On Coloring Of Cycle Union Of R Copies Of (R ≥ 3) Fan Graph , Hanging Pyramid Hj_n Umbrella $U_{n,M}$ Graph

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Abstract:

In this paper, we introduce the new graphs namely drums graph, Barbell graph, and R copies of cycle union of Fan graphs. We investigate proper coloring for Hanging graph , Hanging pyramid and R copies of cycle union of Fan graphs is satisfying coloring condition.

Introduction:

In Graph theory, '**Graph coloring'** is a special case of Graph labeling. It is an assignment of labels traditionally called "colors" to elements of a graph subject to certain constraints [1].In its simplest form it is the way of coloring the vertices of a graph such that no two adjacent vertices share the same color. This is called **"Vertex coloring**". The concept of graph labeling gained a lot of popularity in the area of Graph theory.

Preliminaries:

A coloring using at most k colors is called a proper "k-coloring". The smallest number of colors needed to color a Graph G is called its "**Chromatic number**". A Graph that can be assigned k-coloring is k-colorable, and it is k-chromatic if its chromatic number is exactly k.[2]

A fan graph Fn = Pn + K1, is obtained from a cycle Pn, by attaching a pendant edge at each vertex of the Pn.

Let G be a graph and let G1, G2, G3, , Gr r≥2, be r copies of graph G. Then the graph obtained by adding an edge from Gi to Gi+1 (1≤r≤ i -1) and Gr to G1 is called a cycle union of G and is denoted by C(r. G)[5]

An umbrella graph U(m, n) is the graph obtained by joining a path P_n with the central vertex of a Fan F_n.[4]

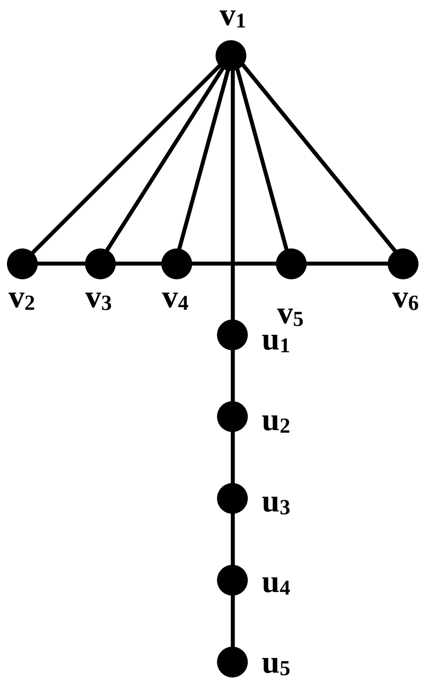

Figure 1 Umbrella $U_{5,5}$

Hanging pyramid graph obtained by attaching the apex of a pyramid graph to a new pendent edge .[3]

A Graph which is obtained by set out of vertices in to a fixed number of line with i vertices in the i th line and every line the jth apex in that line is joined to the j th and (j+1)th vertex of the nextline is called Pyramid graph

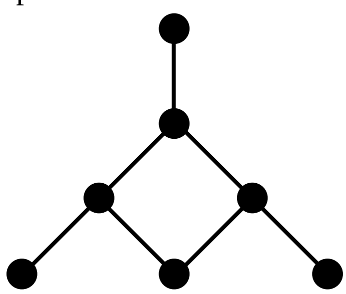

Figure 2 Hanging pyramid

Main results:

Theorem 1:

A cycle union of r copies of (r ≥ 3) fan graph acknowledges proper coloring and whose chromatic number is 3.

Proof:

Consider a fan graph $F_n = P_n + K_1$.

Let $G = C(r \cdot F_n)$. Let u_i $(1 \le i \le r)$ be the apex vertex of i^{th} copy of F_n.
u_{ij} $(1 \le i \le r, 1 \le j \le n)$ be the j^{th} vertex in P_n of i^{th} copy of F_n.
We note that $|V(G)| = r(n+1)$ and $|E(G)| = 2nr$.
Let $f : V(G) \to \{1, 2, 3\}$, coloring has to be classified in to three cases

Case (i): When r = 3, 6, 9 … 3k (k = 1, 2 …), coloring has to be given,

(i) $f(u_i) = 1$, when i = 1, 4, 7 … 3k–2 (k = 1, 2 …)
(ii) $f(u_i) = 2$, when i = 2, 5, 8 … 3k–1 (k = 1, 2 …)
(iii) $f(u_i) = 3$, when i = 3, 6, 9 … 3k (k = 1, 2 …)
(iv) $f(u_{1i}) = 2$, when i = 1, 3
(v) $f(u_{1i}) = 3$, when i = 2, 4
(vi) $f(u_{2i}) = 1$, when i = 1, 3
(vii) $f(u_{2i}) = 3$, when i = 2, 4
(viii) $f(u_{3i}) = 1$, when i = 1, 3
(ix) $f(u_{3i}) = 2$, when i = 2, 4

Illustration 1:

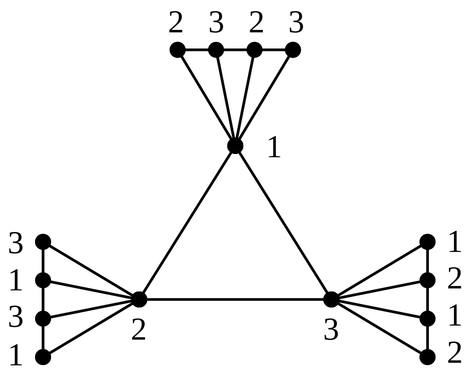

Figure 3. Cycle union of three copies of fan graph

Case (ii): When r = 4, 7, 10 … 3k+1 (k = 1, 2 …)

(i) $f(u_i) = 1$, for i = 1, 4, 7, 10 … 3k–2
(ii) $f(u_i) = 2$, for i = 2, 5, 8 … 3k–1
(iii) $f(u_i) = 3$, for i = 3, 6, 9 … 3k
(iv) $f(u_i) = 2$, for i = n
(v) $f(u_{1i}) = 2$, for i = 1, 3
(vi) $f(u_{1i}) = 3$, for i = 2, 4
(vii) $f(u_{2i}) = 1$, for i = 1, 3
(viii) $f(u_{2i}) = 3$, for i = 2, 4
(ix) $f(u_{3i}) = 1$, for i = 1, 3
(x) $f(u_{3i}) = 2$, for i = 2, 4

Illustration 1

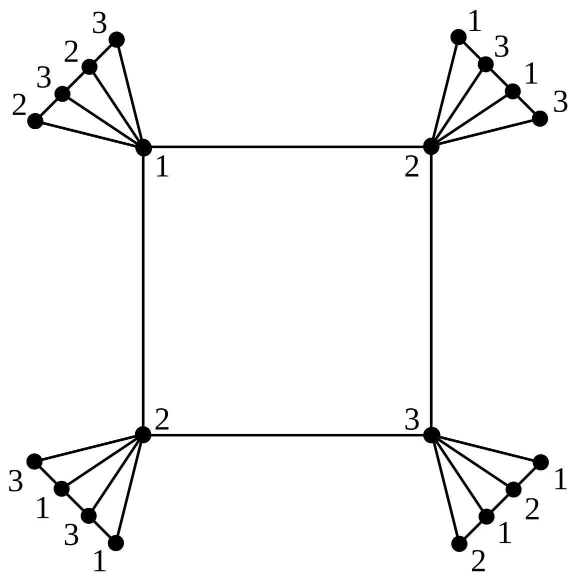

Figure 4. Cycle union of 4 copies of fan graph

Case (iii): When r = 5, 8, 11 … 3k+2 (k = 1, 2 …)

(i) $f(u_i) = 1$, for i = 1, 4, 7, 10 … 3k–2 (k = 1, 2 …)

(ii) $f(u_i) = 2$, for i = 2, 5, 8 … 3k–1 (k = 1, 2 …)

(iii) $f(u_i) = 3$, for i = 3, 6, 9 … 3k (k = 1, 2 …)

(iv) $f(u_i) = 1$, for i = n–1

(v) $f(u_i) = 2$, for i = n

(vi) $f(u_{1i}) = 2$, for i = 1, 3

(vii) $f(u_{1i}) = 3$, for i = 2, 4

(viii) $f(u_{2i}) = 1$, for i = 1, 3

(ix) $f(u_{2i}) = 3$, for i = 2, 4

(x) $f(u_{3i}) = 1$, for i = 1, 3

(xi) $f(u_{3i}) = 2$, for i = 2, 4

Illustration 1:

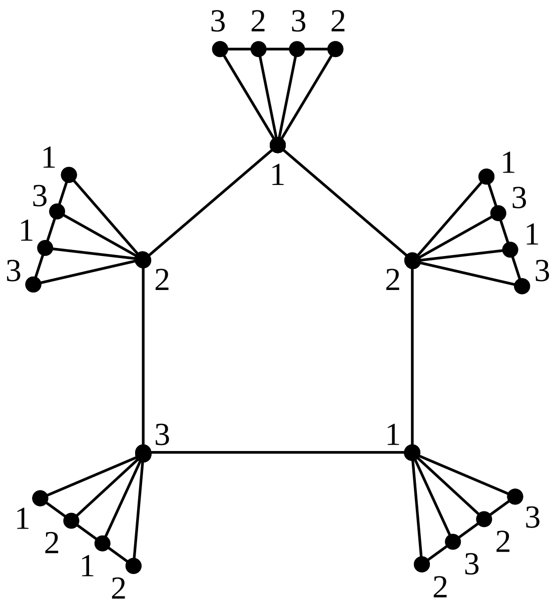

Figure 5. Cycle union of 5 copies of fan graph

Theorem 2

For $n \geq 3$, hanging pyramid HJ_n, whose chromatic number is 3.

Proof:

Let HJ_n be the hanging pyramid graph with $S = U \cup V$, where $U = \{w\}$ and $V = \{x, u_i, v_i, y_i \,;\, 1 \leq i \leq n-1\}$.

Let the function $f : V(G) \rightarrow \{1, 2\}$, the vertex labeling is given by

i) $f(w) = 2$

ii) $f(x) = 1$

iii) $f(u_i) = 1$, when 'i' is an odd

$= 2$, when 'i' is an even

iv) $f(v_i) = 1$, when 'i' is an odd

$= 2$, when 'i' is an even

v) $f(y_i) = 1$, when $i = 1, 4, 5, 6, \ldots$

$= 2$, when $i = 2, 3, 7, 8, \ldots$

Example: HJ_3

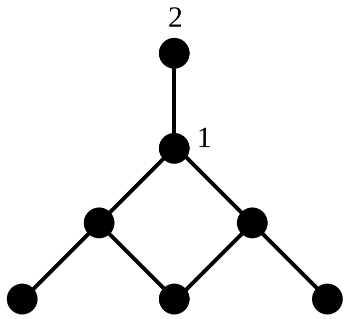

Figure 6 Hanging pyramid HJ3

Example: HJ_5

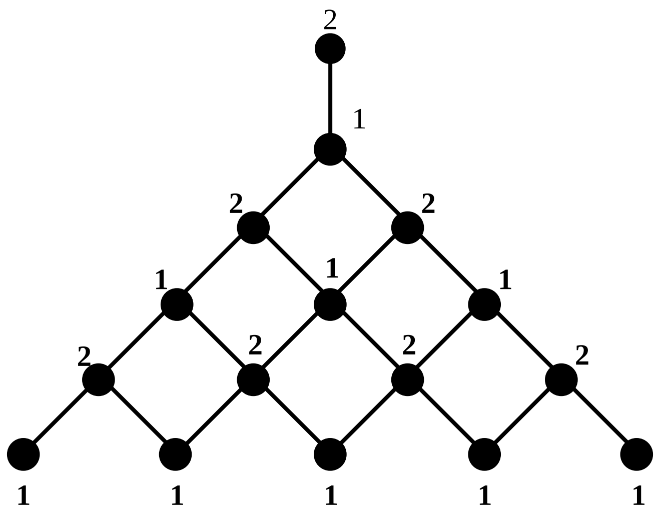

Figure 7 Hanging pyramid HJ5

Chromatic number for the above graph is 2.

Theorem 3

The umbrella $U_{n,m}$ graph where chromatic number is 3.

Proof:

Let V = {u_i, v_j / $1 \leq i \leq n$, $1 \leq j \leq m$} be the vertex set, and E = {ui ui+1, vj vj+1 / $1 \leq i \leq n-1$, $1 \leq j \leq m-1$}$\cup$ {v v_i / $1 \leq i < n$} be the edge set of the graph $U_{n,m}$.

Define $f : V(G) \to \{1, 2, 3\}$ such that

i) $f(v_1) = 1$

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ii) $f(v_i) = 2$, for $i = 2, 4, 6, \ldots, n$

iii) $f(v_i) = 3$, for $i = 3, 5, 7, \ldots, n-1$

iv) $f(u_i) = 2$, for $i = 1, 3, 5, \ldots, n-1$

v) $f(u_i) = 3$, for $i = 2, 4, 6, \ldots, n$

Example: Umbrella $U_{5,3}$

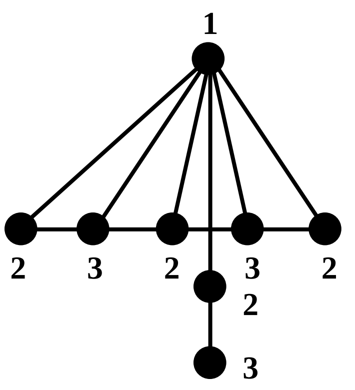

Figure 8 Umbrella graph $U_{5,3}$

Example: Umbrella $U_{6,4}$

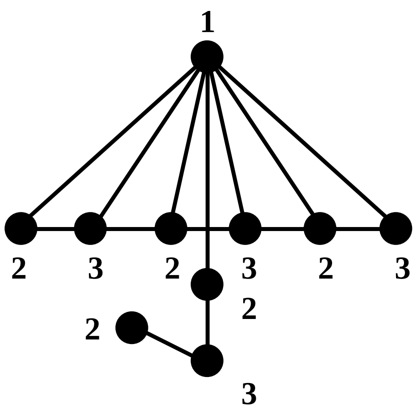

Figure 9 Umbrella graph $U_{6,4}$

In the above graphs, we conclude chromatic number of $U_{n,m}$ is 3.

Conclusion:

The Hanging pyramid graph which holds proper coloring and whose chromatic number is 3, Umbrella graph acknowledges the proper coloring and its chromatic number is 3 where n is any positive integer . A cycle union of r copies of (r ≥ 3) fan graph acknowledges proper coloring and whose chromatic number is 3 have been discussed.

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