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SUB DIVISION OF SOME GRAPHS

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ABSTRACT

A class of graphs is a benefit of graphs closed under isomorphism. We have seen previously a variety of classes of graphs: whole graph, cycle graphs, linked graphs. Many more classes can be distinct by means of a variety of graph parameters. $S(L_n)$ is a pair sum graph, where L_n is a ladder on n vertices.

Keywords:

Vertex, cycle, parameters, ladder, edge, vertices, diagrams, etc.,

Introduction:

The graph obtained by subdividing each edge of a graph G is called the subdivision graph of G. Perfect graphs A graph G is a mathematical object used to model pairwise relations among a collection of entities. This collection of entities is called the vertex set and is denoted by V (G). The edge set, denoted by E(G), represents the relations between pairs of eleme have numerous applications to a wide variety of fi nts of V (G). Graphs elds, from nding the shortest path between two cities on a GPS, to managing inventory in a warehouse or detecting particular molecules in biology. The major part of the thesis will be

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about with structural graph theory. Structural graph theory tries to understand families of graphs. When someone studies a particular problem, it is generally possible to characterize some properties of the underlying family of graphs. One of our main goa ls is to understand what the basic graphs are particular, we want to describe a family in terms of wellin a given family. In understood graphs and construction steps. This description can then lead to a better understanding of how to approach a problem bo th from a theoretical and an algorithmic point of view. Claude Berge stated two conjectures when introducing perfect graphs. The r . e st one, known as the Weak Perfect Graph Conjecture, states that a graph G is perfect if an only if G is perfect It was proved to be true by Lov sz [25]. The second conjecture, known as the Strong PerfectGraph Conjecture, states that a graph is perfect if and only if it is Berge. This conjecture remained open for more than 40 years before Chudnovsky, Robertson, Seym our and Thomas proved it in 2002 [9]. Those two results are stated bellow. 1.1.1 (Weak Perfect Graph Theorem. Lov perfect. sz [25]).

Theorem

$S(L_n)$ is a pair sum graph, where L_n is a ladder on n vertices.

Proof

Let $V(S(L_n))=\{x_i, y_i, z_i, a_j, b_j : 1\le i\le n, 1\le j\le n-1\}$ and

$$E(S(L_n))=\{x_iz_i, z_iy_i : 1\le i\le n\}\cup\{x_ia_i, a_i x_{i+1}, y_i b_i, b_iy_{i+1} : 1\le i\le n-1\}.$$

Case 1: n is even.

When $n = 2$, the proof follows from the Theorem,

For $n > 2$,

Define $g: V(S(L_n))\to\{\pm1,\pm2,\pm3, ..., \pm(5n-2)\}$ by $g(x_{n/2}) = -1$,

$$g(x_{n/2+1}) = -3$$

$$g(x_{n/2-i}) = 10i+3, 1\le i\le \frac{n-2}{2}$$

$$g(x_{n/2+i+1}) = -10i+1, \quad 1\le i\le \frac{n-2}{2}$$

$$g(z_{n/2}) = 5, \quad g(z_{n/2+1}) = -5$$

$$g(z_{n/2-i}) = 10i+1, 1\le i\le \frac{n-2}{2}$$

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$g(z_{n/2+1+i}) = -(10i+1), 1 \le i \le \frac{n-2}{2}$

$g(y_{n/2}) = 3,$

$g(y_{n/2+1}) = 1$

$g(y_{n/2-i}) = 10i-1, 1 \le i \le \frac{n-2}{2}$

$g(y_{n/2+1+i}) = -10i-3, 1 \le i \le \frac{n-2}{2}$

$g(a_{n/2}) = -2$

$g(a_{n/2-i}) = 10i+5,\ 1 \le i \le \frac{n-2}{2}$

$g(a_{n/2+i}) = -10i+3, 1 \le i \le \frac{n-2}{2}$

$g(b_{n/2}) = 2$

$g(b_{n/2-i}) = 10i-3, \quad 1 \le i \le \frac{n-2}{2}$

$g(b_{n/2+i+1}) = -10i-5,\ 1 \le I \le. \frac{n-2}{2}$

When $n = 4$,

$g_e(E(S(L_n))) = \{3, 4, 5, 8, 10, 16, 20, 24, 28\} \cup$
$\{-3, -4, -5, -8, -10, -16, -20, -24, -28\}.$

For $n > 4$,

$g_e(E(S(L_n))) = g_e(E(S(L_4))) \cup \{(26,36,40,44,48,38),$
$(-26, -36, -40, -44, -48, -38), (46,56,60,64,68,58),$
$(-46, -56, -60, -64, -68, -58), (10n-34, 10n-24, 10n-20,$
$10n-16, 10n-12, 10n-22), (-10n+34, -10n+24, -10n+20,$
$-10n+16, -10n+12, -10n+22)\}.$

Therefore g is a pair sum labeling.

Case 2. n is odd.

Clearly $S(L_1) \cong P_3$ and hence $S(L_n)$ is a pair sum graph by Theorem For $n > 1$,

Define

$g: V(S(L_n)) \to \{\pm1, \pm2, \ldots, \pm(5n-2)\}$ by

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$g(x_{(n+1)/2}) = 6$, $g(x_{(n-1)/2}) = 12$

$g(x_{(n+3)/2}) = -12$, $g(a_{(n-1)/2}) = -9$

$g(a_{(n+1)/2}) = 3$

$$g(x_{(n+3)/2+i}) = 10i + 10, \quad 1 \leq i \leq \frac{n-3}{2}$$

$$g(x_{(n-1)/2-i}) = -(10i + 10), \quad 1 \leq i \leq \frac{n-3}{2}$$

$$g(y_{(n+3)/2+i}) = -(6 + 10i), \quad 1 \leq i \leq \frac{n-3}{2}$$

$$g(y_{(n-1)/2-i}) = 6 + 10i, \quad 1 \leq i \leq \frac{n-3}{2}$$

$$g(z_{(n+3)/2+i}) = -10i + 2, \quad 1 \leq i \leq \frac{n-3}{2}$$

$$g(z_{(n-1)/2-i}) = 10i - 2, \quad 1 \leq i \leq \frac{n-3}{2}$$

$g(y_{(n+1)/2}) = 2$, $g(y_{(n-1)/2}) = 10$

$g(y_{(n+3)/2}) = -10$, $g(b_{(n-1)/2}) = -6$

$g(b_{(n+1)/2}) = 4$,

$g(z_{(n+1)/2}) = -4$

$g(z_{(n-1)/2}) = 8$, $g(z_{(n+1)/2}) = -8$

$$g(a_{(n+1)/2+i}) = -(10i + 12), \quad 1 \leq i \leq \frac{n-3}{2}$$

$$g(a_{(n-1)/2-i}) = 10i + 12, \quad 1 \leq i \leq \frac{n-3}{2}$$

$$g(b_{(n+1)/2+i}) = -(10i + 4), \quad 1 \leq i \leq \frac{n-3}{2}$$

$$g(b_{(n-1)/2-i}) = 10i + 4, \quad 1 \leq i \leq \frac{n-3}{2}.$$

Therefore

$g_e(E(S(L_3))) = \{2, 3, 4, 6, 9, 18, 20, -2, -3, -4, -6, -9, -18, -20\}$

and $g_e(E(S(L_5))) = g_e(E(S(L_3))) \cup \{24, 30, 34, 38, 42, 36, -24, -30, -34, -38, -42, -36\}$

When $n > 5$, $g_e(E(S(L_n))) = g_e(E(S(L_5))) \cup \{(40,50,54,58,62,52), (-40, -50, -54, -58, -62, -52),$

$(60,70,74,78,82,72), (-60, -70, -74, -78, -82, -72), \ldots,$

$$(10n-30, 10n-20, 10n-16, 10n-12, 10n-8, 10n-18),$$

$$(-10n+30, -10n+20, -10n+16, -10n+12, -10n+8, -10n+18)\}.$$

Then g is a pair sum labeling.

Theorem

$\boldsymbol{S(C_n K_1)}$ is a pair sum graph

Proof

Let $V(S(C_n K_1)) = \{x_j : 1 \le j \le 2n\} \cup \{z_j y_j : 1 \le j \le n\}$

$E(S(C_n K_1)) = \{x_j x_{j+1} : 1 \le j \le 2n-1\} \cup \{x_{2j-1} z_j : 1 \le j \le n\} \cup \{y_j z_j : 1 \le j \le n\}.$

Case 1. nis even.

Define g: $S(C_n K_1) \to \{\pm 1, \pm 2, \ldots, \pm 4n\}$ by

$g(x_j) = 2j - 1,\ 1 \le j \le n$

$g(x_{n+i}) = -(2j-1),\ 1 \le j \le n$

$g(z_j) = 2n - 1 + 2j,\ 1 \le j \le \frac{n}{2}$

$g(z_{n/2+j}) = -2n + 1 - 2j,\ 1 \le j \le \frac{n}{2}$

$g(y_j) = 3n - 1 - 2j,\ 1 \le j \le \frac{n}{2}$

$g(z_{n/2+j}) = -3n + 1 + 2j,\ 1 \le j \le \frac{n}{2}$

Here $g_e(E) = \{4, 8, 12, \ldots, (4n-4)\} \cup \{-4, -8, -12, \ldots, -(4n-4)\}$

$\cup \{2n+2, 2n+8, 2n+14, \ldots, 5n-4\}$

$\cup \{-(2n+2), -(2n+8), -(2n+14), \ldots, -(5n-4)\} \cup$

$\{5n+2, 5n+6, 5n+10, \ldots, 7n-2\}\{-(5n+2), -(5n+6), -(5n+10), \ldots, -(7n-2)\}$.

Then g is pair sum labeling.

Conclusion

Case nis odd.

Define g: $V(S(C_n K_1)) \rightarrow \{\pm1, \pm2, \ldots, \pm4n\}$ by

$g(x_j) = 4n - 2j + 2, 1 \leq j \leq n$

$g(x_{n/2+j}) = -4n + 2j - 2, 1 \leq j \leq n$

$g(z_j) = -n - 1 + j, 1 \leq j \leq \lceil \frac{n}{2} \rceil$

$g(z_{\lceil n/2 \rceil + j}) = n - j, 1 \leq j \leq \lfloor \frac{n}{2} \rfloor$

$g(y_j) = -2n - 2 + 2j, 1 \leq j \leq \lceil \frac{n}{2} \rceil$

$g(y_{\lceil n/2 \rceil + j}) = 2n + 2 - 2j, 1 \leq j \leq \lfloor \frac{n}{2} \rfloor$

Here $g_e(E(S(C_n K_1))) = \{8n-2, 8n-6, \; 4n+10, 40+6\}$

$\cup \{-(8n-2), -(8n-6), \;, -(4n+10), -(4n+6)\}$

$\cup \{2n-2, -2n+2\} \cup \{3n, 3n-3, 3n-6, \;, 3(n+1)/2\}$

$\cup \{-3n, -(3n-3), -(3n-6), \;, -3(n+1)/2\}$

$\cup \{3n-1, 3n-4, \;, 3(n+7)/2\}$

$\cup \{-(3n-1), -(3n-4), \;, -3(n+7)/2\}$.

Then g is pair sum labeling.

Illustration of theorem is shown in figure

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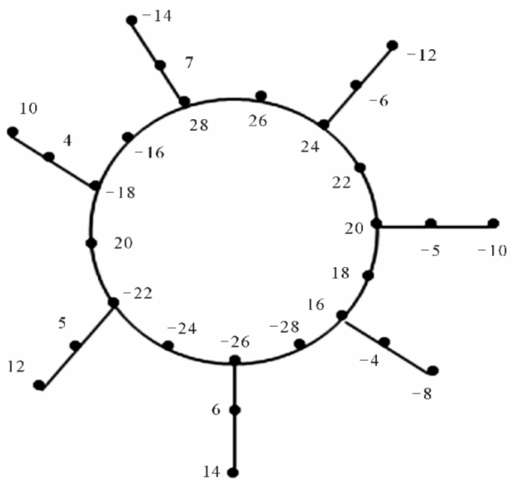

Figure : $S(C_nK_1)$

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