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The Radio Dd-Distance in Lehmer-3 Mean Number of Family of Snake Graphs

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ABSTRACT:

A radio Dd-distance in lehmer-3 mean labeling of a connected graph G is an injective map f from the vertex set V(G) to the ℕ such that for two distinct vertices u and v of G , $D^{Dd}(u,v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^{Dd}(G)$ *where* $D^{Dd}(u,v)$ *denotes Dd- distance between u and v and* $diam^{Dd}(G)$ *denotes the Dd-diameter of G. The radio Dd-distance in lehmer – 3 mean number of f,* $rlmn^{Dd}(f)$ *is the maximum label assigned to any vertex of G. The radio Dd- distance in lehmer -3 mean number of G ,* $rlmn^{Dd}(G)$ *is the minimum value of G. In this paper, we investigate the radio Dd-Distance in lehmer – 3 mean number of family of snake graphs.*

Key Words: Dd-distance, Dd-diameter,radio Dd-distance number, Lehmer-3 mean labeling

1.INTRODUCTION:

By a graph G, we mean a non-trivial finite undirected connected graph without multiple edges and loops. Following standard notation V(G) or V is the vertex set of G and E(G) or E is the edge set of G= G(V,E).

A.Anto Kinsely and P. Siva Ananthi [1] introduced the concept of Dd-distance in graph as follows for two vertices u,v in a graph G, Dd- length of a u-v path is defined as $D^{Dd}(u,v) = D(u,v) + \deg(u) + \deg(v)$ where $D(u,v)$ is the length of the longest path between u and v. The Dd-radius , $r^{Dd}(G)$ is the minimum Dd- eccentricity among all vertices of u and v of G . Similarly the Dd-diameter , $D^{Dd}(G)$ is the maximum Dd- eccentricity among all vertices of G. We observe that for any two vertices u,v of G. we have $d(u,v) \leq D^{Dd}(u,v)$. The equality holds iff u and v are identical. If G is any connected graph then the Dd-distance is metric on the set of vertices of G. We can check easily $r^{Dd}(G) \leq D^{Dd}(G) \leq 2r^{Dd}(G)$.

The concept of lehmer-3 mean labeling was introduced by S.Somasundaram et al. [11]. The concept of radio labeling was introduced by Chatrand et al [3] in 2001. Motivated by the radio

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labelling Ponraj et.al. [9] defined the radio mean labelling of G and found radio mean number of some graphs. In this paper, We are introduced radio Dd-distance in lehmer-3 mean number of family of snake graphs.

2.DEFINITIONS:

Definition 2.1: A radio Dd-distance in lehmer-3 mean labelling of a connected graph G is an injective map f from the vertex set V(G) to the $\mathbb{N}$ such that for two distinct vertices u and v of G , $D^{Dd}(u,v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^{Dd}(G)$ where $D^{Dd}(u,v)$ denotes Dd- distance between u and v and $diam^{Dd}(G)$ denotes the Dd-diameter of G.

Definition 2.2:A graph $G = (V, E)$ with p vertices and q edges is called Lehmer -3 mean graph, if it is possible to label vertices $x \in V$ with distinct label $f(x)$ from $1,2,3, \dots \dots, q+1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ (or) $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, then the edge labels are distinct. In this case f is called Lehmer -3 mean labeling of G.

Definition 2.3: A Triangular snake as a connected graph in which all blocks are triangles and the block-cut-point graph is a path. A Triangular Snake T_n is obtained from a path $v_1, v_2, \dots v_n$ by joining v_i and v_{i+1} to a new vertex u_i for $1 \leq i \leq n$. That is, every edge of a path is replaced by a triangle C_3.

Definition 2.4: A Double triangular snake DT_n consists of two Triangular snakes that have a common path.

Definition 2.5: A Triple triangular snake TT_n consists of three Triangular snakes that have a common path.

Definition 2.6: A quadrilateral snake Q_n is obtained from a path $v_1, v_2, \dots v_{n+1}$ by joining v_i and $v_{i+1}$to new vertices u_i and u_{i+1} respectively and adding edges $u_i u_{i+1}$. That is every edge of a path is replaced by a cycle C_4.

Definition 2.7: A Double quadrilateral snake DQ_n is obtained from two quadrilateral snakes that have a common path.

Definition 2.8: A Triple quadrilateral snake TQ_n is obtained from three quadrilateral snakes that have a common path.

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3.Main Results

Theorem 3.1: The radio Dd- distance in lehmer-3 mean number of Triangular Snake graph,

$$rlmn^{Dd}(T_n) = \begin{cases} 2n+1, & n=1,2 \\ 3n-2, & n=3,4,5 \\ 4n-8, & n=6,8,10\ldots \\ 4(n+1)-12, & n=7,9,11\ldots. \end{cases}$$

Proof: It is obvious that $diam^{Dd}(T_n) = 2n+4, n \geq 1$. Let V={ u_i, v_j / $i = 1,2,\ldots,n, j = 1,2,\ldots,n+1$} be the vertex set and $E = \{ u_iv_j, v_jv_{j+1} / i = 1,2,\ldots,n, j = 1,2,\ldots,n+1\}$ be the edge set .

We must show that the radio Dd- distance in lehmer-3 mean condition $D^{Dd}(u,v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^{Dd}(G) = 2n+5$, for every pair of vertices (u,v) where $u \neq v$

Case (i): $n = 6,8,10\ldots$

Define the function f as $f(v_1) = 3n-8, f(v_{2j}) = 2n-9+j, j = 1,2,\ldots\frac{n}{2}, f(v_{n+1}) = 3n-9, f(v_{n+1-2j}) = 5\left(\frac{n}{2}\right) - 9 + j, j = 1,2,\ldots,\frac{n-2}{2}, f(u_i) = 3n-8+i, i = 1,2,\ldots,n$.

Therefore, the largest label is $4n-8$, $n = 6,8,10\ldots$

Case (ii): $n = 7,9,11\ldots$

Define the function f as $f(v_1) = 3(n+1)-11, f(v_{2i}) = 2(n+1)-11+i, i = 1,2,\ldots\frac{n+1}{2}$,

$$f(v_{n-2i}) = 5\left(\frac{n+1}{2}\right) - 11 + i, i = 1,2,\ldots,\frac{n-3}{2}, f(v_n) = 3(n+1)-12,$$

$f(u_i) = 3(n+1)-11+i, i = 1,2,\ldots,n$.

Therefore, the largest label is $4(n+1)-12$, $n = 7,9,11\ldots$

$$rlmn^{Dd}(T_n) = \begin{cases} 2n+1, & n=1,2 \\ 3n-2, & n=3,4,5 \\ 4n-8, & n=6,8,10\ldots \\ 4(n+1)-12, & n=7,9,11\ldots. \end{cases}$$ ∎

Theorem 3.2: The radio Dd- distance in lehmer-3 mean number of Double triangular snake graph,

$$rlmn^{Dd}(DT_n) = \begin{cases} 3n+1, & n=1,2 \\ 4n-1, & n=3,4,5 \\ 11\left(\frac{n}{2}\right)-9, & n=6,8 \\ 11\left(\frac{n+1}{2}\right)-15, & n=7,9 \\ 11\left(\frac{n}{2}\right)-10, & n=10,12,14\ldots \\ 11\left(\frac{n+1}{2}\right)-16, & n=11,13,15\ldots. \end{cases}$$

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Proof: It is obvious that $diam^{Dd}(DT_n) = 2n+8\,, n \geq 3$. Let V={ $u_i, v_j \,/\, i = 1,2,\ldots,2n, j = 1,2,\ldots,n+1\}$ be the vertex set and $E = \{\ u_iv_j,\ v_jv_{j+1}/\ i = 1,2,\ldots,2n, j = 1,2,\ldots,n+1\}$ be the edge set .

We must show that the radio Dd- distance in lehmer-3 mean condition $D^{Dd}(u,v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^{Dd}(G) = 2n+9$, for every pair of vertices (u,v) where $u \neq v$

Define the function f as

Case (i): $n = 10,12,14 \ldots$

Define the function f as $f(v_1) = 7\left(\frac{n}{2}\right) - 10, f(v_{2j}) = 5\left(\frac{n}{2}\right) - 11 + j, j = 1,2, \ldots \frac{n}{2},$

$$f(v_{n+1-2j}) = 6\left(\frac{n}{2}\right) - 11 + j, j = 1,2,\ldots,\frac{n-2}{2}, f(v_{n+1}) = 7\left(\frac{n}{2}\right) - 11,$$

$$f(u_{2n-1-i}) = 7\left(\frac{n}{2}\right) - 10 + i, i = 1,2,\ldots,2n-2, f(u_{2n-2+i}) = 10\left(\frac{n}{2}\right) - 6 + i, i = 1,2.$$

Therefore, the largest label is $11\left(\frac{n}{2}\right) - 10$, $n = 10,12,14,\ldots$

Case (ii): $n = 11,13,15 \ldots$

Define the function f as $f(v_1) = 7\left(\frac{n+1}{2}\right) - 14, f(v_{2j}) = 5\left(\frac{n+1}{2}\right) - 14 + j, j = 1,2, \ldots \frac{n+1}{2},$

$$f(v_{n-2j}) = 6\left(\frac{n+1}{2}\right) - 14 + j, j = 1,2,\ldots,\frac{n-3}{2}, f(v_n) = 7\left(\frac{n+1}{2}\right) - 15,$$

$$f(u_{2n-1-i}) = 7\left(\frac{n+1}{2}\right) - 14 + i, i = 1,2,\ldots,2n-2,$$

$$f(u_{2n-2+i}) = 11\left(\frac{n+1}{2}\right) - 18 + i, i = 1,2.$$

Therefore, the largest label is $11\left(\frac{n+1}{2}\right) - 16$, $n = 11,13,15,\ldots$

$$rlmn^{Dd}(DT_n) = \begin{cases} 3n+1, & n=1,2 \\ 4n-1, & n=3,4,5 \\ 11\left(\frac{n}{2}\right)-9, & n=6,8 \\ 11\left(\frac{n+1}{2}\right)-15, & n=7,9 \\ 11\left(\frac{n}{2}\right)-10, & n=10,12,14\ldots \\ 11\left(\frac{n+1}{2}\right)-16, & n=11,13,15\ldots. \end{cases}$$ ∎

Theorem 3.3: The radio Dd- distance in lehmer-3 mean number of Triple triangular snake graph, $rlmn^{Dd}(TT_n) = \begin{cases} 5, & n=1 \\ 7n-5, & n=2,3 \\ 5n, & 4 \leq n \leq 8 \\ 6n-8, & n \geq 9. \end{cases}$

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Proof: It is obvious that $diam^{Dd}(DT_n) = 2n + 12\ , n \geq 3$. Let V={ $u_i, v_j\ /\ i = 1,2, \dots ,3n, j = 1,2, \dots , n + 1\}$ be the vertex set and $E = \{\ u_i v_j,\ \ v_j v_{j+1} /\ i = 1,2, \dots ,3n, j = 1,2, \dots , n + 1\}$ be the edge set .We must show that the radio Dd- distance in lehmer-3 mean condition $D^{Dd}(u, v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^{Dd}(G) = 2n + 13$, for every pair of vertices (u, v) where $u \neq v$

For $n \geq 9$ Define the function f as $f(v_1) = 3n - 8,\ f(v_{n-1+j}) = 3n - 11 + j, j = 1,2. f(u_{3n-4+i}) = 3n - 8 + i, i = 1,2, \dots ,3n - 3. f(u_{3n-3+i}) = 3n - 8 + i, i = 1,2,3.$

For $n = 9,11,13 \dots$

$$f(v_{2j}) = 2n - 9 + j, j = 1,2, \dots \frac{n-1}{2}, f(v_{n-2j}) = 5\left(\frac{n+1}{2}\right) - 12 + j, j = 1,2, \dots , \frac{n-3}{2}$$

For n =10,12,14…

$$f(v_{2j}) = 2n - 9 + j, j = 1,2, \dots \frac{n-2}{2}, f(v_{n+1-2j}) = 5\left(\frac{n+1}{2}\right) - 12 + j, j = 1,2, \dots , \frac{n-2}{2}$$

Therefore, the largest label is $6n - 8\ , n \geq 9$.

$$rlmn^{Dd}(TT_n) = \begin{cases} 5, & n=1 \\ 7n-5, & n=2,3 \\ 5n, & 4 \leq n \leq 8 \\ 6n-8, & n \geq 9. \end{cases}$$ ■

Theorem 3.4: The radio Dd- distance in lehmer-3 mean number of Quadrilateral snake graph,

$$rlmn^{Dd}(Q_n) = \begin{cases} 3n + 1, n = 1,2 \\ 5n - 3, n = 3,4,5 \\ 5n - 2, n = 6,7 \\ 6n - 10\ \ , n \geq 8 \end{cases}$$

Proof: It is obvious that $diam^{Dd}(Q_n) = 3n + 4\ , n \geq 1$. Let V={ $u_i, v_j\ /\ i = 1,2, \dots ,2n, j = 1,2, \dots , n + 1\}$ be the vertex set and $E = \{\ u_i v_j,\ \ v_j v_{j+1} /\ i = 1,2, \dots ,2n, j = 1,2, \dots , n + 1\}$ be the edge set .

We must show that the radio Dd- distance in lehmer-3 mean condition $D^{Dd}(u, v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^{Dd}(G) = 3n + 5$, for every pair of vertices (u, v) where $u \neq v$

For $n \geq 8$, Define the function f as $f(v_1) = 4n - 10, f(u_{2n+1-i}) = 4n - 10 + i, i = 1,2, \dots 2n - 1.$

For $n = 8,10,12 \dots$

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$f(v_{2j}) = 3n - 11 + j, j = 1,2, \ldots \frac{n}{2}, f(v_{n+1-2j}) = 7\left(\frac{n}{2}\right) - 11 + j, j = 1,2, \ldots, \frac{n-2}{2}$

$f(v_{n+1}) = 8\left(\frac{n}{2}\right) - 11$

For $n = 9,11,13 \ldots$

$f(v_{2j}) = 3n - 11 + j, j = 1,2, \ldots \frac{n+1}{2}, f(v_{n-2j}) = 7\left(\frac{n+1}{2}\right) - 14 + j, j = 1,2, \ldots, \frac{n-3}{2}$

$f(v_n) = 8\left(\frac{n+1}{2}\right) - 15$

Therefore, the largest label is $6n - 10$, $n \geq 8$.

$$rlmn^{Dd}(Q_n) = \begin{cases} 3n+1, n = 1,2 \\ 5n-3, n = 3,4,5 \\ 5n-2, n = 6,7 \\ 6n-10 \quad , n \geq 8 \end{cases}$$ ■

Theorem 3.5: The radio Dd- distance in lehmer-3 mean number of Double quadrilateral Snake graph, $rlmn^{Dd}(DQ_n) = \begin{cases} 1, n = 1 \\ 7n-2, 2 \leq n \leq 5 \\ 7n-1, 6 \leq n \leq 9 \\ 8n-10 \quad , n \geq 10 \end{cases}$

Proof: It is obvious that $diam^{Dd}(DQ_n) = 3n + 8$, $n \geq 3$. Let V={ u_i, v_j / $i = 1,2, \ldots, 4n, j = 1,2, \ldots, n+1$} be the vertex set and $E = \{ u_i v_j, \; v_j v_{j+1} / \; i = 1,2, \ldots, 4n, j = 1,2, \ldots, n+1\}$ be the edge set .

We must show that the radio Dd- distance in lehmer-3 mean condition $D^{Dd}(u,v) + \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil \geq 1 + diam^{Dd}(G) = 3n + 9$, for every pair of vertices (u, v) where $u \neq v$

Case (i): $n = 10,12,14 \ldots$

Define the function f as $f(v_1) = 4n - 10, f(v_{2j}) = 3n - 11 + j, j = 1,2, \ldots \frac{n}{2}$,

$f(v_{n+1-2j}) = 7\left(\frac{n}{2}\right) - 11 + j, j = 1,2, \ldots, \frac{n-2}{2}, f(v_{n+1}) = 4n - 11$,

$f(u_i) = 4n - 10 + i, i = 1,2, \ldots, 4n$.

Case (ii): $n = 11,13,15 \ldots$

Define the function f as $f(v_1) = 4(n+1) - 10, f(v_{2j}) = 3(n+1) - 11 + i, i = 1,2, \ldots \frac{n+1}{2}$,

$f(v_{n-2j}) = 7\left(\frac{n+1}{2}\right) - 14 + j, j = 1,2, \ldots, \frac{n-3}{2}, f(v_n) = 7(n+1) - 15$,

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$f(u_i) = 4(n+1) - 14 + i, i = 1,2,\ldots,4n.$

Therefore, the largest label is $8n - 10$, $n \geq 10$

$$rlmn^{Dd}(DQ_n) = \begin{cases} 1 , n = 1 \\ 7n - 2, 2 \leq n \leq 5 \\ 7n - 1, 6 \leq n \leq 9 \\ 8n - 10 \quad , n \geq 10 \end{cases} \qquad \blacksquare$$

Theorem 3.6: The radio Dd- distance in lehmer-3 mean number of Triple quadrilateral Snake graph, $rlmn^{Dd}(TQ_n) = \begin{cases} 7n + 1 , n = 1,2 \\ 9n - 3, 3 \leq n \leq 9 \\ 10n - 12 \quad , n \geq 10 \end{cases}$

Proof: It is obvious that $diam^{Dd}(TQ_n) = 3n + 10 , n \geq 3$. Let V={ u_i, v_j / $i = 1,2,\ldots,6n, j = 1,2,\ldots,n+1$} be the vertex set and $E = \{ u_i v_j, \ v_j v_{j+1} / \ i = 1,2,\ldots,6n, j = 1,2,\ldots,n+1\}$ be the edge set .

We must show that the radio Dd- distance in lehmer-3 mean condition $D^{Dd}(u,v) + \left\lceil \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rceil \geq 1 + diam^{Dd}(G) = 3n + 11$, for every pair of vertices (u,v) where $u \neq v$

For $n \geq 10$, Define the function f as $f(v_1) = 4n - 12$,

$f(v_{n-1+j}) = 4n - 15 + j, j = 1,2, f(u_{2i}) = 4n - 12 + i, i = 1,2,\ldots,3n,$

$f(u_{2i-1}) = 7n - 12 + i, i = 1,2,\ldots 3n, n \geq 10$

For n =10,12,14…

$$f(v_{2j}) = 3n - 13 + j, j = 1,2,\ldots \frac{n-2}{2}, f(v_{n+1-2j}) = 7\left(\frac{n}{2}\right) - 14 + j, j = 1,2,\ldots,\frac{n-2}{2}$$

For $n = 9,11,13\ldots$

$$f(v_{2j}) = 2n - 9 + j, j = 1,2,\ldots \frac{n-1}{2}, f(v_{n-2j}) = 7\left(\frac{n+1}{2}\right) - 17 + j, j = 1,2,\ldots,\frac{n-3}{2}$$

Therefore, the largest label is $10n - 12$, $n \geq 10$.

$$rlmn^{Dd}(TQ_n) = \begin{cases} 7n + 1 , n = 1,2 \\ 9n - 3, 3 \leq n \leq 9 \\ 10n - 12 \quad , n \geq 10 \end{cases} \qquad \blacksquare$$

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