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# Sensitiveness of Propagation Characteristics of HermiteCosh Gaussian Laser 

# Beams on Decentered Parameter Dependent Initial Minimum Beam Radius In The Interaction With Collisionless Magnetized Plasma 

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#### Abstract

: It is widely known that decentered parameter dependent Initial beam radius plays an important role in propagation characteristics of Hermite cosh Gaus- sian (HChG) mode laser beams in collisionless magnetized plasma by considering ponderomotive nonlinearity. The standard parabolic equation approach under WKB and paraxial approximation is utilized to investigate propagation char-acteristics of HChG laser beams in collisionless magnetised plasma. The main thrust of present paper lies on analytical investigation of numerical interval and turning points of decentered parameter dependent Initial beam radius which governs self-focusing and defocusing of HChG laser beams. Eventually results are discussed in the light of numerical intervals and turning points of initialbeam radius.


Keywords: Self-focusing, Hermite cosh-Gaussian beam, decentered parameter, magnetized plasma, initial beam radius.

## Introduction:

The interaction of laser radiations with plasma gives various nonlinear phe- nomena such as, self-phase modulation, self-focusing, harmonic generation etc. [1-3]. Investigation of these nonlinear phenomena has been a subject of exper- imental and theoretical research due to their vast applications in laser driven fusion [4], laser based electron accelerators [5,6], X-ray lasers [7], fast ignition for inertial confinement fusion (ICF) [8]. For these applications it is necessary to propagate the laser beam without any divergence and by maintaining a good plasma interaction.

The self- focusing is a genuinely nonlinear phenomenon caused by nonlin-ear response of material medium prompting its reformed refractive index. In the laser-plasma-interaction, the self-focusing concerning the laser beams [9-11] has been the focal point of consideration as it influences some other nonlinear phenomena. Theoretical and experimental investigations on propagation char- acteristics of different laser beam profiles such as Gaussian [12], coshGaussian (ChG) [13,14], Hermite cosh-Gaussian (HChG) [15,16], elliptical Gaussian [17], and

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hollow Gaussian beams [18] have been reported in recent years.
Theoretical researchers have recently focused their attention on the parax-ial wave family of laser beams. The HChG beam is one of the solutions tothe paraxial wave equation, and it is created in the lab by superimposing two decentered HChG beams as Cosh-Gaussian ones [19]. Under Wentzel-Kramers- Brillouin (WKB) and paraxial approximations, the present theoretical analysis uses a parabolic equation technique. According to a review of the literature, the decentered parameter $[20,21]$ and the initial beam radius $[14,22,23]$ are impor- tant factors in the self-focusing phenomena. However, in present paper we have emphasized analytically to set the numerical interval of decentered parameter dependent initial minimum beam radius for various nonlinear phenomena such as self-focusing, self-trapping, and defocusing. We have explored the effect of intersecting points initial minimum beam radius on self-focusing and defocusing of HChG laser beam.

This paper is organised in the following way: Section. 2 gives the beam width parameter differential equation of the HChG laser beam propagating through a collisionless magnetized plasma for mode indices values $m=0,2$, and 4. In section 3, analytical investigation is graphically discussed in detail. In section4, a brief conclusion is included.

## 2. Theoretical Framework

Let us consider the propagation of HChG laser beam through a collisionless magnetized plasma along the $z$ direction along which static magnetic field $B_{0}$ is applied. The electric field of the laser beam propagating in either modes i.e.in $E$ and $O$ modes can be written as [24],

$$
\begin{equation*}
E_{ \pm}=x E_{0 \pm}(r, z) \exp \left[-i\left(\omega t-k_{ \pm} z\right)\right] \tag{1}
\end{equation*}
$$

where is $k_{ \pm} \frac{\omega}{c} \sqrt{\varepsilon_{ \pm}}$the propagation constant of the wave where $\varepsilon_{ \pm}$is the effective dielectric constant of magnetized plasma, $\omega$ is frequency of laser used and $c$ is the speed of light in vacuum.
$\varepsilon_{ \pm}=\varepsilon_{0 \pm}+\varphi_{ \pm}\left(E_{ \pm} E^{*}\right)$
where $\varepsilon_{0 \pm}$ and $\varphi_{ \pm}\left(E_{ \pm} E^{*}\right)$ are the linear and nonlinear parts of the dielectric constant of collisionless magnetized plasma and can be expressed as,
$\varepsilon_{0 \pm}=1-\frac{\omega_{p}^{2}}{\omega\left(\omega \mp \omega_{c}\right)}$
$\varphi_{ \pm}\left(E_{ \pm} E_{ \pm}^{*}\right)=\frac{\omega_{p}^{2}}{\omega\left(\omega \mp \omega_{c}\right)}\left[1-\exp \left(-\alpha E_{ \pm} E_{ \pm}^{*}\right)\right]$
Here, $\omega_{p}=\left(4 \pi N_{0} e^{2} / m\right)^{1 / 2}$ is the plasma frequency and $\omega_{c}=e B_{0} / m c$ is cyclotron frequency.
$e$ and $m$ are the electronic charge and its rest mass respectively.
In the light of Maxwells elctrodynamic equations, the general form of wave equation governing the propagation of laser beam is given as,
$\frac{\partial^{2} E_{ \pm}}{\partial z^{2}}+\delta_{ \pm}\left(\frac{\partial^{2} E_{ \pm}}{\partial r^{2}}+\frac{1}{r} \frac{\partial E_{ \pm}}{\partial r}\right)+\frac{\omega^{2}}{c^{2}}\left(\varepsilon_{ \pm} E_{ \pm}\right)=0(5$
Where, $\delta_{ \pm}=\frac{1}{2}\left(1 \mp \frac{\varepsilon_{0 \pm}}{\varepsilon_{0 z z}}\right)$ and $\varepsilon_{-0 z z}=1-\frac{\omega_{p}^{2}}{\omega^{2}}$
The electric field $E_{ \pm}$given by Eq. (1) satisfies Eq.(5). In circular cylindrical coordinate system, under slowly varying envelope approximation, the evolution of electric field envelope in collisionless magnetized plasma can be expressed as,
$-2 i k_{ \pm} \frac{\partial E_{ \pm}}{\partial z}+\delta_{ \pm}\left(\frac{\partial^{2} E_{ \pm}}{\partial r^{2}}+\frac{1}{r} \frac{\partial E_{ \pm}}{\partial r}\right)+\frac{\omega^{2}}{c^{2}} \varphi_{ \pm}\left(E_{ \pm} E_{ \pm}^{*}\right)=0$
In WKB approximation one can neglect $\partial^{2} E_{ \pm} / \partial z^{2}$ in Eq. (5). The complex amplitude of electric vector may be expressed as,
$E_{0 \pm}(r, z)=A_{ \pm}(r, z) \exp \left[-i k_{ \pm}(z) S_{ \pm}(r, z)\right]$
where, $A_{ \pm}(r, z)$ and $S_{ \pm}(r, z)$ are real functions of $r$ and $z . S_{ \pm}$is known as eikonal of the beam. Substituting above equation in Eq. (6) and separating real and imaginary parts of resulting equations one obtains,
$2\left(\frac{\partial S_{ \pm}}{\partial z_{ \pm}}\right)+\delta_{ \pm}\left(\frac{\partial S_{ \pm}}{\partial r}\right)^{2}+\frac{2 S_{ \pm}}{k_{ \pm}} \frac{\partial k_{ \pm}}{\partial z_{ \pm}}=\frac{\delta_{ \pm}}{k_{ \pm}^{2} A_{ \pm}}\left(\frac{\partial^{2} A_{ \pm}}{\partial r^{2}}+\frac{1}{r} \frac{\partial A_{ \pm}}{\partial r}\right)+\frac{\omega^{2}}{c^{2}} \varphi_{ \pm}\left(A_{ \pm} A_{ \pm}^{*}\right)$
$\frac{\partial A_{ \pm}^{2}}{\partial z}+\delta_{ \pm} \frac{\partial S_{ \pm}}{\partial r} \frac{\partial A_{ \pm}^{2}}{\partial r}+\delta_{ \pm} A_{ \pm}^{2}\left(\frac{\partial^{2} S_{ \pm}}{\partial r^{2}}+\frac{1}{r} \frac{\partial S_{ \pm}}{\partial r}\right)+\frac{A_{ \pm}^{2}}{k_{ \pm}} \frac{\partial k_{ \pm}}{\partial z}=0$
Following approach given by Akhmanov et.al. [25] and developed by Sodha et.al. [26], the solution of Eqs. (8) and (9) for HChG laser beam can be written as
$A_{ \pm}^{2}=I \exp \left[-2\left(\frac{r}{f_{ \pm} r_{0}}+\frac{b}{2}\right)^{2}\right]+\left[-2\left(\frac{r}{f_{ \pm} r_{0}}-\frac{b}{2}\right)^{2}\right]+2 I\left[-2\left(\frac{r}{f_{ \pm} r_{0}}+\frac{b}{2}\right)^{2}\right]$
Where, $I=H_{m}^{2} \frac{E_{0 \pm}^{2}}{f_{0 \pm}^{2}} \exp \left(\frac{b^{2}}{2}\right)$
$S_{ \pm}=\frac{r^{2}}{2} \beta_{ \pm}(z)+\varphi_{ \pm}(z)(11)$
$\beta_{ \pm}(z)=\frac{1}{\delta_{ \pm}} \frac{1}{f_{ \pm}} \frac{d f_{ \pm}}{d z}$
Where, $E_{0 \pm}^{2}$ is an axial laser intensity and $f_{ \pm}$is the dimensionless beam width parameter for $E$ and $O$ modes respectively. $\beta_{ \pm}{ }^{2}$ gives the radius of the curvature of wave front and $\varphi_{ \pm}(z)$ is known as phase. Substituting Eqs.(10), (11) and (12)in Eq. (8) and equating coefficients of $r^{2}$ on both sides we get,

For $m=0$
$\frac{\partial^{2} f_{ \pm}}{\partial \xi_{ \pm}^{2}}=\frac{1}{\varepsilon_{0 \pm}} \frac{1}{f_{ \pm}^{3}}\left\{\frac{\left(12-12 b^{2}-b^{4}\right) \delta_{ \pm}{ }^{2}}{3}-\frac{\left(2-b^{2}\right) \exp \left[-p_{ \pm}\right] p_{ \pm} \rho_{ \pm}^{2} \gamma_{ \pm} \delta_{ \pm}}{2}\right\}$
For $m=2$
$\frac{\partial^{2} f_{ \pm}}{\partial \xi_{ \pm}^{2}}=\frac{1}{\varepsilon_{0 \pm}} \frac{1}{f_{ \pm}^{3}}\left\{-\frac{\left(84+60 b^{2}-b^{4}\right) \delta_{ \pm}{ }^{2}}{3}+\frac{2\left(b^{2}-10\right) \exp \left[-p_{ \pm}\right] p_{ \pm} \rho_{ \pm}^{2} \gamma_{ \pm} \delta_{ \pm}}{2}\right\}$
For $m=4$
$\frac{\partial^{2} f_{ \pm}}{\partial \xi_{ \pm}^{2}}=\frac{1}{\varepsilon_{0 \pm}} \frac{1}{f_{ \pm}^{3}}\left\{\frac{\left(308+108 b^{2}-b^{4}\right) \delta_{ \pm}{ }^{2}}{3}+\frac{72\left(b^{2}-18\right) \exp \left[-p_{ \pm}\right] p_{ \pm} \rho_{ \pm}^{2} \gamma_{ \pm} \delta_{ \pm}}{2}\right\}$
Where, $\gamma_{ \pm}=\frac{\Omega_{p}^{2}}{1 \mp \Omega_{c}}, \quad \Omega_{p}=\frac{\omega_{p}}{\omega}, \Omega_{c}=\frac{\omega_{c}}{\omega}, \quad p_{ \pm}=\frac{\alpha E_{0 \pm}^{2}}{f_{ \pm}^{2}}, \quad \rho_{ \pm}=\frac{r_{0} \omega}{c}$
The quantity $p_{ \pm}$which is dimensionless and proportional to $E_{0 \pm}^{2}$ represents the dimensionless beam power on a suitably chosen scale. The beam width parameters $f_{ \pm}$is a function of $\xi_{ \pm}$, here $\xi_{ \pm}=z / k_{ \pm} r^{2}$ is the normalized propagation distance.

## 1. Results and Discussion

Equations (13)-(15) are second order nonlinear differential equations which gives variation of $f_{ \pm}$with $\xi_{ \pm}$for $m=0, m=2, m=4$ respectively. The first term on the right-hand side gives diffraction divergence of the beam and the second term gives to the convergence of beam which results from the nonlinearity in dielectric constant of collisionless magnetized plasma.

Under initial condition (at $\xi_{ \pm}=0, f_{ \pm=1}$ ) the general power of laser beam $p_{+}$and beam radius $\rho_{+}$are replaced by initial power $p_{0+}$ and initial beam radius $\rho_{0+}$. Consider $m=0$ mode for further investigation. For initial condition one may obtain equation of initial curve [5052].

$$
\begin{equation*}
\rho_{0 \pm}^{2}=\frac{2}{3} \frac{\delta_{ \pm}}{\gamma_{ \pm}} \frac{\exp \left[-p_{0 \pm}\right]}{p_{0 \pm}} \frac{\left(b^{4}+12 b^{2}-12\right)}{\left(b^{2}-2\right)} \tag{16}
\end{equation*}
$$

Here, $\rho_{0 \pm}$ represent initial beam radius coresponding to $E$ and $O$ modes. Only E mode is subject to the following investigation.
By following similar process given by Valkunde et.al.[27] one may obtain value of initial beam power $p_{0+}=1$ for initial minimum beam radius $\rho_{0+\text { min }}$. By using numerical values $N_{0}$ $=1 \times 10^{18} \mathrm{~cm}^{-3}, \omega=1.776 \times 10^{15} \mathrm{rad} / \mathrm{s}, B_{0}=10^{6} \mathrm{gauss}$ equation (16) gives,

$$
\begin{equation*}
\rho_{0+\min }=42.1483\left(\frac{\left(b^{4}+12 b^{2}-12\right)}{\left(b^{2}-2\right)}\right)^{2} \tag{17}
\end{equation*}
$$

From Eq.(17) it is seen that $\rho_{0+m i n}$ is purely $b$ dependent. By using numer-ical computation under the condition that $\rho_{0+\text { min }}>0$ one may obtain domain of decentered parameter. Figure 1 throws light on domain of decentered param- eter for mode index $m=0$. Form Figure 1. it is seen that as initially as value of decentered parameter $b$ increases the value of $\rho_{0+\text { min }}$ decreases and by using simple mathematical calculation one may show that $\rho_{0+\min }$ has minimum value $\rho_{0+\min }=206.515$ at $b=2.4850$. After reaching minimum value of $\rho_{0+\min }$ withincrease in value of decentered parameter $b$ value of $\rho_{0+\min }$ increases.

Hence the above main domain is further divided into two sub domains
$1.4142 \leq b \leq 2.4850$ and $b \geq 2.4850$.


Figure 1: Variation of $\rho_{0+\min }$ with decentered parameter $b$ for mode index $m=0$.
Under initial condition (at $\xi=0, f=1$ ), $\frac{\partial^{2} f_{ \pm}}{\partial \xi_{ \pm}^{2}}=0$ and for $p_{0+}=1$ the right-hand side of the Eq. (13) is represented by function $F\left(\rho_{0+\text { min }}\right)$ withvalue of decentered parameter $b$ from $1.4142 \leq b \leq 2.4850$ and $b \geq 2.4850$.
$F\left(\rho_{0+\min }\right)=-0.33333\left(12-12 b^{2}-b^{4}\right)+0.000146513$
To explore effect of initial minimum beam radius right at the beginning one needs to give careful attention on plots shown in Figures 2a and 2b given here.

The plots shown in Figures 2a and 2 b can be examined for three distinct conditions for propagation characteristics expressed follows and the simple an- alytical approach prompts following limits for initial minimum beam radius,

For Figure 2a (1.4142 $\leq b \leq 2.4850$ )
For Self-trapping

$$
\begin{gathered}
F\left(\rho_{0+\min }\right)=0 \text { for } \rho_{0+\min }=427.291(\mathrm{~b}=1.5) \\
\text { for } \rho_{0+\min }=261.876(\mathrm{~b}=1.8) \\
\text { for } \rho_{0+\min }=238.724(\mathrm{~b}=2.1)
\end{gathered}
$$


(a)

(b)

Figure 2: Graph of $F\left(\rho_{0+m i n}\right)$ against $\rho_{0+\text { min }}$ for mode index $m=0$ (a) for $1.4142 \leq b \leq$ 2.4850 , (b) for $b \geq 2.4850$.

For Self-focusing:
For Defocusing

$$
\begin{gathered}
F\left(\rho_{0+\min }\right)<0 \text { for } \rho_{0+\text { min }} \leq 238.724 \\
F\left(\rho_{0+\text { min }}\right)>0 \text { for } \rho_{0+\text { min }} \geq 427.29
\end{gathered}
$$

For Figure 2b $\quad(b \geq 2.4850)$
For Self-trapping

$$
\begin{array}{r}
F\left(\rho_{0+\min }\right)=0 \text { for } \rho_{0+\min }=239.851(b=3.0) \\
\text { and for } \rho_{0+\min }=246.419(b=3.3) \\
\text { and for } \rho_{0+\text { min }}=254.28(b=3.6)
\end{array}
$$

For Self-focusing:

$$
F\left(\rho_{0+\min }\right)<0 \text { for } \rho_{0+\min } \leq 239.851
$$

For Defocusing

$$
F\left(\rho_{0+\min }\right)>0 \text { for } \rho_{0+\min } \geq 254.28
$$

From Figure 2a (for $1.4142 \leq b \leq 2.4850$ ) and Figure 2b (for $b \geq 2.4850$ ) it is seen that for $1.4142 \leq b \leq 2.4850$, three curves intersects each other at three different points in selffocusing region where as for $b \geq 2.4850$ the three curves intersects each other at three different points in defocusing region. These intersecting points affect the propagation characteristics of laser beams throughmagnetised plasma. The intersecting points are as follow:
Figure 2a (for $1.4142 \leq b \leq 2.4850$ )

$$
\begin{gathered}
\rho_{0+\min }=199.479 \text { for } b=1.50 \text { and } b=1.80 \\
\rho_{0+\min }=211.437 \text { for } b=1.80 \text { and } b=2.10 \\
\rho_{0+\min }=206.0421 \text { for } b=1.50 \text { and } 2.10
\end{gathered}
$$

Figure 2b (for $b \geq 2.4850$ )

$$
\begin{gathered}
\rho_{0+\min }=269.357 \text { for } b=3.00 \text { and } b=3.30 \\
\rho_{0+\min }=285.592 \text { for } b=3.30 \text { and } b=3.60 \\
\rho_{0+\text { min }}=277.962 \text { for } b=3.00 \text { and } 3.60
\end{gathered}
$$

From Figures 3 a and 3 b it is clear that at the turning points, for a given $\rho_{0+m i n}$, there can be two distinct values of decentered parameter $b$ are possible which gives the identical $F\left(\rho_{0+m i n}\right)$ value or one may also say that there can be two identical beam width parameter differential equation for two distinct values of decentered parameter $b$ such that the evolution of laser beam would be identical. The above significance of graph is also evident from the Eq.(18) itself. In Self-focusing region of Figure 3a $\rho_{0+\min } \leq 238.724$ which is the present region of interest. The Figures 4 a and 4 b gives variation of beam width param- eter $f_{+}$againstnormalised
propagation distance $\xi_{+}$before and after turning points respectively.


Figure 3: Graph of $f_{+}$against $\xi_{+}$for mode index $m=0$ in self focusing region of $1.4142 \leq$ $b \leq$
2.4850 (a) before turning points $\left(\rho_{0+\min }=150\right)$ (b) after turning points $\left(\rho_{0+\min }=204\right)$.

In Figures 4 a and 4 b strong self-focusing is observed, it also reveals that initially beam starts converging but after appreciable propagation through mag- netized plasma, the influence of diffraction divergence dominate the converging effect due to nonlinearity and beam starts
diverging. However in Figure 4a ,the self-focusing length is observed over smaller interval of $\xi_{+}$as compared the self- focusing length observed in Figure 4 b over a given range of $\xi_{+}$. Additionally, with increase in values of decentered parameter, the effect of $b$ on self-focusing length in Figure 4 a is exactly opposite to that in the Figure 4b. That is from Figure 4 a it is seen that with increase in a value of decentered parameter $\mathbf{b}$ self- focusing length decreases whereas from Figure 4 b it is seen that with increase in a value of decentered parameter $\mathbf{b}$ self-focusing length increases.

In Defocusing region of Figure $3 \mathrm{~b} \rho_{0+\min } \geq 116.386$ which is the present region of interest the Figures 5 a and 5 b gives variation of beam width parameter $f_{+}$against normalized propagation distance $\xi_{+}$before and after turning points respectively. From Figure 5a. it is seen in that before turning points, oscillatory defocusing is observed whereas after turning points steady state defocusing is observed over given range of $\xi_{+}$as shown in Figure 5b.


Figure 4: Graph of $f_{+}$against $\xi_{+}$for mode index $m=0$ in defocusing region of $b \geq 2.4850$ (a) before turning points $\left(\rho_{0+\text { min }}=240\right)(b)$ after turning points $\left(\rho_{0+\min }=400\right)$.

A similar process is followed for a higher mode indices $m=2$ and $m=4$. The numerical results and the domain are shown in Table and Graphical results are shown in Figures below (for mode index $m=2$ refer Figures 6 to 10 and formode index $m=4$ refer Figures 11 to 15 ).

| Numerical results for mode indices $\mathrm{m}=2$ and $\mathrm{m}=4$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Mo } \\ \text { de } \\ \text { inde } \\ \text { x } \end{gathered}$ | Domain s of $b$. | For Self Trapping $\rho 0+$ min | $\begin{gathered} \text { For } \\ \text { lf } \\ \text { Se } \\ \text {-focusing } \\ \rho_{0+m i n} \end{gathered}$ | For de- focusin g $\rho \mathrm{O}+$ min | Turning points $\rho_{0+\text { min }}$ |
| $\begin{gathered} m= \\ 2 \end{gathered}$ | $\begin{gathered} 3.17< \\ b<6.16 \\ b>6.16 \end{gathered}$ | $\begin{gathered} 175.52(b=4.00) \\ 144.70(b=5.00) \\ 139.14(b=6.00) \\ \\ 145.32(b=8.00) \\ 151.79(b=9.00) \\ 159.41(b=10.00) \end{gathered}$ | $\begin{aligned} & <139.14 \\ & <145.32 \end{aligned}$ | $\begin{aligned} & >175.52 \\ & >159.41 \end{aligned}$ | $\begin{gathered} 119.84(b=4.00,5.00) \\ 126.19(b=4.00,6.00) \\ 131.17(b=5.00,6.00) \\ \\ 170.73(b=8.00,9.00) \\ 178.47(b=8.00,10.00) \\ 185.11(b=9.00,10.00) \end{gathered}$ |
| $\begin{gathered} m= \\ 4 \end{gathered}$ | $\begin{gathered} 4.24< \\ b<8.29 \\ b>8.29 \end{gathered}$ | $\begin{aligned} & 90.55(b=5.0) \\ & 69.43(b=6.0) \\ & 63.85(b=7.0) \\ & \\ & 62.58(b=9.0) \\ & 63.77(b=10.0) \\ & 65.55(b=11.0) \end{aligned}$ | $\begin{aligned} & <63.86 \\ & <62.58 \end{aligned}$ | $\begin{aligned} & >90.55 \\ & >65.55 \end{aligned}$ | $\begin{aligned} & 51.67(b=6.0, b=7.0) \\ & 53.62(b=5.0, b=7.0) \\ & 55.22(b=6.0, b=7.0) \\ & \\ & 67.57(b=9.0, b=10.0) \\ & 72.09(b=9.0, b=11.0) \\ & 72.10(b=9.0, b=11.0) \end{aligned}$ |



Figure 5: Variation of $\rho_{0+\min }$ with decentered parameter $b$ for mode index $m=2$.


Figure 6: critical curves for mode index $m=2$ (a) for $3.17 \leq b \leq 6.16$,(b) for $b \geq 6.16$


Figure 7: Graph of $F\left(\rho_{0+\min }\right)$ against $\rho_{0+\min }$ for mode index $m=2$ (a) for $3.17 \leq b \leq 6.16$ (b) for $b \geq 6.16$


Figure 8: Graph of $f_{+}$against $\xi_{+}$for mode index $m=2$ in self-focusing region of 3 $\leq$

6.16 (a) before turning points $\left(\rho_{0+\min }=80\right)(\mathrm{b})$ after turning points $\left(\rho_{0+\min }=135\right)$

Figure 9: Graph of $f_{+}$against $\xi_{+}$for mode index $m=2$ in defocusing region of $b \geq 6.16$

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(a)before turning points $\left(\rho_{0+\min }=148\right)(b)$ after turning points $\left(\rho_{0+\min }=200\right)$


Figure 10: Variation of $\rho_{0+\min }$ with decentered parameter $b$ for mode index $m=4$.


Figure 11: critical curves for mode index $m=4$ (a) for $4.24 \leq b \leq 8.29$,(b) for $b \geq 8.29$


Figure 12: Graph of $F\left(\rho_{0+\min }\right)$ against $\rho_{0+\min }$ for mode index $m=4$ (a) for $4.24 \leq b \leq 8.29$

(b) for $b \geq 8.29$

Figure 13: Graph of $f_{+}$against $\xi_{+}$for mode index $m=4$ in self-focusing region of $4.24 \leq b$ $\leq$
8.29 (a) before turning points $\left(\rho_{0+\min }=30\right)$ (b) after turning points $\left(\rho_{0+\min }=56\right)$


Figure 14: Graph of $f_{+}$against $\xi_{+}$for mode index $m=4$ in self-focusing region of $b \geq$ 8.29
(a) before turning points $\left(\rho_{0+\min }=60\right)$ (b) after turning points $\left(\rho_{0+\min }=75\right)$

Conclusion
In the present investigation, the domain of value of decentered parameter $b$ for a different mode indices $(m=0, m=2$ and $m=4)$ as well as the numerical interval of $\rho_{0+m i n}$ for a different nonlinear phenomenon such as, self- focusing, self-trapping and defocusing are obtained by using simple calculus. The following important conclusions are drawn from the present analysis:
a. For each mode index there are two domains of decentered parameter $b$.
b. For each domain of decentered parameter $b, \rho_{0+\min }$ show different be- haviour.

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ISSN PRINT 23191775 Online 23207876
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c. With the increase in the mode index of the laser beam, the values of
2. $\quad \rho_{0+\min }$ for decentered parameter $b$ decreases.
a. With increase in values of decentered parameter in Self- focusing region, exactly opposite trends of Self- focusing before and after turning points are observed. That is in Self- focusing region, self-focusing length before turning points and after turning points shows exactly opposite changes..
b. In defocusing region, different trends of defocusing (Oscillatory and Steady state defocusing) before and after turning points are observed.
3. Thus, the present analytical investigations based on decentered parameter and critical beam radius offer a choice to govern the propagation characteristics of the HChG laser beam through collisionless magnetized plasma.

## References:

1. S. A. Akhmanov, A. P. Sukhorukov and R. V. Khokhlov Sov. Phys. Usp., 10 (1968) 609-636.
2. S. Vij, M. Aggarwal, , N. Kant, Opt. Commun..383(2016) 349.
3. [3]N. Kant, D.N. Gupta, and H. Suk, Phys. Lett. A 375 (2011) 31343137
4. F. Winterberg, Laser Part. Beams 26 (2008) 127.
5. H.S.Ghotra and N. Kant, Opt. Commun. 383 (2016) 169.
6. D.N. Gupta, N. Kant, D. E. Kim and H. Suk, Phys. Lett. A 368 (2007)402407
7. J. Parashar, H. D. Pandey, and V. K. Tripathi, Phys. Plasmas 4 (1997) 3040.
8. H. Hora, Laser and Particle Beams 25 (2007) 37-45.
9. T. S. Gill, R. Kaur, and R. Mahajan, Phys. Plasmas 17 (2010) 093101.
10. R. Kaur, T. S. Gill, and R. Mahajan, Optik (Stuttgart)122 (2011) 375.
11. A.T. Valkunde, S.D. Patil, M.V. Takale, B.D. Vhanmore, T.U. Urunkar,
12. K.M. Gavade, D.N. Gupta, Optik 158 (2018) 1034-1039.
13. S.D. Patil, M.V. Takale, S.T. Navare, M.B. Dongare, V.J. Fulari, Optik 124 (2013) 180-183.
14. [13] M. Aggarwal, S. Vij, N. Kant, Optik 125 (2014) 5081-5084.
15. T. U. Urunkar, S. D. Patil, A. T. Valkunde, B. D. Vhanmore, K. M.
16. Gavade and M. V. Takale (2018) Commun. Theor. Phys. 70220
17. A. Belafhal , M. Ibnchaikh. Opt. Commun. 186 (2000) 269-276
18. D. Zhao , H. Mao, H. Liu, J. Opt. A: Pure Appl. Opt. 6 (2004) 77-83.
19. A. Singh, M. Aggarwal, T. S. Gill, Optik 119 (2008) 559-564.
20. A.K.Shafeeque Ali, M .Lakshmanan, Phys. Lett. A 384 (2020) 126527 [19] L.Baida , L. Shirong, Optik 112, No. 11 (2001) 503-506.
21. S.D. Patil, M.V. Takale, V.J. Fulari, M.B. Dongare, J Mod Opt. 21 (2008)3527-3533.

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22. B.D. Vhanmore, S.D. Patil, A.T. Valkunde, T.U. Urunkar, K.M. Gavade,
23. M.V. Takale, Laser Part. Beams 35 (2017) 670-676.
24. A. Sharma, G. Prakash, and M. P. Verma, Phys. Plasmas, 10 (2003) 4079.
25. A. Sharma, I. Kourakis, and M. S. Sodha, Phys. Plasmas 15 (2008) 103103.
26. M. Aggarwal, H. Kumar, N. Kant, Optik 127 (2016) 2212-2216.
27. S. A. Akhamanov, A. P. Sukhorov, and R. V. Khokhlov, Sov. Phys. Usp. 93 (1968) 609.
28. M. S. Sodha, A. K. Ghatak, and V. K. Tripathi, Prog. Opt. 13 (1976) 171.
29. A.T. Valkunde, S.D. Patil, B.D. Vhanmore, T.U. Urunkar, K.M. Gavade,
30. M.V. Takale, V.J. Fulari, Phys. Plasmas 25 (2018) 0331031.

