

Proof

Without loss of generality we assume x is identified to x_1.

Define $g(x)= -1$

$g(x_2)=2$

$g(x_3)=1$

$g(x_4)=-2$

$g(y_j)= -2-j$ $1 \leq j \leq$ n/2 if n is even and $1 \leq j \leq (n+1)/2$ if n is odd

$g(y_{(n/2)+j})=4+j$ $1 \leq j\leq$ n/2if m is even and $1 \leq j \leq (n-1)/2$if n is odd

Then g is a pair sum labeling. Therefore $C_4\hat{O}\, K_{1,n}$ is a pair sum graph.

Theorem

Let $C_n : x_1 x_2 \ldots x_n x_1$be the cycle where $n\equiv 0,1,2(mod 4)$. Let G bethe graph with $V (G)=V (C_n)$and

$E (G)=E (C_n)\cup\{x_1 x_3\}$.Then G is a pair sum graph.

Proof

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Case(i) $n=4m+2$

$\mathbf{g}(x_j)=2(m+1)-j \quad 1 \leq j \leq 2m+1$

$\mathbf{g}(x_{2m+1+j})= -(2(m+1)-j) \quad 1 \leq j \leq 2m+1$

Case(ii) $n= 4m$

$\mathbf{g}(x_1)=2m+1$

$\mathbf{g}(x_{j+1})= 2m-j \quad 1 \leq j \leq 2m-1$

$\mathbf{g}(x_{2m+1})= -(2m+1)$

$g(x_{2m+1+i})= -(2m-j) \quad 1 \leq j \leq 2m-1$

Case(iii) $n= 4m+1$

$\mathbf{g}(x_1)=-4$

$\mathbf{g}(x_{1+j})= j \quad 1 \leq j \leq 2m+1$

$\boldsymbol{g}(x_{n-2j+2})= -(2j-1) \quad 1\leq j\leq m$

$\boldsymbol{g}(x_{n-2j+1})= -(4+2j) \quad 1\leq j\leq m-1$

Then g is a pair sum labeling. Therefore G is a pair sum graph.

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