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Research Paper

A Certain Study on Group A cordial labeling of a graph

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Abstract

Graph labeling is a widely used and fastest growing area in the field of mathematics and computer science. Now a days, when data security is a major area of concern, various researchers and scientist are working to develop the techniques and software that can resolve the issues.

Graph labeling is used in many areas of science and technology. A lot of graph labeling techniques are discussed in [4], we enlist a few of them which are finding their use in different aspects of artificial intelligence [6]

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Most of the graph labeling methods trace their origin to one introduced by Rosa in 1967 or one given by Graham and Sloane in 1980.

Keywords: Cordial labelling, Bipartite Graphs, Wheel Related Graphs and Group $\{1, -1, i, -i\}$

Introduction:

In this Chapter, for any group *A* we define a new labeling, called as Group *A* cordial labeling and give a necessary and sufficient condition for a graph *G* to be group $(Z_p, \oplus)$ cordial where Z_p is the group of integers modulo *p* under addition. We then investigate the group $\{1,-1,i,-i\}$ cordial labeling of some graphs. This is the group of fourth roots of unity, which is cyclic with generators *i* and $-i$. We prove that the Bistar, Path, Cycle, Friendship

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graph and $W(n,n)$ are group $\{1,-1,i,-i\}$ cordial. We also characterize Star and Complete graphs that are group $\{1,-1,i,-i\}$ cordial.

Group A cordial labeling of a graph

Definition:1. Let G be a (p,q) graph and let A be a group. Let $f : V(G) \to A$ be a function. For each edge uv assign the label 1 if $(o(f(u)),o(f(v))) = 1$ or 0 otherwise. f is called a *group A cordial labeling* if $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x)$ and $e_f(y)$ respectively denote the number of vertices labeled with an element x and number of edges labeled with $y(y = 0,1)$. A graph which admits a group A cordial labeling is called a *group A cordial graph*.

Example:1 A simple example of a group $(Z_5,\oplus)$ cordial graph G is given in Fig.2.1.

We have $Z_5 = \{0,1,2,3,4\}$.

Also $o(0) = 1, o(1) = 5, o(2) = 5, o(3) = 5$ and $o(4) = 5$. Define $f : V(G) \to Z_5$ by $f(u) = 1, f(v) = 0, f(w) = 2, f(x) = 3$ and $f(y) = 4$.

So for every $n \in Z_5$ we have $v_f(n) = 1$. As $(1,5) = 1$, the edges uv,vw,vy get label 1 and the edges wx,xy get label 0.

We have $\mathbb{Z}_\omega = \{0,1,2,3,4\}$

Also $o(0) = 1, o(1) = 5, o(2) = 5, o(3) = 5$ and $o(4) = 5$.

Define $f: V(G) \to \mathbb{Z}_6$ by $f(u) = 1, f(v) = 0, f(w) = 2, f(x) = 3$ and $f(y) = 4$

So for every $n \in \mathbb{Z}_5$ we have $v_J(n) = 1$. As $(1,5) = 1$, the edges uv, vw, vy get label 1

and the edges wx, xy get label 0.

This $e_f(0) = 2$ and $e_f(1) = 3$. Since we have $|v_f(a) - v_f(b)| = 0$ for every $a, b \in$

$\mathbb{Z}_0$

and $|e_J(0) - e_J(1)| = 1, f$ is a group $(\mathbb{Z}_\omega,\oplus)$ cordial labeling and G is a group $(\mathbb{Z}_\omega,\oplus)$ cordial graph.

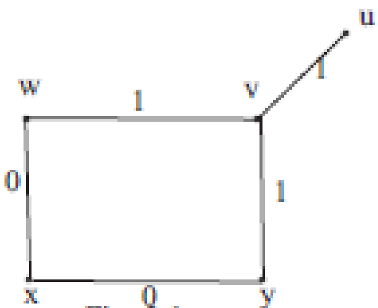

Fig 2.1

We now give a necessary and sufficient condition for a graph G with p vertices where $p$is the power of a single prime, to have a group $(\mathbb{Z}_p,\oplus)$ cordial labeling.

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1. 2 Some results on Group A cordial labeling

Theorem 2.2.1. Let G be a (p, q) graph with $p = z^{\alpha}$ where z is a prime. Then G is agroup $(\mathbb{Z}_p,\oplus)$ cordial graph if and only if G has a vertex v with $\deg v = d$ where $d = \frac{q}{2}, \left\lceil\frac{q}{2}\right\rceil$ or $\left\lfloor\frac{q}{2}\right\rfloor$

Proof. Let $V(G) = \{v, v_1, v_2, \dots, u_{p-1}\}$. Assume that G has a group $(\mathbb{Z}_p,\oplus)$ cordial labeling g. Choose v as the vertex with $g(v) = 0$. Let deg $v = d$. As g is a group $(\mathbb{Z}_p,\oplus)$ cordial labeling, $|e_g(0) - \varepsilon_g(1)| \le 1$

As $p = z^{a}$, by Theorem 1.3.3, order of the labels of all other vertices is z^{j} for some $j \le \alpha$. So the d edges incident at v alone have label 1 and all other edges have label 0 .

$$\text{Thus } |e_g(0) - e_g(1)| \le 1$$
$$\Rightarrow |(q - d) - d| \le 1$$
$$\Rightarrow |q - 2d| \le 1$$
$$\Rightarrow q = 2d, 2d + 1 \text{ or } 2d - 1$$
$$\Rightarrow d = \frac{q}{2}, \left\lceil\frac{9}{2}\right\rceil \text{ or } \left\lfloor\frac{q}{2}\right\rfloor$$

Conversely, assume that G has a vertex v with deg $v = d$ where $d = \frac{q}{2}, \left\lceil\frac{q}{2}\right\rceil$ or $\left\lfloor\frac{q}{2}\right\rfloor$.**Define**$f: V(G) \to \mathbb{Z}_p$, by $f(v) = 0$ and $f(v_i) = i$ for $1 \le i \le p - 1$. The d edges incident with v have label 1 and all other edges have label 0. As $d = \frac{q}{2}, \left\lceil\frac{q}{2}\right\rceil$ or $\left\lfloor\frac{q}{2}\right\rfloor, |e_f(0) - e_f(1)| \le 1$ and so f is a group $(\mathbb{Z}_p,\oplus)$ cordial labeling.

Corollary 2.2.2. If p is a power of a single prime, then P_p is group $(\mathbb{Z}_p,\oplus)$ cordial if and only if $p \le 5$

Proof. For $p \le 5$, a group $(\mathbb{Z}_p,\oplus)$ cordial labeling of P_p is given in Fig. 2.2.

Fig. 2.2

For $p \ge 5$, the proof follows from Theorem 2.2.1.

Corollary 2.2.3. If n is a prime, then the Star $K_{1,n^{a}-1}$ is group $Z_{n^{a}}$ cordial if and only if $n^{\alpha} \le 3$

Proof. The proof follows from Theorem 2.2.1.

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Corollary1.2.4. If n is a prime, then the Bistar $B_{\frac{n^{\alpha}-3}{2}}\frac{n^{\alpha}-1}{2}$ is group $Z_{n^{\alpha}}$ cordial.

Proof. The order and size of the Bistar $B_{\frac{n^{\alpha}-3}{2},\frac{n-1}{2}}$ are n^{α} and $n^{\alpha}-1$ respectively. Notethat the degree of one of the central vertices is $\frac{n^{\alpha}-1}{2}$. Hence the proof follows from Theorem 2.2.1.

1.3 Group $\{1,-1,i,-i\}$ cordial labeling of some graphs

We now investigate the group $\{1,-1,i,-i\}$ cordial labeling of some graphs. This is the group of fourth roots of unity, which is cyclic with generators i and $-i$. We prove that the Bistar, Path, Cycle, Friendship graph and $W(n,n)$ are group $\{1,-1,i,-i\}$ cordial. We also characterize Star and Complete graphs that are group $\{1,-1,i,-i\}$ cordial.

Definition1.3.1. Let G be a (p,q) graph and consider the group $\{1,-1,i,-i\}$ with multiplication. Let $f: V(G) \rightarrow \{1,-1,i,-i\}$ be a function. For each edge uv assign the label 1 if $(o(f(u)),o(f(v)))=1$ or 0 otherwise. f is called a group $\{1,-1,i,-i\}$ cordial labeling if $|v_f(a)-v_f(b)| \leq 1$ and $|e_f(0)-\mathrm{ef}(1)| \leq 1$, where $v_f(x)$ and $e_f(y)$ respectively denote the number of vertices labeled with an element x and number of edges labeled with $y(y=0,1)$. A graph which admits a group $\{1,-1,i,-i\}$ cordial labeling is called a group $\{1,-1,i,-i\}$ cordial graph.

Example 1.3.2. A group $\{1,-1,i,-i\}$ cordial labeling of C_3 is given in Fig. 2.3

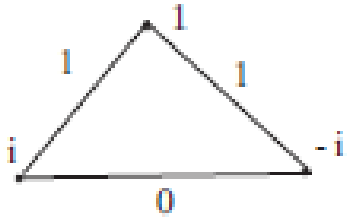

Fig. 2.3

Remark 1.3.3. Let A be any group of order 4. As identity is the only element of order 1, by Theorem 1.3.3, every other element of A is of order 2 or 4 . Thus a graph G is group $\{1,-1,i,-i\}$ cordial if and only if G is group A cordial for any group of order 4 .

Theorem 1.3.4. Every graph is a subgraph of a connected group $\{1,-1,i,-i\}$ cordial graph.

Proof. Let G be a (p,q) graph and $G_i (1 \leq i \leq 4)$ be four copies of the complete graph K_{p-} Let $u_1^i, u_2^i, \ldots, u_p^{t}$ be the vertices of $G_1(1 \leq i \leq 4)$. Let $m=2\binom{p}{2}$. Let G^* be obtained from $G_i(1 \leq i \leq 4)$ as follows:

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The vertex set of G^* is $V(G_1) \cup V(G_2) \cup V(G_3) \cup V(G_4) \cup \{v_i{:}1 \leq i \leq m\}$ and the edgeset is given by

$E(G^*) = E(G_1) \cup E(G_2) \cup E(G_3) \cup E(G_4) \cup \{u_1^1 u_1^2, u_2^2 u_2^3, u_3^3 u_3^4\} \cup \{u_1^1 v_j{:}\, 1 \leq j \leq m\}$. Clearly G^* has $4p + m$ vertices and $4\binom{p}{2} + m + 3$ edges. As G^* contains 4 copies of K_p, G is clearly a subgraph of G^*

Let $f{:}\, V(G^*) \to \{1, -1, i, -i\}$ be any function. Assign label 1 to all the vertices of G_1, i to all the vertioes of $G_2, -i$ to all the vertices of G_3 and -1 to all the vertices of G_4.
Case 1. $m = 4t, t \in \mathbb{Z}$

Assign label 1 to the vertices $v_1, v_2, \dots, v_t, i$ to the vertices $v_{t+1}, \dots, v_{2t} - i$ to the vertices $v_{2t+1}, \dots, v_3$ and finally assign the label -1 to the vertices $v_{3t+1}, v_{3t+2}, \dots, v_{4t}$. In this case $v_p(1) = v_f(-1) = v_f(i) = v_f(-i) = p + t$

Case 1. $m = 4t + 1, t \in \mathbb{Z}$

As in Case 1 , assign labels to the vertices $v_i (1 \leq i \leq m - 1)$ and assign 1 to the vertex v_m. Now $v_f(1) = p + t + 1$ and $v_f(-1) = v_f(i) = v_f(-i) = p + t$

Case 2. $m = 4t + 2, t \in \mathbb{Z}$.

Assign labels to the vertices $v_{\mathrm{i}} (1 \leq i \leq m - 1)$ as in Case 2. Finally assign the label -1 to the vertex v_m. Here $v_f(1) = v_f(-1) = p + t + 1$ and $v_f(i) = v_f(-i) = p + t$.

Case 3. $m = 4t + 3, t \in \mathbb{Z}$.

As in Case 3 , assign labels to the vertioes $v_1 (1 \leq i \leq m - 1)$ and assign i to the vertex v_m. In this case $v_f(1) = v_f(-1) = v_f(i) = p + t + 1$ and $v_f(-i) = p + t$In all cases, $e_f(0) = 3\binom{p}{2} + 2$ and $e_f(1) = 3\binom{P}{2} + 1$. So f is a group $\{1, -1, i, -i\}$ oordial labeling of G^*.

Theorem 1.3.5. The Star $K_{1,n}$ is group $\{1, -1, i, -i\}$ cordial if and only if $n \leq 5$.

Proof. Let $V(K_{1,n}) = \{u, u_i{:}\, 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{uu_i{:}\, 1 \leq i \leq n\}$. Suppose $n \leq 5$. Group $\{1, -1, i, -i\}$ cordial labelings of $K_{1,n}$ are given in Table 2.1.

Conversely, suppose that $K_{1,n}$ is group $\{1, -1, i, -i\}$ cordial. Let f be a group $\{1, -1, i, -i\}$ cordial labeling of $K_{1,n}$

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n	u	u_1	u_2	u_3	u_4	u_5
1	1	-1				
2	-1	1	1			
3	-1	1	1	-6		
4	-1	1	i	-6	1	
5	-1	1	1	-6	1	-1

Table 1. 1

Case 1. $f(u) = 1$

In this case all the edges receive the label 1. Thus $ef(1) = n$ and

$e_J(0) = 0$, a contradiction

Case 2. $f(u) \in \{-1, i, -i\}$

Clearly $v_f(1) = \left\lceil\frac{p}{4}\right\rceil$ or $\left\lceil\frac{p+1}{4}\right\rceil$. This implies $e_f(1) \leq \left\lceil\frac{p+1}{4}\right\rceil$. This contradicts the edgecondition of group $\{1, -1, i, -i\}$ cordial labeling.

Theorem1.3.6. The Bistar $B_{n,n}$ is group $\{1, -1, i, -i\}$ cordial for every n.

Proof. Let $V(B_{n,n}) = \{u, v\} \cup \{u_i v_i : 1 \leq i \leq n\}$ and $E(B_{n,n}) = \{uv\} \cup \{uu_i, vv_y : 1 \leq i \leq n\}$

Let$f: V(B_{n,n}) \to \{1, -1, i, -i\}$ be a function.

Assign the label $1, -1$ to the vertices u and v respectively.

Case 1. $n \equiv 0(\mathrm{mod}4)$.

Let n $= 4$t. Assign the label 1 to the vertices $u_1, u_2, \dots, u_{2t}$ and -1 to the vertices $u_{2t+1}, u_{2t+2}, \dots, u_{4t}$. Assign the label i to the vertices $v_1, v_2, \dots, v_{2t}$ and assign the label $-i$ to the vertices $v_{2t+1}, v_{2t+2}, \dots, v_{4t}-$

Case 2.$n \equiv 1(\mathrm{mod}4)$.

Let n $= 4$t $+ 1$. As in Case 1, assign labels to the vertices $u_i, v_i (1 \leq i \leq n-1)$. Finally assign the label $i, -i$ respectively to the vertices u_n, v_n

Case 3. $n \equiv 2(\mathrm{mod}4)$.

Let n $= 4$t $+ 2$. Assign labels to the vertices $u_i, v_i (1 \leq i \leq n-1)$ as in Case 2. Then assign the label $1, -1$ to the vertices u_n, v_n respectively.

Case 4. $n \equiv 3(\mathrm{mod}4)$

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Let n = 4t + 3. As in Case 3 , assign labels to the vertioes $u_i, v_i (1 \leq i \leq n-1)$. Finally assign the label $i, -i$ to the vertices u_n, v_{n} respectively. That this vertex labeling f is a group $\{1, -1, i, -i\}$ cordial labeling follows from Table 1.2.

n	$v_J(1)$	$v_f(-1)$	$v_g(i)$	$v_f(-1)$	$e_f(0)$	$e_f(1)$
$4t$	$2t+1$	$2t+1$	$2t$	$2t$	$4t$	$4t+1$
$4t+1$	$2t+1$	$2t+1$	$2t+1$	$2t+1$	$4t+1$	$4t+2$
$4t+2$	$2t+2$	$2t+2$	$2t+1$	$2t+1$	$4t+2$	$4t+3$
$4t+3$	$2t+2$	$2t+2$	$2t+2$	$2t+2$	$4t+3$	$4t+4$

'Table 1. 2

Example 1.3.7. Group $\{1, -1, i, -i\}$ cordial labelings of $B(4,4)$ and $B(6,6)$ are given in Fig. 1: 4.

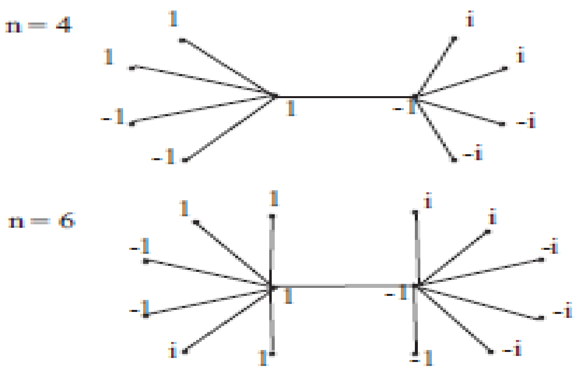

Fig. 2.4

Theorem 1.3.8. Any Path P_n is group $\{1, -1, i, -i\}$ cordial

Proof. Let P_n be the path $u_1, u_2, u_3, \ldots, u_n$. Clearly P_n is group $\{1, -1, i, -i\}$ cordial if $n \leq 3$. Assume $n \geq 4$

Let $f: V(P_n) \rightarrow \{1, -1, i, -i\}$ be a function.

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Case 1. $n \equiv 0(\text{mod}4)$

Let $n = 4t$. Assign the label i to the vertioes $u_i(1 \leq i \leq t)$. Then assign the label $-i$ tothe vertices $u_{t+1}(1 \leq i \leq t)$. Next assign the label -1 to the vertices

$u_{2t+1}, u_{2t+3}, \dots, u_{4t-1-}$ Finally assign the label 1 to the vertioes $u_{2t+2}, u_{2t+4}, \dots, u_{4t}$

Case 2. $n \equiv 1(\text{mod}4)$

Let $n = 4t + 1$. Assign labels to the vertices $u_i(1 \leq i \leq n-1)$ as in Case 1. Finallyassign the label 1 to the vertex u_{n-}

Case 3. $n \equiv 2(\text{mod}4)$

Let $n = 4t + 2$. Assign the label 1 to the vertices $u_1, u_3, u_5, \dots, u_{2t+1}$ and assign the label-1 to the vertices $u_2, u_4, \dots, u_{2t-2,}u_{2t+2-}$ Then assign the label $-i$ to the vertices $u_{2t} + 3, u_{2t+}, \dots, u_{4t+1}$. Finally assign the label i to the vertices $u_{3t+2}, u_{3k+4}, \dots, u_{4t+1}$ and u_{4t+2}

Case 4. $n \equiv 3(\text{mod}4)$

Let $\text{n} = 4\text{t} + 3$. As in Case 3 , assign labels to the vertices $u_i(1 \leq i \leq n-1)$. Finally assign i to the vertex u_n

Table 1.3 and Table 1.4 establish that the above vertex labeling f is a group $\{1, -1, i, -i\}$ cordial labeling of the path P_{n-}

π	$v_\rho(1)$	$v_f(-1)$	$v_f(1)$	$v_f(-1)$
$4t$	t	t	t	t
$4t+1$	$t+1$	t	t	t
$4t+2$	$t+1$	$t+1$	t	t
$4t+3$	$t+1$	$t+1$	$t+1$	t

Table 1. 3

Example 2.3.9. Group $\{1, -1, i, -i\}$ cordial labelings of P_8 and P_9 are given in Fig. 2.5.

$\text{n} = 8$

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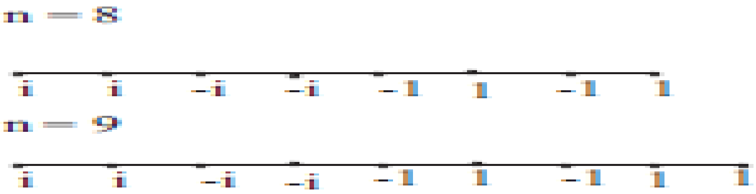

Fig. 1. 5

Corollary 2.3.10. Any Cycle C_n is group $\{1,-1,i,-i\}$ cordial.

Proof. Let the Cycle C_n be $u_1, u_2, u_3, \ldots, u_n, u_1$.

Let $f: V(C_n) \rightarrow \{1,-1,i,-i\}$ be a function.

Case 1. $n \equiv 0,1,3(\mathrm{mod}4)$

The group $\{1,-1,i,-i\}$ cordial labeling given in Theorem 2.3.8 is also a group $\{1,-1,i,-i\}$ cordial labeling of C_n.

Case 2. $n \equiv 2(\mathrm{mod}4)$

Assign labels to the vertioes $u_i (1 \leq i \leq n)$ as in Theorem 2.3.8.

Finally relabel the vertex u_2 with 1 and relabel u_3 with -1

Example 2.3.11. Group $\{1,-1,i,-i\}$ cordial labelings of C_{10} and C_{11} are given in Fig. 2.6.

Theorem 2.3.12. The Complete graph K_n is group $\{1,-1,i,-i\}$ cordial if and only if $n \in \{1,2,3,4,7,14,21\}$

Proof. Let $V(K_n) = \{u_i : 1 \leq i \leq n\}$.

n	u_{11}	u_{12}	u_{18}	u_{14}	u_{15}	u_{16}	u_{12}	u_{18}	v_{19}	u_{20}	u_{21}
1											
2											
3											
4											
7											
14	⋮	-1	-1	-1							
21	-1	1	1	t	1	1	-1	$-i$	-1	-1	-1

'Table 1. 6

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Assume $n > 4$. Suppose f is a group $\{1, -1, i, -i\}$ cordial labeling of K_n.

Case 1. $n \equiv 0(\mathrm{mod}4)$

Let $n = 4t, t \in \mathbb{N}$ and $t > 1$. Obviously $v_f(1) = v_f(-1) = v_f(i) = v_f(-i) = t$. It is easy to verify that $e_f(1) = \binom{t}{2} + t(3t)$. Thus $e_f(1) = \frac{t(t-1)}{2+3[2}$. Also $e_\lambda(0) = \binom{t}{2} + \binom{t}{2} + \binom{t}{2} + t(t) + t(t) + t(t) = 3\binom{t}{2} + 3t^2 = \frac{3x(t-1)}{2+3[x^2}$. So $e_f(0) - e_f(1) = t^2 - t > 1$, a contradiction.

Case 2. $n \equiv 1(\mathrm{mod}4)$

Let $n = 4t + 1, t \in \mathbb{N}, t \neq 5$. Then $v_f(1) = t$ or $t + 1$.

Subcase (i). $v_f(1) = t$

In this case, $e_f(1) = \binom{t}{2} + t^2 + t^2 + t(t + 1) = \frac{t(t-1)}{2} + 3t^2 + t$

Then $e_j(0) = \binom{t}{2} + \binom{t}{2} + \binom{c+1}{2} + t(t) + t(t + 1) + t(t + 1) = 2\binom{t}{2} + \binom{t+1}{2} + t^2 + 2t(t + 1) = \frac{2(t-1)}{2} + \frac{4(t+1)}{2} + 3t^2 + 2t$

Hence $e_f(0) - e_f(1) = \frac{\tau(t-1)}{2} + \frac{t(t+1)}{2} + t = t^2 + t > 1$, a contradiction.

Subcase (ii). $v_f(1) = t + 1$

Here $e_f(1) = \binom{t+1}{2} + 3t(t + 1)$ and $e_j(0) = 3\binom{t}{2} + t^2 + t^2 + t^2 = 3\binom{t}{2} + 3t^2$

Hence $e_f(0) - e_j(1) = t^2 - 5t$, a contradiction.

Case 3. $n \equiv 2(\mathrm{mod}4)$

Let $n = 4t + 2, t \in \mathbb{N}, t \neq 3$. Then $v_f(1) = t$ or $t + 1$

Subcase (i). $v_f(1) = t$

In this case , $e_f(1) = \binom{t}{2} + t^2 + t(t + 1) + t(t + 1)$ and $e_j(0) = \binom{t}{2} + \binom{t+1}{2} + \binom{t+1}{2} + t(t + 1) + t(t + 1) + (t + 1)(t + 1) = \binom{t}{2} + 2\binom{t+1}{2} + 3t^2 + 4t + 1$. Hence $e_j(0) - e_f(1) = t^2 + 3t + 1 > 1, a$ contradiction.

Subcase (ii). $v_f(1) = t + 1$

In this case, $e_f(1) = \binom{t+1}{2} + 2t(t + 1) + (t + 1)^2$ and $e_j(0) = 2\binom{t}{2} + \binom{c+1}{2} + t^2 + 2t(t + 1)$

Hence $e_j(0) - \epsilon_f(1) = t^2 - 3t - 1 > 1$, a contradiction.

Case 4. $n \equiv 3(\mathrm{mod}4)$

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Let $n = 4t + 3, t \in \mathbb{N}, t \neq 1$. Then $v_f(1) = t$ or $t + 1$.

Subcase (i). $v_f(1) = t$.

In this case, $e_f(1) = \binom{t}{2} + 3t(t+1)$ and $e_f(0) = 3\binom{t+1}{2} + 3(t+1)^2$ Thus $e_j(0) - e_f(1) = t^2 + 5t + 3 > 1$, a contradiction.

Subcase (ii). $v_f(1) = t + 1$

In this case, $e_f(1) = \binom{t+1}{2} + 2(t+1)^2 + t(t+1)$ and $e_f(0) = \binom{t}{2} + 2\binom{t+1}{2} + (t+1)^2 + 2t(t+1)$

Hence $e_f(0) - e_f(1) = t^2 + t - 1 > 1$, a contradiction.

Theorem 2.3.13. The Friendship graph $C_3^{(t)}$ is group $\{1, -1, i, -i\}$ cordial if and only if $t \leq 4$.

Proof. Let the vertioes of $C_3^{(t)}$ be labeled as follows: Let u be the vertex common to all 3-cycles and let every 3-cycle be labeled as $uu_iv_s(1 \leq i \leq t)$. The group $\{1, -1, i, -i\}$ cordial labeling of $C_3^{(t)}, t \leq 4$ is given in Table 1.7.

t	u	u_1	u_2	u_3	ι_4	v_1	v_2	v_3	v_4
1	1	-1							
2	-1	1	i			1	-1		
3	-1	1	$-i$	1		1	-1		
4	-1	-1	1	1	1	-4	-1	4	i

Table 1.7

Assume $t > 4$. Suppose f is a group $\{1, -1, i, -i\}$ cordial labeling of $C_3^{(t)}$

Case 1. $f(u) = 1$

Then $e_\gamma(1) \geq 2t$ and $e_j(0) \leq t$. Thus $e_j(1) - e_j(0) \geq t$, a contradiction.

Case 2. $f(u) \neq 1$

Subcase(i). t is even.

Let $t = 2m$. Clearly two $1's$ contribute at most 4 edges with label

1 Therefore $m1's$ contribute at most $2m$ edges with label 1 . Hence $e_f(1) \leq 2(m + 1) = 2m + 2$. But the size of $C_3^{(t)}$ is $6m$, a contradiction.

2 Subcase(ii). t is odd.

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3 Let $t = 2m + 1$. In this case also, $e_f(1) \leq 2m + 2$, a contradiction.

4 Theorem 2.3.14. $W(n, n)$ is group $\{1, -1, i, -i\}$ cordial for every n.

Proof. Let $V(W_{n,n}) = \{u, v, u_i, v_t : 1 \leq i \leq n\}$ and $E(W_{m,n}) = \{uu_1, vv_i : 1 \leq i \leq n\} \cup \{u_i u_{(t+1)} \bmod n, v_1 v_{(t+1)} \bmod n : 1 \leq i \leq n\} \cup \{uv\}$. Note that $W(n, n)$ has $2n + 2$ vertioes and $4n + 1$ edges. Let $f : V(W_{n,n}) \to \{1, -1, i, -i\}$ be a function.

Case 1. n is odd.

Let $n = 2k + 1, k \geq 1, k \in \mathbb{Z}$. Label the vertices $u, u_1, u_3, \ldots, u_{n-2}$ by 1. Label the remaining vertices arbitrarily so that $k + 1$ of them get label $-1, k + 1$ of them get label i and $k + 1$ of them get label $-i$.

Case 2. n is even.

Let $n = 2k, k \geq 2, k \in \mathbb{Z}$

Label the vertices $u, u_1, u_3, u_6, \ldots, u_{n-1}$ by 1. Label the remaining vertices arbitrarily so that $k + 1$ of them get label $-1, k$ of them get label i and k of them get label $-i$.

Conclusion:

The cordial labeling of graphs has been a topic of research for 50 years and it still has many properties to be found. Although its primary interest was the cordial labeling of trees in order to solve Ringl's conjecture, cordial labeling of graph gained over the years its own beauty and interest. This work gives a brief overview of the subject, resenting not only theoretical results form the literature, but also some computational results. Furthermore, we give some contribution to this problem.

Another class of interest is the class of trees, being the main open class on this topic. It is already settled that many classes of trees are graceful, but also there are many classes, even simple one likes lobsters, that are still open. Finally, another approach to the problem is to relax the conditions of cordial labeling graphs. This approach by approximating the labeling is also topic of this research article for both trees and graphs in general.

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