Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 8, Dec 2022

Total Coloring and Intuitionistic Fuzzy Face Magic Labeling of Fan Graph

Naveen.L[1], Uma.N[2]

1.Assistant Professor, Department of Mathematics, Dr.SNS Rajalakshmi College of Arts and Science, Coimbatore, TamilNadu, India.
Email :lnaveen24@gmail.com

2.Associate Professor, Department of Mathematics, Sri Ramakrishna College of Arts and Science, Coimbatore, TamilNadu, India.
Email :umasnr@gmail.com

Abstract:

Use of fuzzy-graphical methods in dealing with ambiguity and vague notions is very natural. Fuzzy graph theory has a large number of applications in modeling various real-time systems where the level of information inherent in the system varies with different levels of precision. Magic-type labelings are useful when a check sum is required or a look up table has to be avoided. In this paper, total coloring of a planar graph, (i.e.) a certain type of planar graph, fan graph is discussed. An algorithm is discussed to build up an Intuitionistic Fuzzy Face Magic Fan Graph.

Keywords: Fan graph, Total Coloring, Magic Graphs, Fuzzy Face-Magic Labeling, Intuitionistic fuzzy planar graph.

1. INTRODUCTION

Lotfi A. Zadeh [1], the inventor of fuzzy sets that have been impressively implemented to clear up the multitudinous real lifestyles choice of difficulties, which is ordinarily erratic, incomplete, precise, uncertain. A fuzzy set is a generalization of a crisp set, in which the elements of the set are divided into membership degrees with an upper bound of 1 and a lower bound of 0. In the fuzzy set, each object's membership grade isn't identical as probability, willingly it establishes each object's belonging grade value that has a single value inside the range of [0, 1]. The basic format of the fuzzy graph and some of its properties expanded through **Asriel Rosenfield** [2] who looked at fuzzy relations on fuzzy sets in 1975.

The ultimate main goal of fuzzy models is to diminish the existing error value in models that cannot be used comprehensively in several fields with long-established mathematical models, especially in the fields of scientific modeling, telecommunications and so on. The notion of connectivity in fuzzy graphs began with **R.T. Yeh and S.Y. Bang** [3]. An ordered pair of a set of vertices V and a set of edges E is known as a crisp graph. Graph theory has many applications in solving various problems of several domains, including networking, communication, data mining, clustering, image capturing, image segmentation, planning, and scheduling. However, in some situations, certain aspects of a graph-theoretical system may be uncertain.

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 8, Dec 2022

Additionally, the cardinality of the set of vertices and the set of edges is called the order and size of the graph. A bijection f in a crisp graph $G = (V, E)$ mapping from $V \cup E$ to N which assigns a completely unique natural number to each vertex and/or edge is known as a Labeling. **B.M. Stewart** [5] launched a new type of graph in 1964 which is a magic graph. He illustrated a graph that is magic when it has an edge-labeling, internal set of real numbers, such that the sum of an edge's labels and their two end points is equal to a constant. If the sum of the labels related to the vertex is consistent, the impartial of the selection of vertex is called vertex-magic. **B.M.Stewart** [5] who implemented super magic into the idea of a magic graph. In the fuzzy magic graph, some values for edges or vertices are commonly used. But in fuzzy magic labeling, the vertices and the edges are distinct. **Gani et al.,** who invented the idea of fuzzy labeling and the novel characteristics of fuzzy labeling graphs [4] and fuzzy magic graphs. **Mordeson et.al** [7] pioneered the notion of "fuzzy labeling" and "fuzzy magic labeling graphs". Additionally various authors discussed the numerous forms of magic graphs and its labeling.

2. PRELIMINARIES

Definition 2.1

In graph theory, a planar graph is a graph that can be embedded in the plane. In other words, it can be drawn in such a way that no edges cross each other. Such a graph is called a **planar graph** or planar embedding of the graph.

Definition 2.2

A **fuzzy graph** $G = (\sigma, \mu)$ is a pair of functions $\sigma: V \rightarrow [0, 1]$ and $\mu: V \times V \rightarrow [0, 1]$, where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 2.3

The graph $G = (V, E)$ is a crisp graph, a **coloring function** is a mapping C: V$(G) \rightarrow N$(where N is set of positive integers) such that $C(u) \neq C\,(v)$ if u and v are adjacent in G.

The graph $G = (V, E)$ is a crisp graph, a **k-coloring function** is a mapping $C^k: V(G) \rightarrow \{1,2, \dots, k\}$ such that $C^k(u) \neq C^k(v)$ if u and v are adjacent in G. A graph G is k-colorable if it admits k-coloring. The **chromatic number** (G), of a graph G is the minimum k for whichG is k-colorable.

Definition 2.4

If $G = (V, \mu)$ is such a fuzzy graph where $V = \{1,2,3, \dots, n\}$ and μ is a fuzzy number on the set of all subsets of $V \times V$. Assume $I = A \cup \{0\}$ where $A = \{\alpha_1 < \alpha_2 < \cdots < \alpha_k\}$ is the fundamental set (level set) of G. For each $\alpha \in I$, G_α denote the crisp graph $G_\alpha = (V, E_\alpha)$ where $\{(i, j)/\ 1 \leq i < j \leq n, (i, j) \geq \alpha\}$ and $\chi_\alpha = \chi\,(G_\alpha)$, denote the chromatic number of crisp graph.

By this definition the **chromatic number of the fuzzy graph** G is the fuzzy number $\chi\,(G) = \{(i, v(i))/i \in X\}$ where $v(i) = \max\{\alpha \in I/i \in A_\alpha\}$ and $A_\alpha = \{1, \dots, \chi_\alpha\}$.

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Definition 2.5

A graph $G = (V, \mu, \sigma)$ is said to be a fuzzy labelling graph if $\mu : V \rightarrow [0,1]$ and $\sigma : V \times V \rightarrow [0,1]$, is bijective such that the membership value of edges and vertices are distinct and $\sigma(x, y) \leq \mu(x) \wedge \mu(y)$ for all $x, y \in V$.

3. FAN GRAPH

Definition 3.1

A **fan graph** $F_{m,n}$is defined as the graph join $K_m + P_n$, where K_m is the empty graph on m vertices and P_n is the path graph on n vertices. Here, as an example Fan graph $F_{1,5}$ is given below (figure 1).

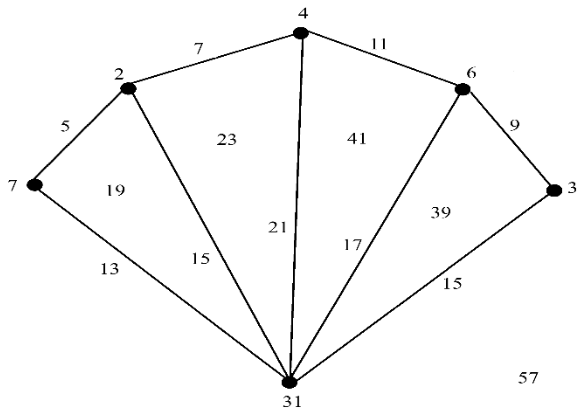

Figure 1. Fan Graph $F_{1,5}$

Definition 3.2

A fan in a fuzzy graph is called a **fuzzy fan graph** $F_{1,n}$ is defined as a graph $K_1$with a label U attaching all the verticdes v_j of P_n through the edges e_j such the $\mu(U, v_i) > 0$ and $\mu(v_i, v_{i+1}) < 1$ and$|e_i| > 1$, where$1 \leq j \leq n$.(Figure 2)

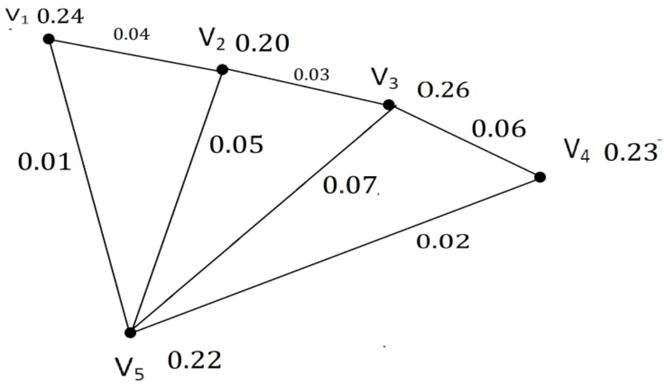

Figure 2. Fuzzy fan graph $F_{1,4}$

IJFANS International Journal of Food And Nutritional Sciences
Official Publication of International Association of Food and Nutrition Scientists

ISSN PRINT 2319 1775 Online 2320 7876

 UGC CARE Listed (Group -I) Journal Volume 11, Iss 8, Dec 2022

Definition 3.3

Graph coloring is to assign a color to elements (vertex, edge, and face) of graph such that two adjacent elements have different colors, then the **total coloring of a planar graph** is defined as

- **(i)** **Vertex coloring** of a planar graph is a way of coloring the vertices of a graph such that no two adjacent vertices are of the same color.
- **(ii)** **Edge coloring** of a planar graph is an assignment of colors to the edges of the graph so that no two incident edges have the same color.
- **(iii)** **Face coloring** of a planar graph assigns a color to each face so that no two faces that share an edge have the same color.

Total coloring of a Planar graph is an assignment of colors to all the elements of vertices, edges and faces such that (i) no two adjacent vertices are of the same color (ii) no two incident edges have the same color and (iii) no two faces that share an edge have the same color.

Definition 3.4

The **chromatic number of a graph** is the smallest numbers of color needed in graph coloring.

Definition 3.5

The **fuzzy vertex chromatic number** of a fuzzy graph G is the minimum number of colors needed for a proper fuzzy vertex coloring of G. It is denoted by $\boldsymbol{\chi(G)}$.

Definition 3.6

The **fuzzy edge chromatic number** of a fuzzy graph G is the minimum number of colors needed for a proper fuzzy edge coloring of G. It is denoted by $\boldsymbol{\chi'(G)}$.

Definition 3.7

The **fuzzy face chromatic number** of a fuzzy graph G is the minimum number of colors needed for a proper fuzzy face coloring of G. It is denoted by $\boldsymbol{\chi''(G)}$.

4. TOTAL COLORING OF FAN GRAPH

4.1 Procedure for the total coloring of a planar graph

The following are the steps involved to color the vertices, edges and faces of the planar graph.

Step 1: Exhibit the vertices of the graph in the same order.

Step 2: Choose the first vertex and provide it with the first color for vertex.

Step 3: Choose the next vertex and color it with the lowest numbered color that has not been colored on any vertices adjacent to it. If all the adjacent vertices are colored with this color, assign a new color to it.

ISSN PRINT 2319 1775 Online 2320 7876

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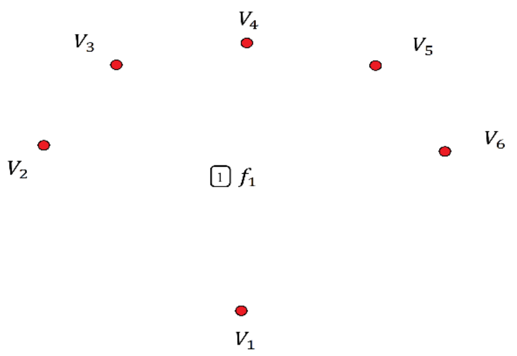
Step 4: Connect the two vertices with an edge and provide it with the first color for edge. If new edge is attained color it with the lowest numbered color that has not been colored on any edges adjacent to it. If all the adjacent edges are colored with this color, assign a new color to it.

Step 5: Repeat Step 3 and Step 4 until the graph configure a face. If new face is attained color it with the lowest numbered color that has not been colored on any faces adjacent to it. If all the adjacent faces are colored with this color, assign a new color to it.

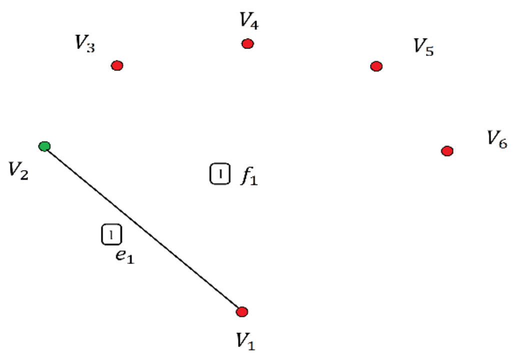
Step 6: Repeat the above steps until all the vertices, edges and faces are colored.

Example 4.1.1:

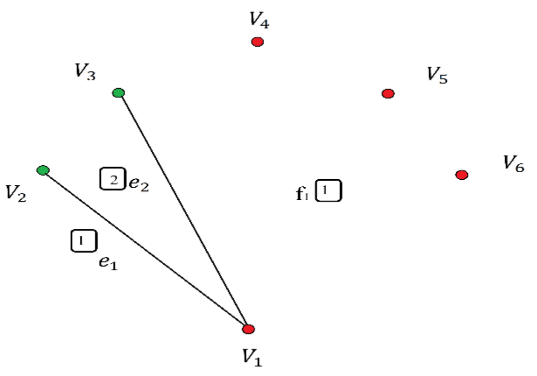
Here, we considered the specific type of planar graph, say **Fan Graph $F_{1,5}$** as an example to perform the total coloring.



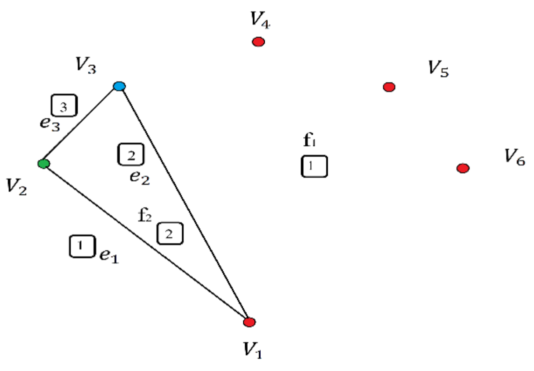
$G_{1,0}$; $\chi_{1,0} = 1$; $\chi'_{1,0} = 0$; $\chi''_{1,0} = 1$



G_{e_1} ; $\chi_{e_1} = 2$; $\chi'_{e_1} = 1$; $\chi''_{e_1} = 1$



G_{e_2} ; $\chi_{e_2} = 2$; $\chi'_{e_2} = 2$; $\chi''_{e_2} = 1$



G_{e_3} ; $\chi_{e_3} = 3$; $\chi'_{e_3} = 3$; $\chi''_{e_3} = 2$

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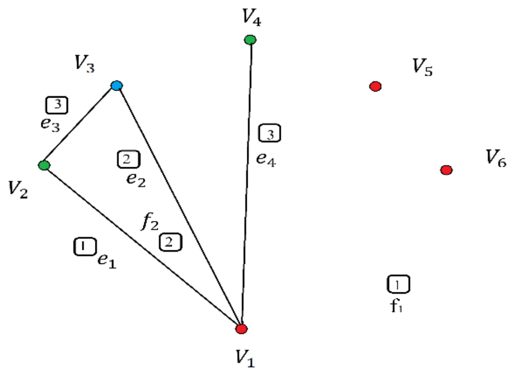
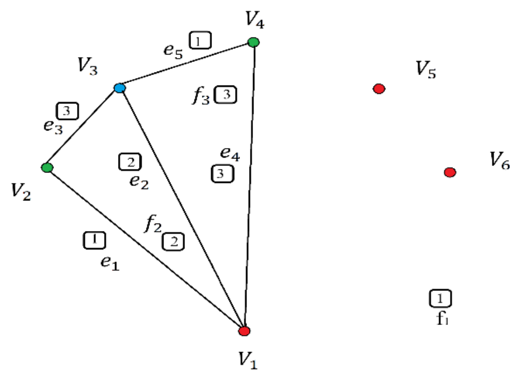
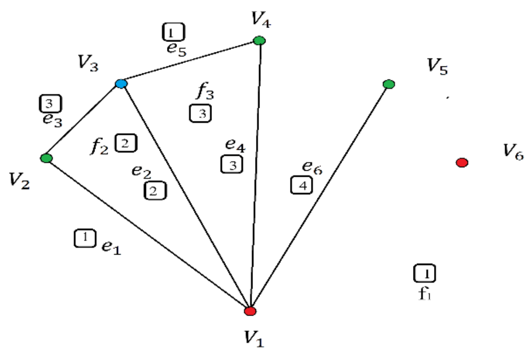


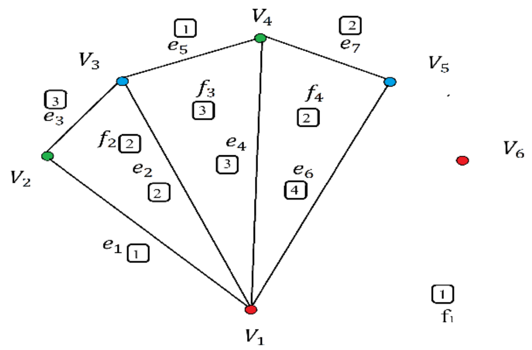
G_{e_4} ; $\chi_{e_4} = 3$; $\chi'_{e_4} = 3$; $\chi''_{e_4} = 2$



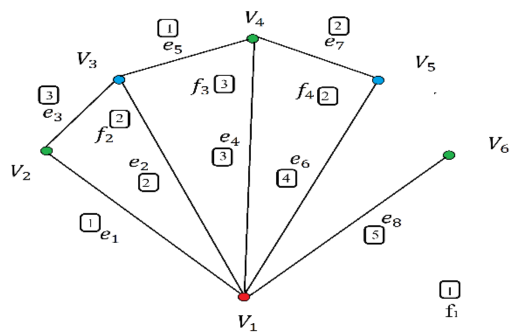
G_{e_5} ; $\chi_{e_5} = 3$; $\chi'_{e_5} = 3$; $\chi''_{e_5} = 3$



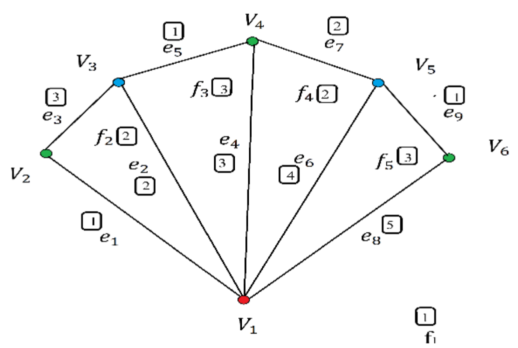
G_{e_6} ; $\chi_{e_6} = 3$; $\chi'_{e_6} = 4$; $\chi''_{e_6} = 3$



G_{e_7} ; $\chi_{e_7} = 3$; $\chi'_{e_7} = 4$; $\chi''_{e_7} = 3$



G_{e_8} ; $\chi_{e_8} = 3$; $\chi'_{e_8} = 5$; $\chi''_{e_8} = 3$



G_{e_9} ; $\chi_{e_9} = 3$; $\chi'_{e_9} = 5$; $\chi''_{e_9} = 3$

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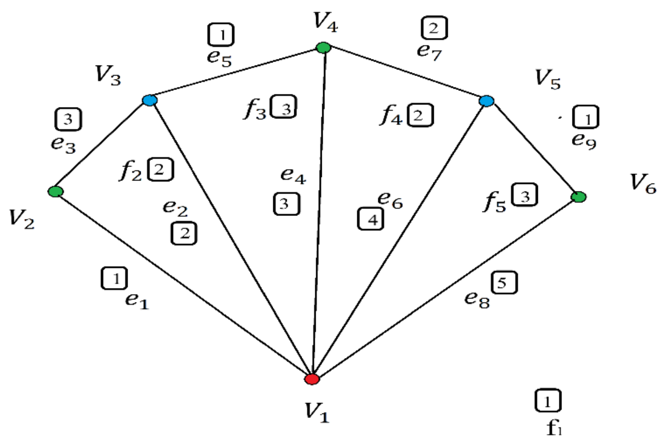
Thus, the total (vertex, edge and edge) coloring of the fan graph is shown in the following figure 3.



$$F_{1,5} \;\; ; \chi_{F_{1,5}} = 3 \, ; \, \chi'_{F_{1,5}} = 5 \; ; \, \chi''_{F_{1,5}} = 3$$

Figure 3. Total coloring of Fan graph $F_{1,5}$

5. FUZZY FACE MAGIC LABELING OF FAN GRAPHS

Definition 5.1

A fuzzy graph G=(V, μ, σ) is called a **fuzzy magic graph** if there are two bijective functions μ:V → [0,1] and $\sigma : V \times V \rightarrow [0,1]$, with restricted the conditions σ (u,v) < μ (u) + μ (v) and μ (u)+ σ (uv)+ μ (v) = m(G) ≤ 1 where , m(G) is a real constant for all u,v∈ G

(i) A fuzzy labelling graph is said to be a **fuzzy vertex magic graph** if $\mu\,(u) + \sigma\,(uv) + \mu\,(v)$ has a same magic value for all $u, v \in V$ which is denoted as $m_0(G)$.

(ii) A fuzzy labelling graph is said to be a **fuzzy edge magic graph** if $\mu\,(u) + \sigma\,(uv) + \mu\,(v)$ has a same magic value for all $u, v \in V$ which is denoted as $M_0(G)$.

Definition 5.2

A fuzzy labelling graph is said to be a **fuzzy face magic graph** if the weights of all s-sided faces are all equal to some constant ω(s) then the labeling is considered to be face-magic where, the weight of a face is the sum of the face's own label and the labels of vertices and edges enclosing the face.

Definition 5.3

The **Fan graph** $F_n \cong P_n + K_1$ is a graph with the vertex set $V = \{v_i : 1 \le i \le n\} \cup \{h\}$ where h is referred to hub, edge set $E = R \cup S$ where $R = \{v_i v_{i+1} : 1 \le i \le n - 1\}$are referred to as rims and $S = \{hv_i : 1 \le i \le n\}$ are referred to as spokes and a set of face, say F.

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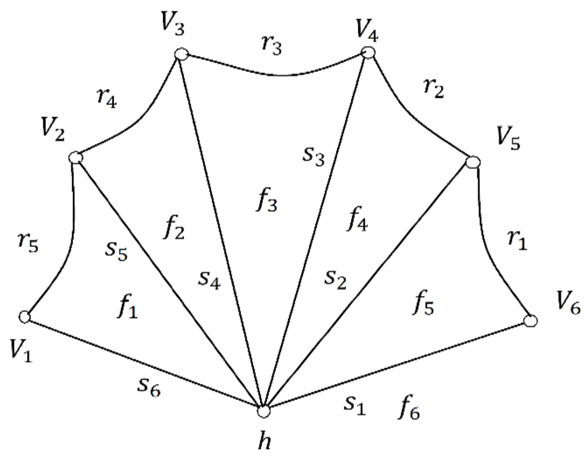

Figure 4. Fan Graph $F_{1,6}$ with the representations as vertex, hub, spoke, rim and face.

In the above figure 4,

(i) The vertex set is $V = \{v_1, v_2, \ldots . v_5, h\}$ where h is referred as hub.

(ii) The edge set is $E = S \cup R$ where $S = \{s_1, s_2, \ldots . s_6\}$ where S is referred as a set of spokes and $R = \{r_1, r_2, \ldots . , r_5\}$ where R is referred as a set of rims.

(iii) The face set is $F = \{f_1, f_2, \ldots, f_6\}$.

5.4 Construction of Fuzzy Face Magic Fan graph

Step 1: Begin the labeling with the blank fan graph.

Step 2: Label the n-interior faces consecutively across the fan. The exterior face label can be assigned freely since it does not affect the weight of the face.

Step 3: Observe the face with the highest weight. Calculate weight needed for each rim (to be balanced).

Step 4: Label the rims in complement with the random value, say w. (i.e.) Assign the value to rim with respect to the calculated weight of face to be balanced and the random value, w.

Step 5: Label the remaining edges and vertices in compliment pairs.

Step 6: Now, label the left label (hub) freely since it is neighbour with every face.

Note: The hub and exterior face labels can be assigned freely but it is needed to be noticed, such that sum of each face weight must be less than or equal to 1.(≤ 1)

Example : Here, we considered the specific type of planar graph, say Fan Graph $F_{1,5}$ as an example to perform the total coloring.

Step 1: Beginning with blank $F_{1,5}$. (Figure 5.1)

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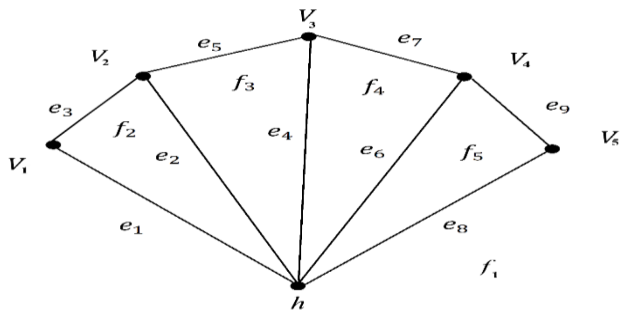

Figure 5.1 Blank $F_{1,5}$

Step 2: Choose any random face labels. (Figure 5.2)

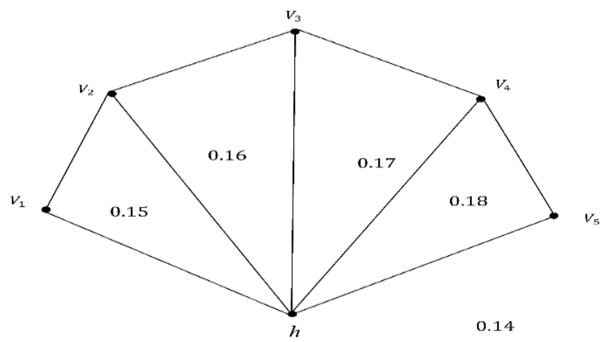

Figure 5.2 Face Labels

Step 3: Observe the face with the highest weight. Calculate weight needed for each rim (to be balanced).Face with highest face: f_5= 0.18
Weight needed to be balanced: $f_2 = 0.03$; $f_3 = 0.02$; $f_4 = 0.01$; $f_5 = 0$

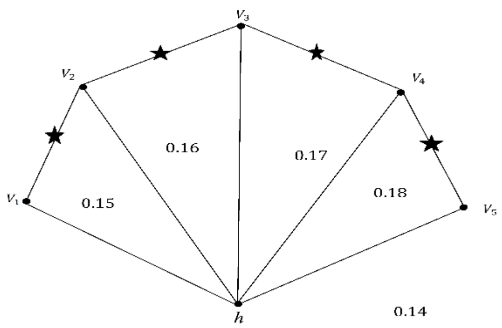

Figure 5.3 Weight of each face so far

Step 4: Labeling rims with the random value, w.
(w + weight needed for each rim to be balanced)
Here, $w = 0.38$.

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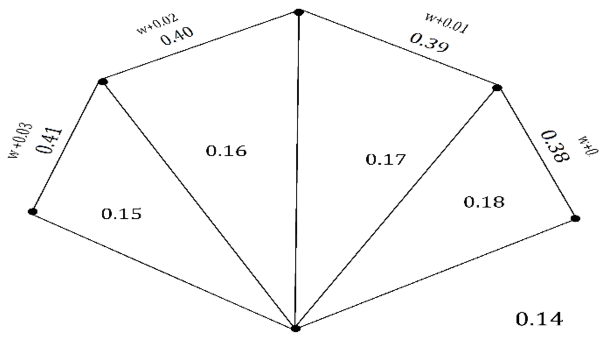

Figure 5.4 Weight needed for each rim.(Labeling rims).

Now, each face weights the same, say 0.56.

Step 5: Now each face needed to be added with equal weight. (Figure 5.5)

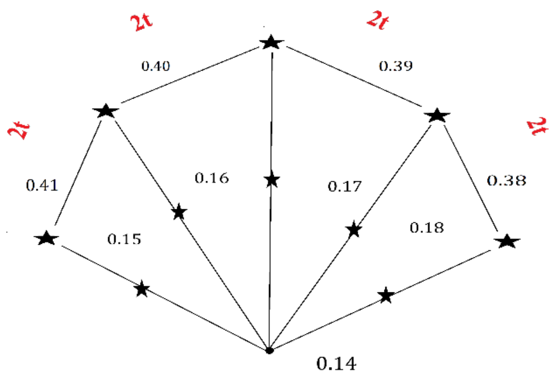

Figure 5.5 Each face needs equal weight added.

Each edge-vertex pair should equal to t, where t is a random value.

Step 6: Label in compliments with the help of random value, say t. Here, $t = 0.15$.
And update the face weights.

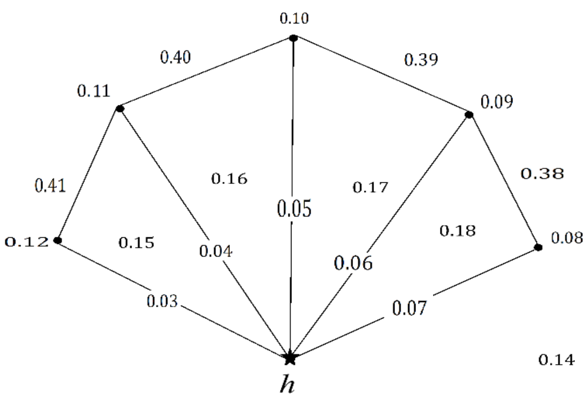

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Figure 5.6 Label in compliments and update the weights.

Now, each face weights the same, say 0.86.

Step 7: Now label the left labels (hub), such that sum of each face weight ≤ 1. since, it is a Fuzzy Graph.

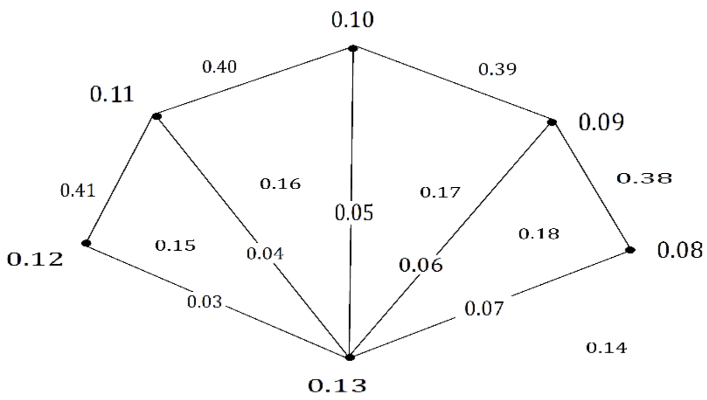

Figure 5.7 Labeling hub.

Here, the weight of the each face is 0.99. (≤ 1)

6. INTUITIONISTIC FUZZY FACE MAGIC LABELING OF FAN GRAPHS

Definition 6.1

Intuitionistic fuzzy sets are the sets whose elements have degrees of membership and non-membership. Intuitionistic fuzzy sets have been introduced by **Krassimir Atanassov** (1983) as an extension of **Lotfi Zadeh's** [1] notion of fuzzy set, which itself extends the classical notion of a set.

The theory of intuitionistic fuzzy sets further extends both concepts by allowing the assessment of the elements by two functions: μ for membership and ν for non-membership, which belong to the real unit interval [0, 1] and whose sum belongs to the same interval, as well.

Intuitionistic fuzzy sets generalize fuzzy sets, since the indicator functions of fuzzy sets are special cases of the membership and non-membership functions μ and ν of intuitionistic fuzzy sets, in the case when the strict equality exists: $\nu=1-\mu$, i.e. the non-membership function fully complements the membership function to 1, not leaving room for any uncertainty.

Definition 6.2

Let X be a fixed universe set. Let A be a subset of X. Then **Intuitionistic Fuzzy set** A* in a set X is defined as an object of the form

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$A^* = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$ where $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 6.3

We know that, the intuitionistic fuzzy set is of the form,

$A^* = \{(x, \mu_A(x), (x)) / x \in X\}$ where $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$, for every $x \in X$, $0 \leq (x) + \nu_A(x) \leq 1$. At first, let us define the degree of membership of the element for the fuzzy face magic labeling of fan graph.(i.e.) for membership function, let us consider the above example 5.4.2.

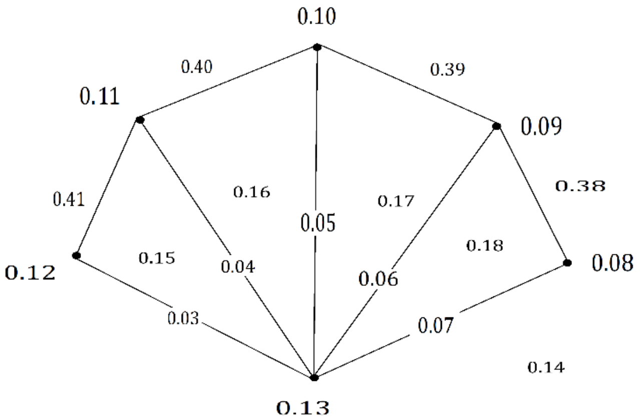

Figure 6. Membership function of fuzzy face magic labeling of the fan graph $F_{1.5}$

The weight of each face of Membership function of fuzzy face magic labeling of the fan graph is 0.99. Similarly, consider another example of fuzzy face magic labeling of the fan graph for non-membership function. Thus, we get the following graph. (Figure 7.)

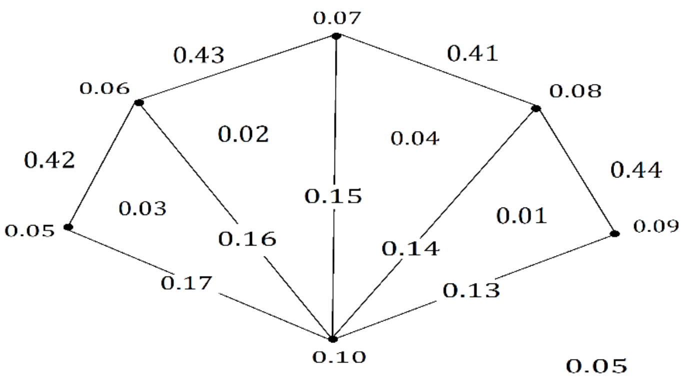

Figure 7. Non-membership function of fuzzy face magic labeling of the fan graph $F_{1.5}$

The weight of each face of Non-membership function of fuzzy face magic labeling of the fan graph is also 0.99. Thus, **the intuitionistic fuzzy face magic labeling of the fan graph $F_{1.5}$** with the above obtained membership and non-membership function id given as, (figure 8).

ISSN PRINT 2319 1775 Online 2320 7876

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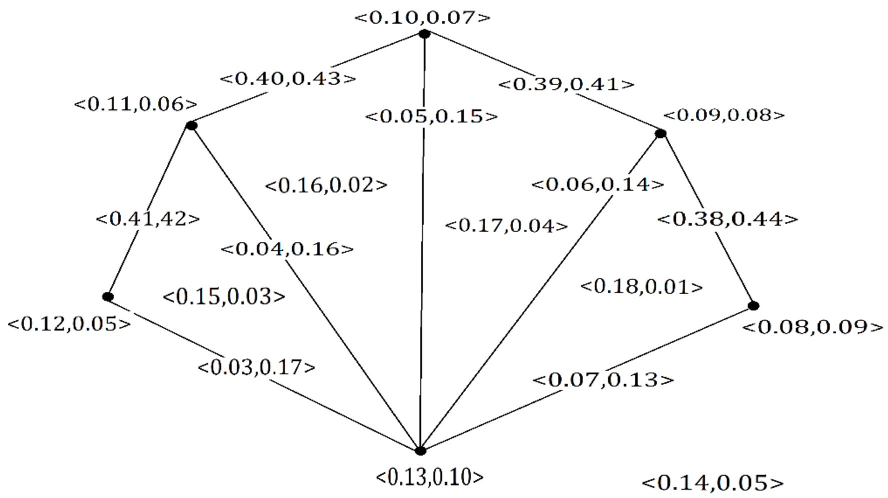

Figure 8. Intuitionistic fuzzy face magic labeling of the fan graph $F_{1.5}$

Here, the fuzzy face magic value of the intuitionistic fan graph is (0.99, 0.99).

7. CONCLUSION

In this paper, we have discussed about the total (vertex, edge and face) coloring of the specific type of planar graph, (i.e. Fan graph) and also an algorithm for labeling the fuzzy face magic of the fan graph is discussed with an example. Eventually, Intuitionistic fuzzy face magic labeling of the fan graph is also defined with the same algorithm. To disclose, the Fan graph $F_{1,5}$ is studied and formatted as a graph for the discussion in this paper.

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