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Research Paper

ON SYMMETRIZATION INCLUDING ROOTS OF NONLINEAR EIGENVALUE PROBLEMS

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Abstract

The operator $T(\alpha) = \alpha^2 + A\alpha + B$ bundle is being considered and that has a polynomial component and static operator constants. One uncovers that perhaps the system enables that needs to be factored $T(\alpha) = (z - \alpha_1)(z - \alpha_2)$ under certain fractional conditions. whereby each element α_1 and α_2 are eigenvalues. Besides that, the roots represent interpretations of a fundamental operator described throughout the function space. These activities are essential for wave equation.

Keywords: Relatively Solid Damping, Eigenvalue Problem, Density constructor, Fractional Conditions, Self-Adjoint Operators.

Mathematics Subject Classification: 15A60, 46A08.

1. Introduction

It was discovered by P.H.Muller [1] that the nonlinear problem can be put in the form

$$\alpha^2 + A(z)\alpha + B(z) = 0 \ldots\ldots\ldots\ldots(1)$$

Here A as well as B become restricted self-adjoint operators in some kind of a Hilbert space.

$$\text{Also if } (Bz, z) \geq 0 \ldots\ldots\ldots\ldots(2)$$

thereafter, this same matrix generator

$$M = \begin{bmatrix} 0 & \sqrt{B} \\ \sqrt{B} & A \end{bmatrix} \ldots\ldots\ldots\ldots (3)$$

is bounded, constrained as well as self-adjoint in the Hilbert Space H.

[3-6] demonstrated that perhaps the non - linear framework occurs under certain settings upon that bounded self-adjoint regulators H and I

$$\alpha^2 + H(z)\alpha + I = 0 \ldots\ldots\ldots\ldots(4)$$

Together under alternate presumption regarding relatively heavy damping, certain roots α_1 and α_2 become demonstrated too really be symmetrizable

$$(Bz, z) + (F_z, z) \geq 0,\ \|z\| > 1 \ldots\ldots\ldots\ldots(5)$$

Throughout this article we examine the eigenvalve problem areas of something like the type

$$\alpha^2 - A(z)\alpha - B(z) = 0 \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

We demonstrate that one for certain eigenvalve cases, a factor conversion converts the dilemma within (1) whereby B accepts (2).

Instead, upon this dimension with H and I, we add some extra constraints that guarantee that perhaps the accompanying regulator [4] becomes self-adjoint.

2. MINIMAL FORM OF MILLER'S

This should use the potentially the best findings that perhaps a Hermite, all over everything described regulator S becomes persistent however a self-adjoint, probability density constructor S seems to have an anywhere specified, consistent equivalent [7,8,10]. The whole strategic alliance that H as well as I become Hermites. The subsequent theorem seems to be a gross generalisation with Kren and Laner's [3] and Laner's [5] findings, which have been generalisations with R. J. Dufer's [6] results. Although this result demonstrates that unless H and I are both strong in some kind of a non - negative range which fulfil their absolute Inconsistency [12-15].

Theorem 2.1. Suppose H and I be Hermite and L(α) be neutrally solid damping, then $R_+(a) \geq R_-(b)$, when a,b ϵ Z(L), $\|a\| = \|b\| = 1$.

Proof: We employ the method of contradiction to prove the above claim.
On the contrary, let us suppose that $R_+(a) > R_-(b)$
We consider an infinitesimally small positive quantity ϵ, in such a manner $R_+(a, \epsilon) > R_-(b, \epsilon)$.

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for any v ϵ d(L) and $\|a\| = 1$. These really are associated towards the complete set

$$T_e(a) = \alpha^2 - A(z)\alpha - (I - 0.25H) \ldots\ldots\ldots\ldots (7)$$

Which of the following needs to fulfils the condition [5]. P is indeed the perpendicular representation upon this subspace H spanned across a and b.

Hence $T'(\alpha) = \alpha^2 F + A'(z)\alpha + B(z) = MT'_e(\alpha)$ validates characteristics at (7) in H, but instead F' as well as G' being restricted self-adjoint operations with H.

We get such a combination $T''(\alpha) = \alpha^2 F + A''(z)\alpha + B'(z) = T'_e(z + \alpha)$ through rendering one reasonably high, whereby F" as well as G" were favourable about H. We get a combination whilst making Ө relatively larger $T''(\alpha) = \alpha^2 F + A''(z)\alpha + B' = T'(\alpha + \theta)$, where A'' and $B'$are absolute on Hermite H. When $R'_+(a)$ and $R'_-(a)$ are indeed the components that refer to $T''(\alpha)$. Thus $R_+(a) < R_-(b)$, which contradicts the results of Dufer's.

Thus $R_+(a) \geq R_-(b)$, hence the proof.

Theorem 2.2. Suppose self-adjoint operations P and Q to be everywhere stated on Hermite H and thereby making B to have been positive definite, then M is all over stated well-defined, self-adjoint, however optimistic definite onto H. Also, $\delta^-(\delta^+)$ comprises some restricted, α-measurable subset that's still embedded only within progressive semiaxis as well as represents another component of both the distribution of T that would be embedded within in the valid semiaxis. Then $Y_+ = Y(\delta^-), Y_- = Y(\delta^-)$ are one-to-one transformation of H onto itself and furthermore Y is continuous. Also Y_+ as well as Y_- are positive fixed in terms including its scalar product rule.

Proof: Assuming each Y does indeed have a continuous inverse

$$M^{-1} = \begin{bmatrix} -\sqrt{Q}P\sqrt{Q} & \frac{1}{\sqrt{Q}} \\ \frac{1}{\sqrt{Q}} & 0 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The continuum among M is narrowed gone from the current but also turns the embeddings on Y_+ and Y_-. Thus the choice of $\delta(Y_+)$ onto oneself in some kind of a one-to-one association. Also Y_+ remain illustrations of M on Y_+. Thus, Y_+ converts $\delta(Y_+)$ onto the focus in 1-1 correspondence. Furthermore, this same representation including its connected operators being denoted by R.

Therefore Y_+ and Y_- stand compactly well-defined in Hermite H. This indicates about Y_+ and Y_- are indeed a linear transformation. Moreover, Y_+ and Y_- are too thoroughly and compactly demarcated towards H, M(Y_+) = H. This same dimensions a,b throughout H regarding random vectors a,b. Thus, we get

$$(a\sqrt{Q},b\sqrt{Q}) + (aY_+, bY_-) = 0, a, b \in \text{H} \text{.........................} (9)$$

$$\text{Thus, we get } -Q = Y_+ * Y_- \text{............................} (10)$$

Pre-multiplying respectively sideways of (9), in addition to respectively side of the following by $Y_+^2 = PY_+ +$ Q, subsequently taking its adjoints, we change to the subsequent characteristics

$$Y_+' = P - Y_+ \text{ and } Y_+' = P - Y_+, \text{..................} (11)$$

which stand comparable towards (7).

Thus accomplishes the proof.

Theorem 2.3. Suppose A as well as B remain self-adjoint but rather well-defined anywhere else on H, and $T(a) = \alpha^2 - A(z)\alpha - B$ be a solid damping. Then there are two consecutive and continuous roots.

Proof: Given $T(a) = \alpha^2 - A(z)\alpha - B$, the above equation is quadratic, hence it will yield two roots of M, Y_+ and Y_-, such that

$$T(\alpha) = (z - Y_+)(z - Y_-) = (z - Y_-)(z - Y_+) \text{ ....} (12)$$

on Hermite H. Also, the operator K = $Y_+ - Y_-$ is positive definite. Furthermore, ϵ remain a real quantity termed to be operators $(\epsilon z - Y_+)$ and $(\epsilon z - Y_-)$ remain self-adjoint with reference towards the scalar product.

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