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A VIEW ON LABELING OF STAR RELATED GRAPHS

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ABSTRACT

The labeling of graphs has been applied in the fields such as circuit design, communication network, coding theory, and crystallography. In the domain of Mathematics, graph theory is the study of graphs that concerns with the relationship among edges and vertices. Graphs are one of the prime objects of study in discrete Mathematics. Labeling of graphs is an active research area, it has been widely studied by several researchers. In a wide area network (WAN), several systems are connected to the main server, the labeling technique plays a vital role to label the cables.

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KEYWORDS:

Labeling, graph, edge, definition, injective, map, elements, sets, circuit, network, theory definition, etc.,

INTRODUCTION

A graph labeling is an assignment of integers to the vertices or the edges, or both, subject to certain conditions. If the domain is the set of vertices we speak about the vertex labeling. If the domain is the set of edges, then the labeling is called the edge labeling. If the labels are assigned to the vertices and also the edge of a graph such a labeling is called total.

Theorem

Let G be the tree with $V(G) = V(B_{n,m}) \cup \{z_j : 1 \le j \le 4\}$ and $E(G) = E(B_{n,m}) \cup \{xz_1, z_1z_2, z_2y, yz_3, z_3z_4\} / \{xy\}$. Then G is a pair sum graph.

Proof

Define $g: V(G) \to \{\pm1, \pm2, \dots, \pm(n+m+6)\}$
by $g(x) = -1$, $g(y) = 2$, $g(z_1) = -4$,
$g(z_2) = 1$, $g(z_3) = 3$, $g(z_4) = 4$.

Case 1): $n=m$.

$g(x_1) = -6$,

$g(x_{j+1}) = -6 - j$, $1 \le j \le m-1$ and $g(y_j) = 5 + j$, $1 \le j \le m$.

case 2): n>m

Assign the label to x_j, $y_j (1 \le j \le m)$ as in case 1.

Define $g(x_{m+j}) = -5-m-j$, $1 \le j \le \lceil \frac{n-m}{2} \rceil$ and $g(x_{\lceil (n+m)/2 \rceil + j}) = 8+m+j$, $1 \le j \le \lfloor \frac{n-m}{2} \rfloor$.

case 3): n<m

Assign the label to x_j, $y_j (1 \le j \le m)$ as in case 1.

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Define $g(y_{m+j})=-8-n-j,\ 1\leq j\leq\lceil \frac{m-n}{2} \rceil$ *and*

$g(y_{\lceil (n+m)/2\rceil+j})= 5+n+j,\ 1\leq j \leq\lfloor \frac{m-n}{2} \rfloor.$

Then G is a pair sum graph.

Theorem

The tree G with vertex set $V(G)=(B_{n,m})\cup\{z_j:1\leq j\leq 5\}$ and edge set $E(G)= E(B_{n,m})\cup\{xz_1, z_1 y, yz_2, z_2 z_3, yz_4, z_4 z_5\}/\{xy\}$. Then G is a pair sum tree.

Proof

Define $g:V(G)\rightarrow\{\pm1,\pm2,\ldots,\pm(n+m+7)\}$

by $g(x)=-1, g(y)=1, g(z_1)=-4,$

$g(z_2)=2, g(z_3)=3, g(z_4)=-3,$

$g(z_5)=5.$

Case 1): $n=m$

$g(x_j)=-5-j,\ 1\leq j\leq m$

and $g(y_j)=5+j,\ 1\leq j\leq m$

case 2): n>m

Assign the label to x_j and, y_j as in case 1 for $1\leq j\leq m$

Define

$g(x_{m+j})= -5-m-j,\ 1\leq j\leq\lceil \frac{n-m}{2} \rceil$ and

$g(x_{\lceil (n+m)/2\rceil+j})=6+m+j,\ 1\leq j\leq\lceil \frac{n-m}{2} \rceil.$

case 3): n<m

Assign the label to x_j, $y_j(1\leq j\leq m)$ as in case 1.

Define $g(y_{m+j})=5+n+j,\ 1\leq j\leq\lceil \frac{m-n}{2} \rceil$

$g(y_{\lceil (n+m)/2\rceil+j})= -7-n-j,\ 1\leq j\leq\lceil \frac{m-n}{2} \rceil.$

Then G is a pair sum graph.

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Theorem

The trees G_j, $(1\leq j\leq 5)$ with vertex set and edge set given below are pair sum.

(i) $V(G_1)= V(K_{1,m})\cup\{y_j:1\leq j\leq 6\}$ and

$E(G_1)= E(K_{1,m})\cup\{xy_1, y_1y_2,y_2y_3,y_3y_4,y_4y_5,y_5y_6\}$.Then G_1 is a pair sum graph.

(ii) $V(G_2)= V(K_{1,m})\cup\{y_j:1\leq j\leq 7\}$

and$E(G_2)=E(K_{1,m})\cup\{xy_6,y_6y_7,y_1y_2,y_2y_3,y_3y_4,y_4y_5,y_5x\}$.

Then G_2 is a pair sum graph.

(iii) $V(G_3) = V(K_{1,m}) \cup \{y_j: 1 \leq j \leq 7\}$ and

$E(G_3) = E(K_{1,m}) \cup \{xy_5,y_5y_6,y_6y_7,y_1y_2,y_2y_3,y_3y_4,y_4x\}$

Then G_3 is a pair sum.

(iv) $V(G_4) = V(K_{1,m}) \cup \{y_j\, z_j: 1 \leq j \leq 4\}$ and

$E(G_4) = E(K_{1,m}) \cup \{xz_1,z_1z_2,xz_3,z_3z_4,y_1y_2,y_2y_3,y_3y_4,y_4x\}$.

Then G_4 is a pair sum graph.

(v) $V(G_5) = V(K_{1,m})\cup\{y_jz_j:1\leq j\leq 3\}$

and $E(G_5)=E(K_{1,m})\cup\{xy_1,xy_2,xy_3,y_1z_1,y_2z_2,y_3z_3\}$.Then G_5 is a pair sum graph.

Proof (i)

$V(K_{1,m})=\{x,x_j:1\leq j\leq m\}$ and

$E(K_{1,m})=\{xx_j 1\leq j\leq m\}$.

g: $V(G_1)\rightarrow\{\pm1,\pm2,\ldots,\pm(m+7)\}$

by $g(x)=-1$, $g(y_1)=-7$, $g(y_2)=-5$,

$g(y_3)=1$, $g(y_4)=3$, $g(y_5)=5$,

$g(y_6)=7$ $g(x_j)=-2j-4$, $1\leq j\leq\lceil \frac{m}{2}\rceil$and

$g(x_{\lfloor (m+1)/2+j\rfloor}) = 2i+6$,$1\leq j\leq\lceil \frac{m}{2}\rceil$

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Then G_1 is a pair sum tree.

Proof (ii)

Define a map

$g: V(G_2) \to \{\pm 1, \pm 2, \ldots, \pm(m+8)$

by $g(y_1) = -3$, $g(y_2) = -6$, $g(y_3) = -1$,

$g(y_4) = -4$, $g(y_5) = 1$, $g(y_6) = 3$,

$g(y_7) = 4$, $g(x) = 2$, $g(x_1) = 7$,

$g(x_{1+j}) = 4+2i, \quad 1 \le j \le \lfloor \frac{m}{2} \rfloor$ and

$g(x_{\lceil (m/2)+j \rceil + j}) = -2j-8$, $1 \le j \le \lceil \frac{m-2}{2} \rceil$

Proof (iii)

Define a map

$g: V(G_3) \to \{\pm 1, \pm 2, \ldots, \pm(m+8)\}$

$g(y_1) = -3$, $g(y_2) = -6$, $g(y_3) = -1$,

$g(y_4) = -4$, $g(y_5) = 2$, $g(y_6) = 3$,

$g(y_7) = 4$, $g(x) = 1$,

$g(x_1) = 7+j, \quad 1 \le j \le \lceil \frac{m}{2} \rceil$

$g(x_{\lceil (m/2) \rceil + j}) = -j-10$, $1 \le j \le \lfloor \frac{m}{2} \rfloor$

Then G_3 is a pair sum graph.

Proof (iv)

Define a map

$g: V(G_4) \to \{\pm 1, \pm 2, \ldots, \pm(m+9)\}$

by $g(x) = -1$, $g(y_1) = 3$, $g(y_2) = 2$,

$g(y_3) = 1$, $g(y_4) = -4$, $g(z_1) = -5$,

$g(z_2) = -6$, $g(z_3) = 7$,

$g(z_4) = 4$. For the other vertices we define,

$g(x_j) = -5-2j$, $1 \le j \le \lceil \frac{m}{2} \rceil$

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$g(x_{\lfloor (m+1)/2 \rfloor + j})=7+2j,\ 1 \le j \le \lfloor \frac{m}{2} \rfloor$.

Obviously g is a pair sum labeling.

Proof (v)

Define

g: $V(G_5) \to \{\pm1, \pm2, \ldots, \pm(m+7)\}$

by $g(x)=-1$, $g(y_1)=2$, $g(y_2)=3$,

$g(y_3)=4$, $g(z_1)=-3$, $g(z_2)=-5$,

$g(z_3)=-7$,

$g(x_j)=2j+4,\ 1 \le j \le \lceil \frac{m}{2} \rceil$

$g(x_{\lfloor (m+1) \rfloor + j})=-2-2j,\ 1 \le j \le \lfloor \frac{m}{2} \rfloor$.

Obviously g is a pair sum labeling.

Theorem

Let G be the tree with $V(G)=V(B_{n,m}) \cup \{z_j : 1 \le j \le 5\}$ and

$E(G)=E(B_{n,m}) \cup \{z_1 z_2, z_2 x, x z_3, z_3 y, y z_4, z_4 z_5\} / \{xy\}$.Then G is a pair sum tree.

Proof

Define a function g: $V(G) \to \{\pm1, \pm2, \ldots, \pm(n+m+7)\}$

By $g(x)=-4$,

$g(y)=2$,

$g(z_1)=-6$,

$g(z_2)=-1$,

$g(z_3)=1$,

$g(z_4)=3$,

$g(z_5)=4$.

Case 1): $n=m$.

$g(x_j)=-6-j,\ 1 \le j \le m$ and

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$g(y_j)=8+j$, $1\leq j \leq m$.

Case 2): $n<m$.

Assign the label to x_j, $y_j(1 \leq j\leq n)$ as in case 1).

Define $g(y_{n+j})=8+n+j$, $1\leq j\leq \lceil \frac{m-n}{2} \rceil$

and $g(y_{\lceil (m-n)/2 \rceil+j})= -12-n-j$, $1\leq j \leq \lfloor \frac{m-n}{2} \rfloor$.

Then G is a pair sum graph.

Theorem

Let G be the tree with $V(G)= V(B_{n,m})\cup\{z_j:1\leq j \leq 6\}$

and $E(G)=E(B_{n,m})\cup\{yz_1, z_1 z_2, z_2 z_3, yz_4, z_4 z_5, z_5 z_6\}$.

Then G is a pair sum graph.

Proof

Define $V(B_{n,m})=\{x,y,y_i, v_i:1\leq i \leq n,1\leq i \leq m\}$ and

$E(B_{n,m})=\{xy,xx_i,yy_j:1\leq i \leq n,1\leq j \leq m\}$.

$g:V(G)\rightarrow\{\pm1,\pm2,..., \pm(n+m+ 8)\}$ and

$g(z_1)=4$, $g(z_2)=5$, $g(z_3)=6$,

$g(z_4)= -2$, $g(z_5)= -7$, $g(z_6)= -4$,

$g(x)=2$, $g(y)= -1$.

Case 1): n=m $g(x_1)= -3$,

$g(x_{1+j})=9+ j$, $1\leq j \leq m -1$,

$g(y_j)= -10- j$, $1\leq j\leq m$

case 2): n>m

Assign the label to x_j, $y_j(1\leq j\leq m)$ as in case 1.

Define $g(x_{m+j})=8+m+j$, $1\leq j\leq \lceil \frac{n-m}{2} \rceil$ and

$g(x_{\lceil (n+m)/2 \rceil+j})= -13-m-j$, $1\leq j \leq \lfloor \frac{n-m}{2} \rfloor$.

case 3): n<m

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Assign the label to x_j, $y_j(1\leq j\leq n)$ as in case 1.

Define $g(x_{n+j})=11+n+j, \quad 1\leq j \leq \lceil \frac{m-n}{2} \rceil$

$g(x_{\lceil (n+m)/2 \rceil +j})= -10-n-j$, $1\leq j \leq \lfloor \frac{m-n}{2} \rfloor$.

Then G is a pair sum graph.

Theorem

If G is the tree with $V(G)= V(B_{n,m}) \cup \{z_j:1\leq j\leq 3\}$ and

$$E(G)= E(B_{n,m})\cup\{xz_1, z_1 z_2, z_2 z_3, z_3 y\}/\{xy\} .$$

Then G is a pair sum tree.

Proof

Define a function $g:V(G)\rightarrow\{\pm1,\pm2,\ldots, \pm(n+m+ 5)\}$ βψ

$g(x)= -1, g(y)=3, g(z_1)= -4,$

$g(z_2)=1, g(z_3)=2.$

Case 1): $n=m$.

$g(x_j)= -5- j$, $1\leq j \leq m$ and

$g(y_i)=3+j$, $1\leq j \leq m$.

case 2): n>m

Assign the label to x_j, $y_j(1\leq j\leq m)$ as in case 1.

Define $g(x_{m+j})=-5-m-j$, $1\leq j\leq\lceil \frac{n-m}{2} \rceil$

and $g(x_{\lfloor (n+m)/2 \rfloor +j})=7+m+j$, $1\leq j \leq \lfloor \frac{n-m}{2} \rfloor$.

Then G is a pair sum graph

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