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A Study on Colouring and chromatic number of graphs

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***Abstract*:**

In this paper, we discuss the significance of various graph colouring approaches using pictures. We determine the graph colouring characteristics such as chromatic, achromatic, and pseudoachromatic number for a variety of graphs in this study. We compute chromatic and achromatic numbers for a novel class of graphs, such as the central graph of a cycle graph $C(Cn)$ and the central graph of a jellyfish J, in this study (m, n). The applications of various graph colouring approaches in domains such as automated differentiation, mobile networks, optical networks, medical data mining, game theory, and radio networks are discussed in this research article..

***Index Terms*:** Graph colouring, central graph, chromatic number, achromatic number

I. Introduction

A graph is a visual depiction of a group of things connected by lines. The lines are called edges, while the vertices are called vertices. In a variety of fields, such as network analysis, finance, chemistry, engineering, and satellite navigation, graph theory is used to represent a problem. The graph colouring issue is a well-known problem that is used to resolve disagreements in real-life settings. [1], [2],[3].

II. Motivation and Significance of Graph Colouring

The problem of allocating colours to the nations of a geographical map in such a way that no two countries with a shared boundary receive the same colour is the goal of graph colouring theory. When we mark the nations with points in the plane and connect each pair of points with a line, we get a planar graph, which is a well-known Four Color Problem that may be coloured with four different colours. If we assume that adding a new colour to the map incurs a cost factor, then a map with n colours will be more costly than a map with $(n-1)$ colours, and so on. The theory of graph colouring is primarily concerned with resolving disagreements. In order to solve the conflict problem, neighbouring vertices must avoid having the same colour, resulting in a persistent conflict. As a result, the four-color dilemma was the driving force behind our investigation. [9].

III. Colouring Techniques of Graphs

Graph colouring is a strategy for resolving real-world problems. Graph colouring is a subset of graph labelling in graph theory. A suitable coloration of a graph is when colours are assigned to its members in such a way that no two adjacent elements have the same colour. An incorrect or pseudo-coloring of a graph occurs when colours are assigned to the elements of the graph in such a way that neighbouring members may share a same colour. If there is an edge (u, v) such that vertex u is coloured I and vertex v is coloured j for every pair of unique colours i, j, then it is not always appropriate colouring. If a graph colouring is both correct and pseudo complete, it is said to be complete. [9],[10],[16].

Graph colouring is broadly classified into three types such as vertex colouring, edge colouring and face colouring [2],[16],[17].

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Vertex colouring:

Vertex colouring is the process of assigning colours to the vertices of a graph in such a way that neighbouring vertices are coloured differently.

Edge Colouring:

Edge colouring is the process of assigning colours to the vertices of a graph in such a way that neighbouring vertices are coloured differently.

Face Colouring:

Face colouring is the process of assigning colours to the faces of a planar graph in such a way that neighbouring faces are coloured with two separate colours and no two faces share the same colour boundary.

IV. Colouring Parameters of Graph With Illustration

Chromatic number:

The minimum number of colours used in complete colouring is called chromatic number of G. It is denoted by $\chi\,(G)$.

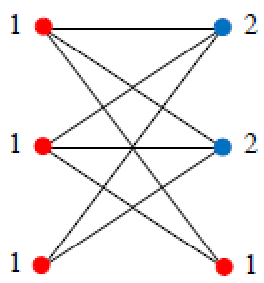

Figure 1: Chromatic number of $G(K3,3 - e)$, $(K_{3,3-e}) = 2$

Achromatic number:

The maximum number of colours used in complete colouring is called chromatic number of G. It is denoted by $\alpha(G)$.

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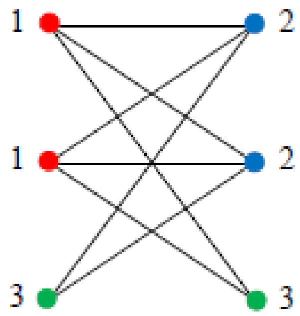

Figure 2: Achromatic number of $G(K_{3,3\text{-}e})$, $(K_{3,3\text{-}e}) = 3$

V. Computation of Chromatic Numbers for New Class of Graphs

In this paper, we compute the chromatic for new class of graphs like central graphs of cycle graph Cn and jelly fish graph $J(m,n)$ [16],[17].

Central graph:

Let G be a undirected graph with no loops and parallel edges. The graph is formed by subdividing the each edge exactly once and joining all the non-adjacent vertices of the graph is called the central graph. It is denoted by the symbol $C(G)$.

Theorem 5.1: Prove that the chromatic number for central graph of cycle of length n is 4. (i.e) $\chi(C(Cn)) = 4$.

Proof:

Construction of Central graph of cycle Cn, C(Cn):

The cycle graph is denoted by the symbol Cn. The vertex set $V(Cn) = \{v1, v2, v3, \ldots, vn\}$ and edge set $E(Cn) = \{q1, q2, q3, \ldots, qn\}$ where $qi = vi, vi + 1, 1 \leq i \leq n - 1, qn = vn, v1$. The central graph of cycle of length n is denoted by $C(Cn)$. The central $C(Cn)$ is formed by subdividng each edge $vi, vi + 1, 1 \leq i \leq n - 1$ of Cn exactly once by adding a new vertex ui and subdividing $vn, v1$ by un and joining vi with $vj, 1 \leq i, j \leq n, i \neq j$ and $vi, vj \quad E(Cn)$. The new vertex set formed is $V(C(Cn)) = \{V1UU1\}$ where $V1 = \{v1, v2, v3, \ldots, vn\}$ and $U1 = \{u1, u2, u3, \ldots, un\}$. The new edge set formed is

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$E(C(Cn)) = \{E1UE2\}$ such that $E1 = \{e1', e2', e3', \ldots, en'\}$ where $ek' = vi, vj;\ 1 \leq i, j \leq n, k = 1, 2, \ldots, n;\ i \neq j;\ vi, vj$ □ $E(Cn)$ and $E2 = \{e1'', e2'', e3'', \ldots, en''\}$ where $ei'' = ui, vi + 1 \leqq i \neq j;\ ei = vi, uj\ for\ i = j$.

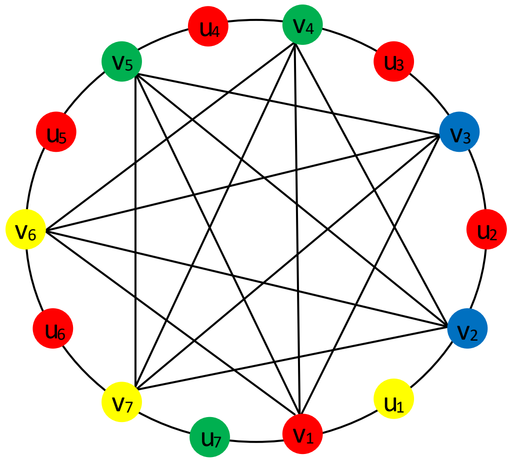

Figure 3: Central graph of cycle graph C_n,$C(C_n)$

Define the mapping $\psi : V(C(C_n))$ -> $\{1, 2, 3, 4\}$ such that $\psi(v_1) = 1$, $\psi(v_2) = \psi(v_3) = 2, \psi(v4) = \psi(v5) = 3, \psi(v6) = \psi(v7) =\ and\ \psi(u2) = \psi(u3) = \psi(u4) = \psi(u5) = \psi(u6) = 1;\ \psi(u1) = 2, \psi(u7) = 3$. Clearly it is easy to check that it is a proper colouring on vertices and hence we have $\chi(C(Cn)) \geq 4$. Suppose assume that $\psi(C(Cn)) = 5$ by some optimal colouring β. Then the colouring β assigns distinct colours to higher degree non-adjacent vertices. Therefore colours on $v1, v2, v4, v7$ must be distinct. Now the fifth colour must appear on any of the remaining vertices. If this happens, then there exists any one pair of colour with non-adjacent vertices does not have edge between them which is a contradiction. So we have $\psi(C(C_n)) \leq 4$. Therefore from the inequalities we arrive the result. Hence the chromatic number for central graph of cycle of length n is 4.(i.e) $\chi(C(Cn)) = 4$.

Theorem 5.2: Prove that the chromatic number for central graph of jelly fish graph 4. (i.e) $\chi(C(J(m, n))) = 4$.

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Proof:

Construction of Central graph of jelly fish graph, $C(J(m,n))$: The Jelly fish graph is denoted by $J(m,n)$. The vertex set V(Jm,n) = $\{x1, x2, x3, x4\ U\ u1, u2, \dots\ um\ U\ v1, v2, v3, \dots$ $v_n\}$ and edge set $E(C_n)$ = $\{(x1, x2), (x2, x3), (x3, x4), (x4, x1), (x1, x3)$ U (x_4, u_i) U $(x_2, v_j)\}$ where $i = 1, 2, 3 \dots n;\ j = 1, 2, 3 \dots m\}$. The central graph of jelly fish graph is denoted by the symbol $C(J(m,n))$. The central graph of $C(J(m,n))$ is formed by subdividng each edge x_i, $xi + 1, 1 \leq i \leq n$ exactly once by adding a new vertex c_i and joining x_i with $xi + 2;\ i = 1, 2, 3, 4$ and inclusion of vertex c_5 between the edge (x_1, x_3). Also subdividing the pendant vertices connected by x_4 to p_i and x_2 to q_j and joining x_4 with p_i and x_2 with q_j where i = 1, 2, 3 … m; j = 1, 2, 3 … n. The new vertex set formed is $V(C(J(mn)) = \{x1, x2, x3, x4\ U\ c1, c2, c3, c4, c5\ U\ pi\ U\ ui\ U\ qj\ U\ v_j\}$. The new edge set is E(C(J(m,n)) = $\{\{(x1, c1), (x1, c2), (c2, x2), (x2, x3), (c3, x3), (c4, x3), (c4, x1), (c4, x4), (x1, c5),$ (x_3, c_5) U (x_4, p_i) U $(pi, ui)\ U\ (x2, qj)\ U\ (qj, vj)\}\ where\ i = 1, 2, 3, \dots, n;\ j = 1, 2, 3, \dots, m$.

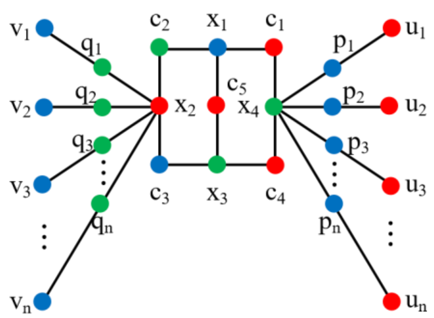

Figure 4: Central graph of jelly fish graph $C(J(m,n))$

Define the mapping $\psi : V(C(Jm,n))$ -> $\{1, 2, 3\}$ such that $\psi(x_1) = 1, \psi(x_2) = 2, \psi(x_3) = 3$. The remaining vertices can be assigned any one of these colours with the proper colouring without loss of generality. Clearly it is easy to check that it is a proper vertex colouring on vertices and hence we have $\chi(C(Jm,n)) \geq 3$. Suppose assume that $\psi(C(Jm,n)) = 4$ by some optimal colouring μ. Then the colouring μ assigns different colours to higher degree non-

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adjacent vertices. Therefore colours on $x1, x2, x_3$ must be distinct. Now the fourth colour must appear on any of the remaining vertices. If this happens, then there exists any one pair of colour which does not have edge between them which is a contradiction to our assumption. So we have $\psi(C(Jn)) \leq 3$. Hence from the inequalities we arrive the required result. Hence the chromatic number for central graph of jellyfish graph is 3. $(i.e)$ $\chi(C(Jm,n)) = 3$.

VI. A Tour on Selected Kinds of Graph Colouring Techniques with Applications in Diversified Arena

In this paper, we look at a variety of graph colouring techniques, as well as graph colouring parameters such as chromatic and achromatic numbers, and their applications in a variety of fields. Graph colouring is a fundamental idea in graph theory that has a wide range of applications in daily life. In this work, we present several graph colouring ideas and their real-time applications in a variety of domains.

6.1. Acyclic colouring and its Application in Automated Differentiation

If neighbouring vertices in a graph are coloured with two distinct colours and there is no two-colored cycle in G, the vertex colouring is said to be acyclic. The acyclic chromatic number is the least number of colours in an acyclic coloration. The symbol represents it. $\chi_A(G)$.

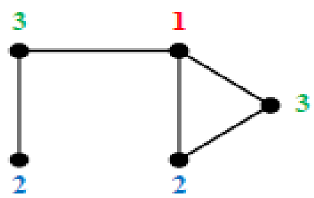

Figure 5: Acyclic chromatic number, $A(G)$ = 3

In the realm of computer science, the concept of acyclic colouring is commonly used. Acyclic colouring is used extensively in the automatic differentiation software approach for

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computing sparse hessians. Automatic differentiation is a technique for converting a programme from one domain to a programme in a different domain that determines the function's derivatives. The automatic differentiation data is the outcome of the tool's execution. The method is similar to split differences in that instead of assessing the original data, it creates a new dataset in which the derivatives are computed alongside the original data. The automated differentiation process has multiple modes, including forward mode, backward mode, and combination mode. The use of multiple methods, such as source-to-source translation and operator overloading, to create software processes. [1].

6.2. *R*-distinguishing colouring and its application in Game Theory

If no automorphism of the graph retains all of the vertex colours or breaks all of the graph's nontrivial symmetries, the graph's vertex colouring is R-distinguishing. The least R such that graph G has a suitable R-distinguishing colouring is the graph's distinguishing chromatic number. The symbol represents it. $\chi D(G)$.

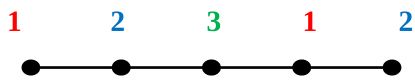

Figure 6: *R*-distinguishing chromatic number, □$D(P5)$ = 3

In the discipline of game theory, the R-distinguishing colouring is used. Game theory is a self-contained study that is widely employed in several fields of applied science. The goal of the game is to learn about reasonable behaviour. The differentiating game is a two-player game with opposing goals. The two players are represented as vertices with a graph's distinctive colouring, which assigns colours to the graph's vertices that aren't necessarily appropriate and violates all of the graph's non-trivial symmetries. The game ends when all of the vertices have been coloured, and the winning strategy is determined by who starts the game. [6],[13].

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6.3. B-colouring and its Application in Medical Data Mining

B-coloring is a valid vertex coloration of a graph in which each colour class has a vertex that has a neighbour in all other colour classes. The B-chromatic number is the maximum number of colours in B-coloring. The symbol represents it. $\chi b(G)$.

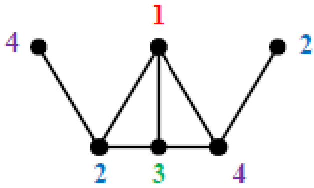

Figure 7: B-colouring chromatic number, □$B(G)$ = 4

The concept of B-coloring is used in the field of medical data mining, particularly in clustering. The process of extracting hidden or disguised patterns from medical data is known as medical data mining. Clustering is the process of segmenting data into subsets with elements that are likely to be different. The partition problem of reducing data sets into subclasses to the minimum colouring problem, where each colour represents a different class and the amount of colours utilised must be optimised, is known as B-coloring based clustering. This approach prefers to split the data set into compact clusters with a lower weighting for cluster separation. [5],[12].

6.4. Fractional colouring and its Application in Optical Networks:

A fractional coloration of G is a non-negative real function $f{:}\, I\; I(G, u)$, where I is the independent set that satisfies for each vertex u in G, and the weight of f is the total of its values. The fractional chromatic number is the lowest weight. The symbol represents it. $\chi\; f(G)$.

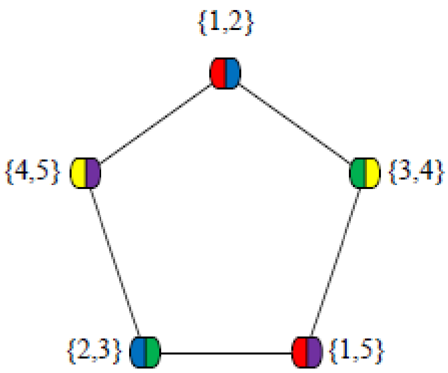

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Figure 8: Fractional chromatic number $f(C5) = 5/2$

In the field of optical networks, fractional coloration is used. The graph colouring issue is an optimization problem in which the colour set is minimised. In other words, it's the challenge of determining the cheapest option. An optical network consists of a collection of sets with end-to-end communication requests and n number of nodes. Each request must have its own light route or link, and each link must have its own colour so that there are no conflicts with the light path. The goal is to optimise the colour palette. Because optical communications are one-way, fractional colouring decides the output with natural linear programming for directed graphs. The wavelength assignment problem is the name given to this problem. [4],[8].

6.5. List colouring and its Application in Mobile Network

A correct vertex coloration is one in which each vertex v is allocated a colour from the list $L(v)$, where $L(v)$ is a list of colours for each vertex v. If a graph has a correct list colouring, it is termed k list colorable. The list chromatic number is the number of colours in a list that is the smallest. It's indicated by $\chi\ l\ (G)$.

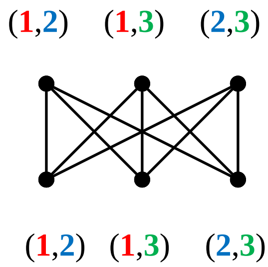

Figure 9: List chromatic number, $\chi_l(K_{3,3}) = 3$

In the field of mobile networks, list colouring is used. In a mobile network, the graph colouring issue is the task of colouring a graph with the fewest possible colours. The mobile network addresses the issue of network communication frequency assignment. Each vertex of the graph is assigned a colour associated with a given list of colours in the list colouring problem, so that each vertex is coloured with one of the colours in the list, resulting in correct vertex colouring of the graph. The frequency assignment problem is represented by an

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interference graph, in which the graph represents the transmitter and each edge represents a transmitter interference. The mobile cellular network is separated into cells, which are tiny areas. Each cell serves as a network-connected service station. The problem's goal is to reduce the maximum frequency channel used in a communication network to the lowest possible value. [10],[11],[15].

6.6. Radio colouring and its Application in Radio Network

A proper vertex colouring of a graph G is said to be radio colouring such that two colours i and j can be assigned to two distinct vertices u and v only if it satisfies the conditions, $|f(x) - f(y)| \geq 1, if\ d(x,y) = 2\ and\ |f(x) - f(y)| \geq 2, if\ d(x,y) = 1$. The least number of colours in radio colouring is called the radio chromatic number. It is denoted by χ_r (G).

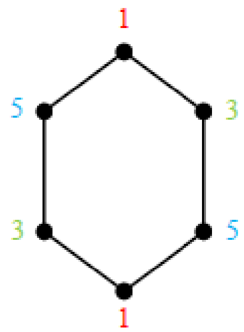

Figure 10: Radio chromatic number, □r(C6) = 3

In the realm of radio networks, the radio colouring concept is used. The radio colouring problem occurs when two distant emitters are given distinct channels. A channel separation might be a cause of interference. A collection of distances can be used to determine the interference limitations. The interference graph restriction specifies the minimum distance between channels assigned to each pair of transmitters. Signal interference in radio networks can be avoided by using physically nearby transmitters on different frequency channels. The goal of the radio coloration problem is to reduce the number of unique colours utilised in radio coloration, which means reducing the maximum frequency channel used between neighbouring

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transmitters. The problem of frequency assignment in radio networks is analogous to the graph colouring problem in the interference graph, with the exception that radio colouring uses the smallest amount of colours. [7],[14].

CONCLUSION

We have investigated the numerous forms of graph colouring approaches in this study, with extensive explanations provided through various figures, which will serve as a motivator for new researchers. We compute chromatic and achromatic numbers for a novel class of graphs, such as the central graph of a cycle graph $C(Cn)$ and the central graph of a jellyfish J, in this study (m, n). The purpose of this research is to identify various graph colouring settings that will pique the reader's attention. In this essay, we underline the necessity of using graph colouring approaches to handle conflict problems in real life. This study serves as a source of inspiration for future graph theory researchers.

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