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Total Face Edge Sum Divisor Cordial Graphs

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Abstract

In this paper, we introduce and investigate the total face edge sum divisor cordial labeling of fan graph with n vertices, wheel graph, gear graph and closed helm graph.

Keywords : face edge sum divisor cordial graph, total face edge sum divisor cordial labeling, total face edge sum divisor cordial graph, wheel graph.

Introduction

We begin with simple, finite, planar, undirected graph. A (p,q) planar graph G means a graph G = (V,E), where V is the set of vertices with |V| = p, E is the set of edges with |E| = q and F is the set of interior faces of G with |F| = number of interior faces of G. For standard terminology and notations related to graph theory we refer to Harary [3]. For graph labeling we refer to Gallian [2]. In [1], Cahit introduced the concept of cordial labeling of graph. In [10], Yilmaz et al introduced the concept of E-cordial labeling of graph. Varatharajan et al.[7] introduced the concept of divisor cordial labeling of graphs. The concept of sum divisor cordial labeling was introduced by Lourdusamy et al.[5]. Lawrence et al introduced the concept of face edge product cordial labeling of graph in [4]. Mohamed Sheriff et al. introduced the concept of face sum divisor cordial labeling of graph in [6]. In [8], Vijayalakshmi et al. introduced the concept of edge sum divisor cordial labeling of graph. In [9], Vijayalakshmi et al. introduced the concept of face edge sum divisor cordial labeling of graph. The present work is focused on some new families of total face edge sum divisor cordial labeling of fan graph with n vertices, wheel graph, gear graph and closed helm graph.

Definition 1.1 Let a and b be two integers. If a divides b means that there is a positive integer k such that b = ka. It is denoted by $a|b$. If a does not divide b, then we denote $a \nmid b$.

Definition 1.2 Let G = (V(G),E(G)) with p vertices and q edges and $f : E(G) \to \{0,1\}$. Define f* on V(G) by $f^*(v) = \Sigma\{f(uv)/uv \in E(G)\}(\text{mod}2)$. The function f is called an E-cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differs by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 differs by at most 1. A graph that admits E-cordial labeling is called E-cordial.

Definition 1.3 Let G = (V(G), E(G)) be a simple graph and $f : V(G) \to \{1,2,...,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if $f(u)|f(v)$ or $f(v)|f(u)$ and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0)-e_f(1)| \leq 1$. A graph with a divisor cordial labeling is called a divisor cordial graph.

Definition 1.4 Let G = (V(G), E(G)) be a simple graph and $f : V(G) \to \{1,2,\ldots, |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if $2|(f(u)+f(v))$ and the label 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.

Definition 1.5 Let G = (V(G),E(G)) be a simple graph and $f : E(G) \to \{1,2,...,|E(G)|\}$ be a bijection. For each vertex v, assign the label 1 if $2 \mid f(a_1)+f(a_2)+\ldots+f(a_s)$ and the label 0 otherwise where $a_1,a_2,\ldots,a_s$ are edges incident with the vertex v. The function f is called a edge sum divisor cordial labeling if the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1. A graph which admits an edge sum divisor cordial labeling is called an edge sum divisor cordial graph.

Definition 1.6 A face sum divisor cordial labeling of a graph G with vertex set V is a bijection f from V(G) to $\{1,2,\ldots, |V(G)|\}$ such that an edge uv is assigned the label 1 if 2 divides f(u)+f(v) and 0 otherwise and for face f is assigned the label 1 if 2 divides $f(u_1)+f(u_2)+\ldots+ f(u_k)$ and 0 otherwise, where $u_1,u_2,\ldots,u_k$ are vertices corresponding to the face. Also the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1 and the number of faces labeled with 0 and the number of faces labeled with 1 differ by at most 1. A graph which admits a face sum divisor cordial labeling is called a face sum divisor cordial graph.

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Definition 1.7 A face edge sum divisor cordial labeling of a graph G with edge set E is a bijection f from E(G) to $\{1,2,\ldots,|E(G)|\}$ such that a vertex v is assigned the label 1 if 2 divides $f(a_1)+f(a_2)+\ldots+f(a_s)$ and 0 otherwise where $a_1,a_2,\ldots,a_s$ are edges incident with the vertex v and for face f is assigned the label 1 if 2 divides $f(b_1)+f(b_2)+\ldots+f(b_t)$ and 0 otherwise, where $b_1,b_2,\ldots,b_t$ are edges corresponding to the face f. Also the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of faces labeled with 0 and the number of faces labeled with 1 differ by at most 1. A graph which admits a face edge sum divisor cordial labeling is called a face edge sum divisor cordial graph.

Definition 1.8 A total face edge sum divisor cordial labeling of a graph G with edge set E is a bijection g from E(G) to $\{1,2,\ldots,|E(G)|\}$ such that a vertex v is assigned the label 1 if 2 divides $f(a_1)+f(a_2)+\ldots+f(a_s)$ and 0 otherwise where $a_1,a_2,\ldots,a_s$ as are edges incident with the vertex v and for face f is assigned the label 1 if 2 divides $f(b_1)+f(b_2)+\ldots+f(b_t)$ and 0 otherwise, where $b_1,b_2,\ldots,b_t$ are edges corresponding to the face f. Also the number of vertices and faces labeled with 0 and the number of vertices and faces labeled with 1 differ by at most 1. A graph which admits a total face edge sum divisor cordial labeling is called a total face edge sum divisor cordial graph.

MAIN RESULTS

Theorem 1.1 The fan graph with n vertices is a total face edge sum divisor cordial graph for $n \geq 2$.

Proof. Let $v_1,v_2,\ldots,v_n,v_{n+1}$ be vertices, $e_1,e_2,\ldots,e_{2n-1}$ be edges and $f_1, f_2, \ldots, f_{n-1}$ interior faces of fan graph with n vertices, where $e_i = v_iv_{i+1}$ for $i = 1,2,\ldots,n-1$, $e_{n+i-1} = v_{n+1}v_i$ for $i = 1,2,\ldots,n$ and $f_i = v_iv_{n+i}v_{i+1}v_i$ for $i = 1,2,\ldots,n-1$.

Let G be the fan graph with n vertices.

Then $|V(G)| = n+1$, $|E(G)| = 2n-1$ and $|F(G)| = n-1$.

Define $f : E(G) \to \{1,2,\ldots,|E(G)|\}$ as follows.

For $n \equiv 1 \pmod 4$

$f(e_i) = i$, for $1 \leq i \leq 2n-3$

$f(e_{2n-2}) = 2n-1$

$f(e_{2n-1}) = 2n-2$

Then induced vertex labels are

$f^*(v_{2i-1}) = 1$, for $1 \leq i \leq \frac{n+1}{2}$

$f^*(v_{2i}) = 0$, for $1 \leq i \leq \frac{n-3}{2}$

$f^*(v_{n-2}) = 1$

$f^*(v_{n+1}) = 0$

Also the induced face labels are

$f^{**}(f_{2i-1}) = 1$, for $1 \leq i \leq \frac{n-3}{2}$

$f^{**}(f_{2i}) = 0$, for $1 \leq i \leq \frac{n-1}{2}$

$f^{**}(f_{n-2}) = 0$

In view of the above defined labeling pattern we have

$v_f(1) = \frac{n+3}{2}$, $v_f(0) = \frac{n-1}{2}$, $f_g(0) = \frac{n+1}{2}$ and $f_g(1) = \frac{n-3}{2}$.

Thus $(v_g(0)+f_g(0)) = (v_g(1)+f_g(1)) = n$.

Then $|\,(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))\,| \leq 1$.

For $n \equiv 2 \pmod 4$

$f(e_i) = i$, for $1 \leq i \leq 2n-1$

Then induced vertex labels are

$f^*(v_{2i-1}) = 0$, for $1 \leq i \leq \frac{n}{2}$

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$f^*(v_{2i}) = 1$, for $1 \le i \le \frac{n}{2}$

$f^*(v_{n+1}) = 0$

Also the induced face labels are

$f^{**}(f_{2i-1}) = 1$, for $1 \le i \le \frac{n}{2}$

$f^{**}(f_{2i}) = 0$, for $1 \le i \le \frac{n-2}{2}$

In view of the above defined labeling pattern we have

$v_f(0) = \frac{n+2}{2}$, $v_f(1) = \frac{n}{2}$, $f_g(1) = \frac{n}{2}$ and $f_g(0) = \frac{n-2}{2}$.

Thus $(v_g(0)+f_g(0)) = (v_g(1)+f_g(1)) = n$.

Then $|(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))| \le 1$.

For $n \equiv 3 (\text{mod } 4)$

$f(e_i) = i$, for $1 \le i \le 2n-1$

Then induced vertex labels are

$f^*(v_{2i-1}) = 1$, for $1 \le i \le \frac{n-1}{2}$

$f^*(v_{2i}) = 0$, for $1 \le i \le \frac{n-1}{2}$

$f^*(v_n) = 0$

$f^*(v_{n+1}) = 1$

Also the induced face labels are

$f^{**}(f_{2i-1}) = 1$, for $1 \le i \le \frac{n-1}{2}$

$f^{**}(f_{2i}) = 0$, for $1 \le i \le \frac{n-1}{2}$

In view of the above defined labeling pattern we have

$v_f(1) = v_f(0) = \frac{n+1}{2}$ and $f_g(0) = f_g(1) = \frac{n-1}{2}$.

Thus $(v_g(0)+f_g(0)) = (v_g(1)+f_g(1)) = n$.

Then $|(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))| \le 1$.

For $n \equiv 0 (\text{mod } 4)$

$f(e_i) = 2i-1$, for $1 \le i \le n-1$

$f(e_{n-1+i}) = 2i$, for $1 \le i \le n-1$

$f(e_{2n-1}) = 2n-1$

Then induced vertex labels are

$f^*(v_1) = 0$

$f^*(v_i) = 1$, for $2 \le i \le n$

$f^*(v_{n+1}) = 0$

Also the induced face labels are

$f^{**}(f_i) = 0$, for $1 \le i \le n-2$

$f^{**}(f_{n-1}) = 1$

In view of the above defined labeling pattern we have

$v_f(0) = 2$, $v_f(1) = n-1$, $f_g(1) = 1$, $f_g(0) = n-2$.

Thus $(v_g(0)+f_g(0)) = (v_g(1)+f_g(1)) = n$.

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Then $|(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))| \leq 1$.
Thus fan graph with n vertices is total face edge sum divisor cordial graph for $n \geq 2$.

Illustration 1.1 The fan graph with 6 vertices and its total face edge sum divisor cordial labeling is shown in figure 1.1.

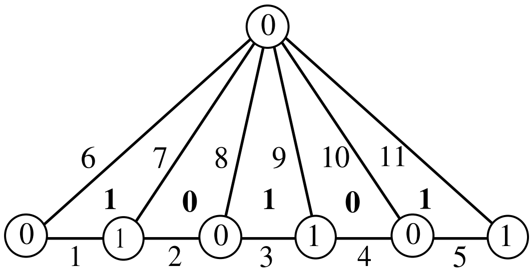

Figure 1.1

Theorem 1.2 The Wheel W_n is a total face edge sum divisor cordial graph for $n \geq 3$.

Proof. Let $v_0, v_1, v_2, \ldots, v_n$ be vertices, $e_1, e_2, \ldots, e_{2n}$ be edges and $f_1, f_2, \ldots, f_n$ interior faces of W_n, where $e_i = v_i v_{i+1}$ for $i = 1,2,\ldots,n-1$, $e_n = v_n v_1$, $e_{n+i} = v_0 v_i$ for $i = 1,2,\ldots,n$, $f_i = v_i v_0 v_{i+1} v_i$ for $i = 1,2,\ldots,n-1$ and $f_n = v_n v_0 v_1 v_n$.

Let G be the graph W_n.

Then $|V(G)| = n+1$, $|E(G)| = 2n$ and $|F(G)| = n$.

Define $f : E(G) \rightarrow \{1,2,\ldots,|E(G)|\}$ as follows.

For $n \equiv 1 \pmod 4$

$f(e_i) = i$, for $1 \leq i \leq 2n$

Then induced vertex labels are

$f^*(v_0) = 1$

$f^*(v_1) = 1$

$f^*(v_{2i}) = 1$, for $1 \leq i \leq \frac{n-1}{2}$

$f^*(v_{2i-1}) = 0$, for $2 \leq i \leq \frac{n+1}{2}$

Also the induced face labels are

$f^{**}(f_{2i-1}) = 1$, for $1 \leq i \leq \frac{n-1}{2}$

$f^{**}(f_{2i}) = 0$, for $1 \leq i \leq \frac{n-1}{2}$

$f^{**}(f_n) = 0$

In view of the above defined labeling pattern we have

$v_f(1) = \frac{n+3}{2}$, $v_f(0) = \frac{n-1}{2}$, $f_g(0) = \frac{n+1}{2}$ and $f_g(1) = \frac{n-1}{2}$.

Thus $(v_g(0)+f_g(0)) = n$ and $(v_g(1)+f_g(1)) = n+1$.

Then $|(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))| \leq 1$.

For $n \equiv 2 \pmod 4$

$f(e_i) = i$, for $1 \leq i \leq 2n$

Then induced vertex labels are

$f^*(v_0) = 0$

$f^*(v_{2i-1}) = 1$, for $1 \leq i \leq \frac{n}{2}$

$f^*(v_{2i}) = 0$, for $1 \leq i \leq \frac{n}{2}$

Also the induced face labels are

$f^{**}(f_{2i-1}) = 1$, for $1 \leq i \leq \frac{n}{2}$

$f^{**}(f_{2i}) = 0$, for $1 \leq i \leq \frac{n}{2}$

In view of the above defined labeling pattern we have

$v_f(0) = \frac{n+2}{2}$, $v_f(1) = \frac{n}{2}$, $f_g(1) = \frac{n}{2}$ and $f_g(0) = \frac{n}{2}$.

Thus $(v_g(0)+f_g(0)) = n+1$ and $(v_g(1)+f_g(1)) = n$.

Then $|(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))| \leq 1$.

For $n \equiv 3 (\mathrm{mod}\ 4)$

$f(e_i) = i$, for $1 \leq i \leq 2n$

Then induced vertex labels are

$f^*(v_0) = 0$

$f^*(v_1) = 1$

$f^*(v_{2i}) = 1$, for $1 \leq i \leq \frac{n-1}{2}$

$f^*(v_{2i-1}) = 0$, for $2 \leq i \leq \frac{n+1}{2}$

Also the induced face labels are

$f^{**}(f_{2i-1}) = 1$, for $1 \leq i \leq \frac{n-1}{2}$

$f^{**}(f_{2i}) = 0$, for $1 \leq i \leq \frac{n-1}{2}$

$f^{**}(f_n) = 0$

In view of the above defined labeling pattern we have

$v_f(1) = \frac{n+1}{2}$, $v_f(0) = \frac{n+1}{2}$, $f_g(0) = \frac{n+1}{2}$ and $f_g(1) = \frac{n-1}{2}$.

Thus $(v_g(0)+f_g(0)) = n+1$ and $(v_g(1)+f_g(1)) = n$.

Then $|(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))| \leq 1$.

For $n \equiv 0 (\mathrm{mod}\ 4)$

$f(e_i) = i$, for $1 \leq i \leq 2n$

Then induced vertex labels are

$f^*(v_0) = 1$

$f^*(v_{2i-1}) = 1$, for $1 \leq i \leq \frac{n}{2}$

$f^*(v_{2i}) = 0$, for $1 \leq i \leq \frac{n}{2}$

Also the induced face labels are

$f^{**}(f_{2i-1}) = 1$, for $1 \leq i \leq \frac{n}{2}$

$f^{**}(f_{2i}) = 0$, for $1 \leq i \leq \frac{n}{2}$

In view of the above defined labeling pattern we have

$v_f(0) = \frac{n}{2}$, $v_f(1) = \frac{n+2}{2}$, $f_g(1) = \frac{n}{2}$ and $f_g(0) = \frac{n}{2}$.

Thus $(v_g(0)+f_g(0)) = n$ and $(v_g(1)+f_g(1)) = n+1$.

Then $|(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))| \leq 1$.

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Thus W_n is total face edge sum divisor cordial graph for $n \geq 2$.

Illustration 1.2 The Wheel graph W_5 and its total face edge sum divisor cordial labeling is shown in figure 1.2.

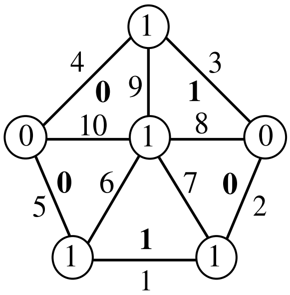

Figure 1.2

Theorem 1.3 The gear graph G_n is a total face edge sum divisor cordial graph for $n \geq 3$.

Proof. Let $v_0,v_1,v_2,...,v_{2n}$ be vertices, $e_1,e_2,...,e_{3n}$ be edges and $f_1, f_2, ..., f_n$ interior faces of W_n, where $e_i = v_iv_{i+1}$ for $i = 1,2,\ldots,2n-1$, $e_{2n} = v_{2n}v_1$, $e_{2n+i} = v_0v_{2i-1}$ for $i = 1,2,\ldots,n$, $f_i = v_0v_{2i-1}v_{2i}v_{2i+1}v_0$ for $i = 1,2,\ldots,n-1$ and $f_n = v_0v_{2n-1}v_{2n}v_1v_0$.

Let G be the graph W_n. Then $|V(G)| = 2n+1$, $|E(G)| = 3n$ and $|F(G)| = n$.

Define $f : E(G) \rightarrow \{1,2,\ldots, |E(G)|\}$ as follows.

For $n \equiv 0 \pmod 4$

$f(e_i) = i$, for $1 \leq i \leq 3n$

Then induced vertex labels are

$f^*(v_0) = 1$

$f^*(v_{2i}) = 0$, for $1 \leq i \leq n$

$f^*(v_{1+4(i-1)}) = 1$, for $1 \leq i \leq \frac{n}{2}$

$f^*(v_{3+4(i-1)}) = 0$, for $1 \leq i \leq \frac{n}{2}$

Also the induced face labels are

$f^{**}(f_i) = 1$, for $1 \leq i \leq n$

In view of the above defined labeling pattern we have

$v_f(1) = \frac{n}{2} + 1$, $v_f(0) = \frac{3n}{2}$, $f_g(0) = 0$ and $f_g(1) = n$.

Thus $(v_g(0)+f_g(0)) = \frac{3n}{2}$ and $(v_g(1)+f_g(1)) = \frac{3n}{2} + 1$.

Then $|\,(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))\,| \leq 1$.

For $n \equiv 2 \pmod 4$

$f(e_i) = i$, for $1 \leq i \leq 3n$

Then induced vertex labels are

$f^*(v_0) = 0$

$f^*(v_{2i}) = 0$, for $1 \leq i \leq n$

$f^*(v_{1+4(i-1)}) = 1$, for $1 \leq i \leq \frac{n}{2}$

$f^*(v_{3+4(i-1)}) = 0$, for $1 \leq i \leq \frac{n}{2}$

Also the induced face labels are

$f^{**}(f_i) = 1$, for $1 \leq i \leq n$

In view of the above defined labeling pattern we have

$v_f(1) = \frac{n}{2}$, $v_f(0) = \frac{3n}{2} + 1$, $f_g(0) = 0$ and $f_g(1) = n$.

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Thus $(v_g(0)+f_g(0)) = \frac{3n}{2}+1$ and $(v_g(1)+f_g(1)) = \frac{3n}{2}$.

Then $|(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))| \le 1$.

For $n \equiv 3 \pmod 4$

$f(e_i) = i$, for $1 \le i \le 3n$

Then induced vertex labels are

$f^*(v_0) = 1$

$f^*(v_{2i}) = 0$, for $1 \le i \le n$

$f^*(v_{1+4(i-1)}) = 1$, for $1 \le i \le \frac{n+1}{2}$

$f^*(v_{3+4(i-1)}) = 0$, for $1 \le i \le \frac{n-1}{2}$

Also the induced face labels are

$f^{**}(f_i) = 1$, for $1 \le i \le n-1$

$f^{**}(f_n) = 0$

In view of the above defined labeling pattern we have

$v_f(1) = \frac{n+3}{2}$, $v_f(0) = \frac{3n-1}{2}$, $f_g(0) = 1$ and $f_g(1) = n-1$.

Thus $(v_g(0)+f_g(0)) = (v_g(1)+f_g(1)) = \frac{3n+1}{2}$.

Then $|(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))| \le 1$.

For $n \equiv 1 \pmod 4$

$f(e_i) = i$, for $1 \le i \le 2n-1$

$f(e_{2n}) = 2n+1$

$f(e_{2n+1}) = 2n$

$f(e_i) = i$, for $2n+2 \le i \le 3n$

Then induced vertex labels are

$f^*(v_0) = 1$

$f^*(v_{2i}) = 0$, for $1 \le i \le n-1$

$f^*(v_{2n}) = 1$

$f^*(v_{1+4(i-1)}) = 1$, for $1 \le i \le \frac{n+1}{2}$

$f^*(v_{3+4(i-1)}) = 0$, for $1 \le i \le \frac{n-1}{2}$

Also the induced face labels are

$f^{**}(f_1) = 0$

$f^{**}(f_i) = 1$, for $2 \le i \le n-1$

$f^{**}(f_n) = 0$

In view of the above defined labeling pattern we have

$v_f(1) = \frac{n+5}{2}$, $v_f(0) = \frac{3n-3}{2}$, $f_g(0) = 2$ and $f_g(1) = n-2$.

Thus $(v_g(0)+f_g(0)) = (v_g(1)+f_g(1)) = \frac{3n+1}{2}$.

Then $|(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))| \le 1$.

Thus G_n is total face edge sum divisor cordial graph for $n \ge 2$.

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Illustration 1.3 The gear graph G_6 and its total face edge sum divisor cordial labeling is shown in figure 1.3.

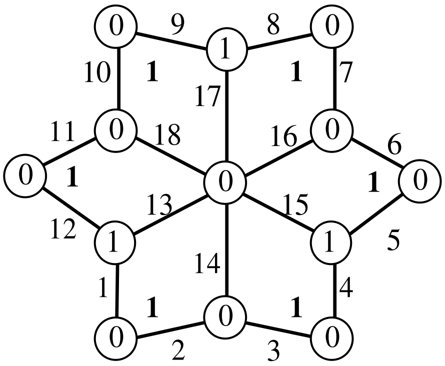

Figure 1.3

Theorem 1.4 The Closed Helm CH_n is a total face edge sum divisor cordial graph for $n \geq 3$.

Proof. Let $v_0,v_1,v_2,...,v_{2n}$ be vertices, $e_1,e_2,...,e_{4n}$ be edges and f_1, f_2, ..., f_{2n} interior faces of CH_n, where $e_i = v_0v_i$ for $i = 1,2,\ldots,n$, $e_{n+i} = v_iv_{i+1}$ for $i = 1,2,\ldots,n-1$, $e_{2n} = v_nv_1$, $e_{2n+i} = v_iv_{n+i}$ for $i = 1,2,\ldots,n$, $e_{3n+i} = v_{n+i}v_{n+i+1}$ for $i = 1,2,\ldots,n-1$, $e_{4n} = v_{2n}v_{n+1}$, $f_i = v_iv_0v_{i+1}v_i$ for $i = 1,2,\ldots,n-1$, $f_n = v_nv_0v_1v_n$, $f_{n+i} = v_iv_{n+i}v_{n+i+1}v_{i+1}v_i$ for $i = 1,2,\ldots,n-1$ and $f_{2n} = v_nv_{2n}v_{n+1}v_1v_n$.

Let G be the graph CH_n. Then $|V(G)| = 2n+1$, $|E(G)| = 4n$ and $|F(G)| = 2n$.

Define $f : E(G) \to \{1,2,\ldots, |E(G)|\}$ as follows.

$f(e_i) = 2i-1$, for $1 \leq i \leq n$

$f(e_{n+i}) = 2i$, for $1 \leq i \leq n$

$f(e_{2n+i}) = 2n+2i$, for $1 \leq i \leq n$

$f(e_{3n+i}) = 2n+2i-1$, for $1 \leq i \leq n$

Then induced vertex labels are

$f^*(v_0) = 1$, if n is odd

$f^*(v_0) = 0$, if n is even

$f^*(v_i) = 1$, for $1 \leq i \leq n$

$f^*(v_{n+i}) = 0$, for $1 \leq i \leq n$

Also the induced face labels are

$f^{**}(f_i) = 1$, for $1 \leq i \leq n$

$f^{**}(f_{n+i}) = 0$, for $1 \leq i \leq n$

In view of the above defined labeling pattern we have

$v_f(1) = n$, $v_f(0) = n+1$, $f_g(0) = n$ and $f_g(1) = n$, when n is odd

Thus $(v_g(0)+f_g(0)) = n+1$ and $(v_g(1)+f_g(1)) = n$.

Then $|\,(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))\,| \leq 1$.

$v_f(1) = n+1$, $v_f(0) = n$, $f_g(0) = n$ and $f_g(1) = n$, when n is odd

Thus $(v_g(0)+f_g(0)) = n$ and $(v_g(1)+f_g(1)) = n+1$.

Then $|\,(v_g(0)+f_g(0)) - (v_g(1)+f_g(1))\,| \leq 1$.

Thus CH_n is total face edge sum divisor cordial graph for $n \geq 2$.

Illustration 1.4 The Closed Helm CH_4 and its total face edge sum divisor cordial labeling is shown in figure 1.4.

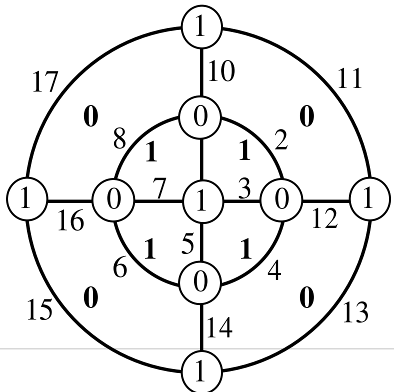

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Figure 1.4

Conclusion

In this paper, we introduced total face edge sum divisor cordial labeling of graph and investigated the total face edge sum divisor cordial labeling of fan graph with n vertices, wheel graph, gear graph and closed helm graph.

References

[1]. I. Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, Ars Combinatoria, 23(1987), 201-207.

[2]. J.A. Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 25(2022), #DS6.

[3]. F. Harary, Graph theory, Addison Wesley, Reading, Massachusetts, 1972.

[4]. P. Lawrence Rozario Raj and R. Lawrence Joseph Manoharan, Face and Total face edge product cordial graphs, International Journal of Mathematics Trends and Technology, 15(2) (2015), 136-149.

[5]. A. Lourdusamy and F. Patrick, Sum divisor cordial graphs, Proyecciones Journal of Mathematics, 35(1) (2016), 119-136.

[6]. M. Mohamed Sheriff and G. Vijayalakshmi, Face Sum Divisor Cordial Graphs, Intern. J. Fuzzy Mathematical Archive, 15(2) (2018), 197-204.

[7]. R. Varatharajan, S. Navanaeethakrishnan and K. Nagarajan, Divisor cordial graphs, International J. Math. Combin., 4(2011), 15-25.

[8]. G. Vijayalakshmi and M. Mohamed Sheriff, Edge Sum Divisor Cordial Graphs, Communicated.

[9]. G. Vijayalakshmi and M. Mohamed Sheriff, Face Edge Sum Divisor Cordial Graphs, Communicated.

[10]. R. Yilmaz and I. Cahit, E-Cordial Graphs, Ars Combinatoria 46 (1997), 251-266.