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ON PROPER COLORING OF DURER GRAPH SQURE GRAPH OF COMB GRAPH,GENERALIZED THETA GRAPH AND TADPOLE GRAPH

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ABSTRACT:

To exhibit the proper coloring of Durer graph, Square graph of comb graph, Theta graph, and Tadpole graph have been discussed by unique way.

Key words: Durer graph, Comb graph, Theta graph, Tadpole graph, Chromatic number

INTRODUCTION

Graph theory is an old subject with many modern applications. Its basic ideas were introduced in the eighteenth century by the great Swiss mathematician Leonhard Euler. In the last three decades graph theory has established itself a worthwhile mathematical discipline and there are many applications of graph theory to a wide variety of subjects such as operation research, physics etc. [Rosa 2006]. [Francis Guthrie 1851]

In Graph theory, ‘Graph coloring’ is a special case of Graph labeling. It is an undertaking of labels historically referred to as “colors” to elements of a graph issue to positive constraints .In its best form it's miles the manner of coloring the vertices of a graph such that no two adjoining vertices share the identical shade. This is called “Vertex coloring”. [Bollobas, B 1988]

Preliminaries

Generalized theta graph

A generalized theta graph consisting of $k \geq 2$ internal disjoint path of length ℓ with the same end points u & v and denoted as $\theta[\ell^{(m)}]$. [Shiama,2012]

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square graph

The square graph G^2 of an undirected graph G is another graph that has the same set of vertices but in which two vertices are adjacent when their distance in G is at most 2. [Esakkiammal,2006]

Durer graph

The Durer graph is the graph formed by the vertices and edges of the Durer solid. It is a cubic graph of girth 3 and diameter 4. The Durer graph is Hamiltonian. It has exactly 6 Hamiltonian cycles, each pair of which may be mapped into each other by a symmetry of the graph

Tadpole graph

The (m, n)-Tadpole graph is a special type of graph consisting of a cycle graph on m vertices and path graph on n vertices, connected with a bridge [Sankari 2008]

MAIN RESULTS

Theorem 1

The Durer graph acknowledges the proper coloring and $\chi(G) = 3$

Proof

Consider the graph G with 12 vertices and 18 edges. Let v_i be the vertex set for $0 \leq j \leq 11$. Now we define the function $f : V \rightarrow \{1, 2, 3\}$ as follows:

(i) $f(v_0) = 3$
(ii) $f(v_1) = 1$
(iii) $f(v_2) = 3$
(iv) $f(v_3) = 2$
(v) $f(v_4) = 1$
(vi) $f(v_5) = 2$
(vii) $f(v_6) = 3$
(viii) $f(v_7) = 2$
(ix) $f(v_8) = 2$
(x) $f(v_9) = 1$
(xi) $f(v_{10}) = 1$
(xii) $f(v_{11}) = 3$

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Illustration:

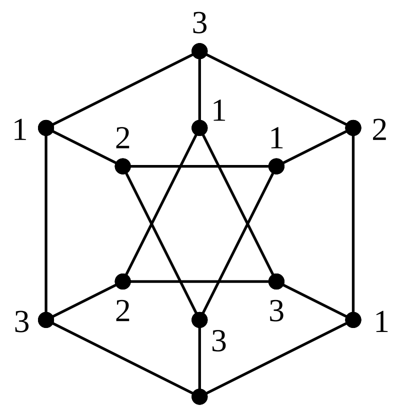

Figure 1: Durer graph

Theorem 2:

The square graph of comb admits proper coloring and its chromatic number is 4.

Proof:

Let G be the square graph of comb and denoted by $(P_n \odot K_1)^2$ with the vertex $V = \{x_i, y_i \,/\, 1 \leq i \leq n\}$ and the edge $E = E_1 \cup E_2 \cup E_3$ where

$E_1 = \{x_i x_{i+1}, x_i y_{i+1}\}$, $i = 1, 2 \ldots n-1$

$E_2 = \{x_i x_{i+2}, y_i x_{i+2}\}$,

$E_3 = \{x_i y_i\}$

The number of vertices and edges are $2n$ & $5n-1$.

Let $f : V(G) \to \{1, 2, 3, 4\}$ as follows:

(i) $f(x_i) = 4$, for $i = 1, 4, 7 \ldots 3n-2$
(ii) $f(x_i) = 3$, for $i = 2, 5, 8 \ldots 3n-1$
(iii) $f(x_i) = 1$, for $i = 3, 6, 9 \ldots 3n$
(iv) $f(y_i) = 2$, for all i

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Illustration 1:

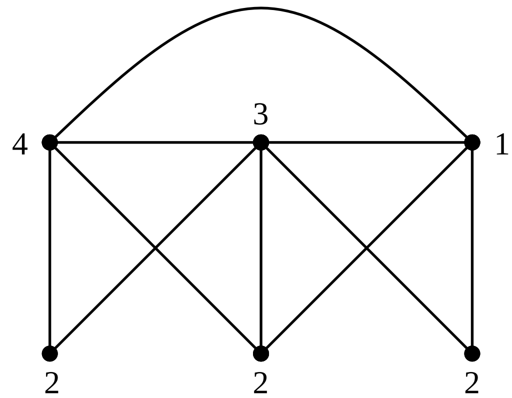

Figure 3 $(P_3 \ \Theta \ K_1)^2$

Illustration 2:

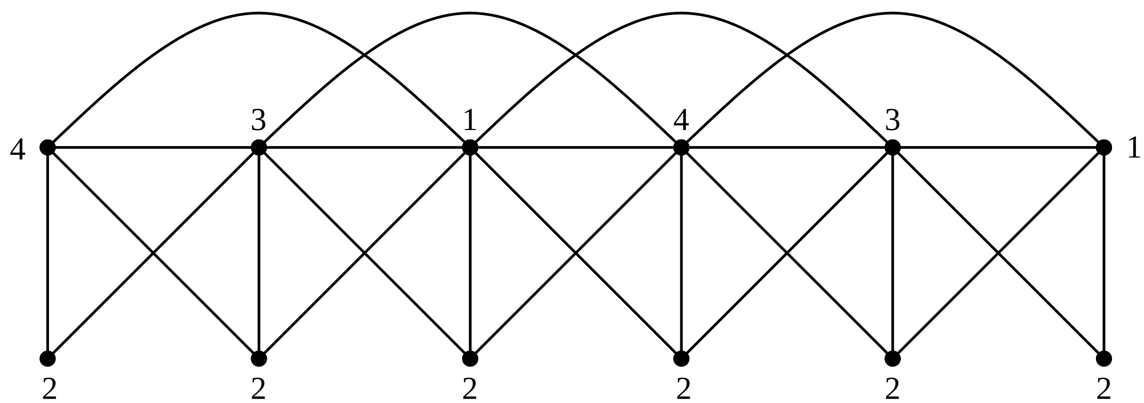

Figure 4: $(P_6 \ \Theta \ K_1)^2$

Theorem 3:

The generalized theta graph $\theta[\ell^{(m)}]$ acknowledges proper coloring whose chromatic number is 2.

Proof:

Consider $G = \theta[\ell^{(m)}]$ with $|V(G)| = m(\ell-1)$ and $|E(G)| = \ell m$, and let

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$\ell_{11}, \ell_{12} \ldots \ell_{1n}$ be the first set of vertices

$\ell_{21}, \ell_{22} \ldots \ell_{2n}$ be the second set of vertices

Similarly

$\ell_{m1}, \ell_{m2} \ldots \ell_{mn}$ be the last set of vertices and

ℓ be the left most end vertex

m be the right most end vertex

Let us consider the following cases:

Case (i): When ℓ is odd

(i) $f(\ell) = 1$
(ii) $f(\ell_{1n}) = 2$ when n is odd
(iii) $f(\ell_{1n}) = 1$ when n is an even
(iv) $f(m) = 2$

$f(\ell_{mn}) = 2$ when n is odd

$f(\ell_{mn}) = 1$ when n is an even

Illustration:

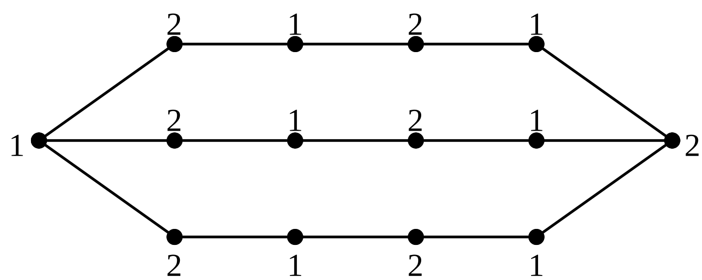

Figure 5: $\theta[5^{(3)}]$ – Theta graph

Case (ii): When ℓ is even

(i) $f(\ell) = 1$
(ii) $f(\ell_{1n}) = 2$ when n is odd
(iii) $f(\ell_{1n}) = 1$ when n is even

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(iv) $f(m) = 1$

Similarly

$f(\ell_{mn}) = 2$ when n is odd

$f(\ell_{mn}) = 1$ when n is even

Illustration:

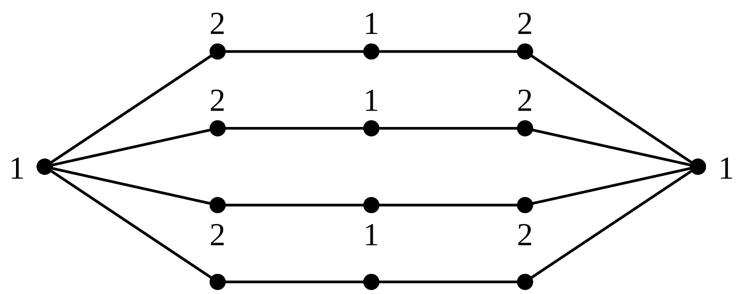

Figure 6: $\theta[4^{(4)}]$ – Theta graph

Theorem 4

Every tadpole T(n, m) admits proper coloring whose chromatic number 2 when n is even and 3 when n is odd.

Proof:

Let $x_1, x_2 \ldots x_n$ be the cycle vertices of C_n. $x_{n+1}, x_{n+2} \ldots x_{n+m}$ be the path vertices of P_n. Coloring has to be given into following two cases:

Case (i): When the cycle is even number of vertices

(i) $f(x_i) = 1$, when $i = 1, 3, 5 \ldots n-1$

(ii) $f(x_i) = 2$, when $i = 2, 4, 6 \ldots n$

(iii) $f(x_{n+i}) = 1$, when $i = 1, 3, 5 \ldots n-1$

(iv) $f(x_{n+i}) = 2$, when $i = 2, 4, 6 \ldots n$

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Illustration:

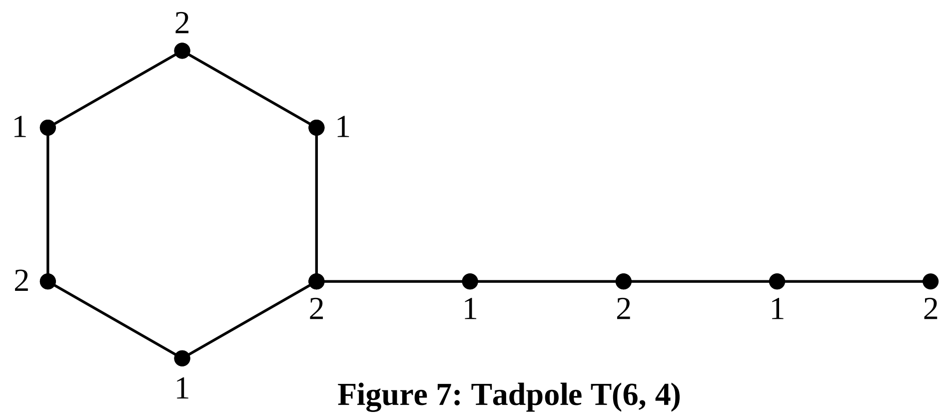

Figure 7: Tadpole T(6, 4)

Case (ii): When the cycle has odd number of vertices

(i) $f(x_i) = 1$, when $i = 1, 4 \ldots 3n{-}2$
(ii) $f(x_i) = 2$, when $i = 2, 5 \ldots 3n{-}1$
(iii) $f(x_i) = 3$, when $i = 3, 6 \ldots 3n$

For cycle vertices coloring has to be given 1, 2, 3 periodically.

(iv) $f(x_{n+i}) = 1$, when $i = 1, 3, 5 \ldots n{-}1$
(v) $f(x_{n+i}) = 2$, when $i = 2, 4, 6 \ldots n$

Illustration:

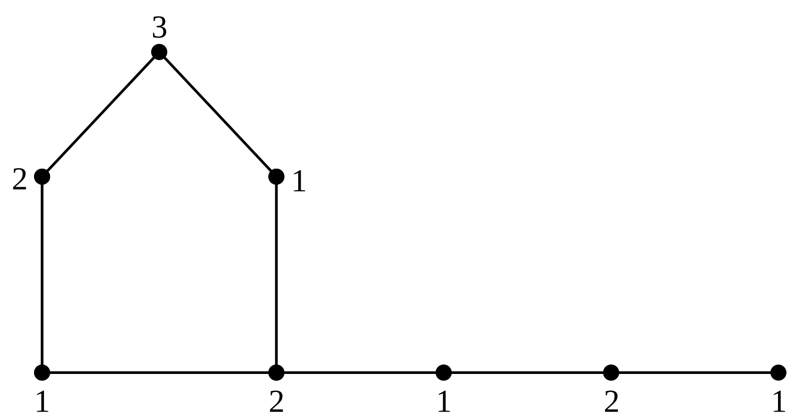

Figure 8: Tadpole T(5, 3)

CONCLUSION:

Here we discussed Chromatic number of Durer graph, Square graph of comb graph, Generalized Theta graph is 3,4,2 respectively.And also Chromatic number of Tadpole graph is 2, when number of vertices is an even and 3 , when number of vertices is an odd.

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It is of interest to find different types of coloring , such as Harmonious coloring, Grundy coloring, k-coloring, star coloring for various classes of graphs.

REFERENCES:

1.Bollobas, B., "Modern graph theory", Graduate Texts in Mathematics, Vol. 184, Springer-Verlag, 1998

2.Francis Guthrie, "Four Color theorem", 1851.

3.Esakkiammal, E., Deepa, B. and Thirusangu, K., "Some Labelings on Square Graph of Comb", International Journal of Mathematics Trends and Technology (IJMTT) – Special Issue NCCFQET

4.Rosa, A, "On certain valuations of the vertices of graph", Theory of graphs, International Symposium, Rome, Gordan and Breach, Newyork and Dunod Paris, 2006.

5.Sankari, G. and Lavanya, S., "Odd-even graceful labeling of umbrella and tadpole graphs", Int. Journal of Pure and Applied Mathematics, Vol. 115, pp. 139–143.2008

6.Shiama, J., "Square sum labeling for some middle and total graphs", International Journal of Computer Applications, Vol. 37, No. 4 (2012).