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SOME NEW RESULTS ON GENERALIZED BS-HOMEOMORPHISMS

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Abstract

we introduce two classes of maps called BS-sgs-homeomorphisms and BS-gsg homeomorphisms and study their properties. These bitopological notions are generalized from the topological notions in bitopological spaces. These generalizations are substantiated with suitable examples and investigated with utmost care.

1. Introduction

Levine has generalized the concept of closed sets to generalized closed sets. Bhattacharyya and Lahiri have generalized the concept of closed sets to semi-generalized closed sets with the help of semi-open sets and obtained various topological properties. Arya and Nour have defined generalized semi-open sets with the help of semi-openness and used them to obtain some characterizations of s-normal spaces. Devi et al defined two classes of maps called semi-generalized homeomorphisms and generalized semi-homeomorphisms and also defined two classes of maps called sgc-homeomorphisms and gsc-homeomorphisms. sgs-homeomorphisms and gsg-homeomorphisms were recently introduced and investigated by Ozcelik and Narli.

In this chapter, we introduce two classes of maps called BS -sgs-homeomorphisms and BS -gsg homeomorphisms and study their properties. These bitopological notions are generalized

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from the topological notions in [14]. These generalizations are substantiated with suitable examples and investigated with utmost care.

2. New Definitions

Definition 1. Let P be a subset of X. Then P is said to be

(1) BS -semi-open if $P \subseteq$ BS -cl(BS -int(P));

(2) BS -semi-closed if BS -int(BS -cl(P)) $\subseteq$ P.

The complement of BS -semi-open set is called BS -semi-closed.

Remark 1. (1) Every BS -closed set is BS -semi-closed but not conversely.
(2) Every BS -open set is BS -semi-open but not conversely.

Definition 2. A map f: X → Y is called

(1) BS -closed if f(F) is BS -closed in Y for each BS -closed set F in X;

(2) BS -open if f(F) is BS -open in Y for each BS -open set F in X;

(3) BS -semi-closed if f(F) is BS -semi-closed in Y for each BS -closed set F in X.

Remark 2. Every BS -closed map is BS -semi-closed but not conversely.

Definition 3. Let P be a subset of X. Then BS -sint(P) = ∪{Gi : Gi is BS -semi-open in X and Gi $\subset$ P} (2) BS -scl(P) = ∩{Hi : Hi is BS -semi-closed in X and Hi $\supset$ P}.

Definition 4. Let P be a subset of X. Then P is said to be BS -sg-closed if BS -scl(P) $\subseteq$ U whenever P $\subseteq$ U and U is BS -semi-open.

The complement of BS -sg-closed set is called BS -sg-open.

The family of all BS -sg-closed sets of X is denoted by BS -sgc(X).

Remark 3. Every BS -semi-closed set is BS -sg-closed but not conversely.

Definition 5. Let P be a subset of X. Then P is said to be BS -gs-closed if BS -scl(P) $\subseteq$ U whenever P $\subseteq$ U and U is BS -open. The complement of BS -gs-closed set is BS -gs-open. The family of all BS -gs-closed sets of X is denoted by BS -gsc(X).

Remark 4. Every BS -sg-closed set is BS -gs-closed but not conversely.

Definition 6. A map f: X → Y is called

(1) BS -continuous if $f^{-1}(V)$ is BS -closed in X for each BS -closed set V in Y;

(2) BS -sg-continuous if $f^{-1}(V)$ is BS -sg-closed in X for each BS -closed set V of Y;

(3) BS -gs-continuous if $f^{-1}(V)$ is BS -gs-closed in X for each BS -closed set V of Y;

(4) BS -sg-closed if f(F) is BS -sg-closed in Y for each BS -closed set F of X;

(5) BS -sg-open if f(F) is BS -sg-open in Y for each BS -open set F of X.

Remark 5. Every BS -semi-closed map is a BS -sg-closed.

Definition 7. A map f: X → Y is called

(1) BS -gs-open if f(F) is BS -gs-open in Y for each BS -open set F of X;

(2) BS -gs-closed if f(F) is BS -gs-closed in Y for each BS -closed set F of X.

Remark 6. Every BS -sg-closed map is BS -gs-closed.

Definition 8. A map f: X → Y is called

(1) BS -sg-irresolute if $f^{-1}(V)$ is BS -sg-closed in X for each BS -sg-closed set V in Y;

(2) BS -gs-irresolute if $f^{-1}(V)$ is BS -gs-closed in X for each BS -gs-closed set V in Y.

Definition 9. A bijective map f: X → Y is called

(1) BS -homeomorphism if f is both BS -continuous and BS -open;

(2) BS -sg-homeomorphism if f is both BS -sg-continuous and BS -sg-open;

(3) BS -sgc-homeomorphism if f is BS -sg-irresolute and f^{-1} is BS -sg-irresolute;

(4) BS -gs-homeomorphism if f is both BS -gs-continuous and BS -gs-open;

(5) BS -gsc-homeomorphism if f is BS -gs-irresolute and f^{-1} is BS -gs-irresolute.

Remark 7. (1) Every BS -sgc-homeomorphism is BS -sg-homeomorphism but not conversely;

(2) Every BS -sg-homeomorphism is BS -gs-homeomorphism but not conversely;

(3) Every BS -gsc-homeomorphism is BS -gs-homeomorphism but not conversely.

Definition 10. A space X is called

(1) BS -$T_{1/2}$ if and only if every BS -gs-closed set is BS -semi-closed;

(2) BS -Tb if every BS -gs-closed set is BS -closed.

Definition 11. A map f: X → Y is called BS -gsg-irresolute if $f^{-1}(F)$ is BS -sg-closed in X for each BS -gs-closed set F of Y.

Definition 12. A bijective map f: X → Y is called BS -gsg-homeomorphism if f and f^{-1} are both BS -gsg-irresolute.

If there exists a BS -gsg-homeomorphism from X to Y, then the bitopological spaces X and Y are said to be BS -gsg-homeomorphic.

The family of all BS -gsg-homeomorphisms of X is denoted by BS -gsg h(X).

Definition 13. A map f: X → Y is called BS -sgs-irresolute if f−1(M) is BS -gs-closed in X for each BS -sg-closed set M of Y.

Definition 14. A bijective map f: X → Y is called BS -sgs-homeomo- rphism if f and f−1 are both BS -sgs-irresolute.

If there exists a BS -sgs-homeomorphism from X to Y, then the bitopological spaces X and Y are said to be BS -sgs-homemorphic.

3. Properties of BS-gsg-Homeomorphisms

Remark 8. The following two examples show that the concepts of BS -homeomorphisms and BS -gsg-homeomorphisms are independent of each other.

Example 1. Let X = {α, β, γ}, τ_1 = {ϕ, X} and τ_2 = {ϕ, X, {α}}. Then the sets in {ϕ, X, {α}} are called BS -open and the sets in {ϕ, X, {β, γ}} are called BS -closed. Let IX: (X, τ_1,τ_2) → (X, τ_1, τ_2) be the identity map. Clearly, I_X is BS -homeomorphism but it is not BS -gsg homeomorphism.

Example 2. Let X = {α, β}, τ_1 = {ϕ, X, {β}} , τ_2 = {ϕ, X, {α}}, σ_1 = {ϕ, X} and σ_2 = {ϕ,X}. Then the sets in {ϕ, X,{α},{β}} are called BS -open and BS -closed; and the sets in {ϕ, X} are BS -open and BS -closed. Let I_X: (X, τ_1, τ_2) → (X,σ_1,σ_2) be the identity map. Clearly, I_X is BS -gsg-homeomorphism but it is not BS -homeomorphism.

Example 3. Every BS -gsg-homeomorphism implies both BS -gsc-homeomorphism and BS -sgc homeomorphism. However the converse is not true as shown by the following example.

Example 4. Let X = {α,β,γ}, τ_1 = {ϕ, X,{β}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {β}} are called BS -open and the sets in {ϕ, X,{α,γ}} are called BS -closed. We have BS -sgc(X) = {ϕ,X, {α}, {γ}, {α, γ}} and BS -gsc(X) = {ϕ, X, {α}, {γ}, {α, β}, {α, γ}, {β, γ}}.

Let I_X: (X, τ_1, τ_2) → (X, τ_1, τ_2) be the identity map. Clearly I_X is both BS -gsc-homeomorphism and BS -sgc-homeomorphism. Since the set {β, γ} is BS -gs-closed but the set I_X^{-1} ({β,γ}) = {β,γ} is not BS -sg-closed, the identity map I_X is not BS -gsg-homeomorphism on X.

Remark 9. Every BS -gsg-homeomorphism implies both a BS -gs-homomorphism and a BS -sg homeomorphism.

However the converse is not true as shown by the following example.

Example 5. In Example 4, clearly I_X is both BS -gs-homeomorphism and BS -sg-homeomorphism. However, I_X is not BS -gsg-homeomorphism.

4. Properties of BS-sgs-Homeomorphisms

Remark 10. Every BS -sgc-homeomorphism and BS -gsc-homeomorphism implies BS -sgs-homeomorphism.

However the converse is not true as shown by the following examples.

Example 6. Let X = Y = {α, β, γ}, τ_1 = {ϕ, X, {α}, {α, β}}, τ_2 = {ϕ, X, {β}, {β, γ}}, σ_1 = {ϕ, Y, {β}} and σ_2 = {ϕ, Y, {α, β}}. Then the sets in {ϕ, X, {α}, {β}, {α, β}, {β, γ}} are called BS -open and the sets in {ϕ, X, {α}, {γ}, {α, γ}, {β, γ} are called BS -closed. Moreover the sets in {ϕ, Y, {β}, {α, β}} are called BS -open and the sets in {ϕ, Y, {γ}, {α, γ}} are called BS -closed. We have BS -sgc(X) = BS -gsc(X) = P(X) \ {{β}, {α, β}} where P(X) is the power set of X and BS -sgc(Y) = {ϕ, X, {α}, {γ}, {α, γ}} and BS -gsc(Y) = P(Y) \ {{β}, {α, β}}.
Clearly the identity map I_X: (X, τ_1, τ_2) → (Y,σ_1, σ_2) is BS -sgs-homeomorphism but it is not BS -sgc-homeomorphism.

Example 7. Let X = Y = {α, β, γ}, τ_1 = {ϕ, X, {α}}, τ_2 = {ϕ, X}, σ_1 = {ϕ, Y, {β}} and σ_2 = {ϕ, Y, {α, β}}. We have BS -sgc(X) = {ϕ, X, {β}, {γ}, {β, γ}}, BS -gsc(X) = {ϕ, X, {β}, {γ},{α, β}, {α, γ}, {β, γ}}, BS -sgc(Y) = {ϕ, Y, {α}, {γ}, {α, γ}} and BS -gsc(Y) = {ϕ, Y, {α}, {γ},{α, γ}, {β, γ}}. Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(α) = β; f(β) = α; f(γ) = γ. Clearly f is BS -sgs-homeomorphism but it is not BS -gsc-homeomorphism.

Remark 11. Every BS -homeomorphism is BS -sgs-homeomorphism. However the converse is not true as seen from the following example.

Example 8. In Example 7, clearly f is BS -sgs-homeomorphism but it is not BS -homeomorphism.

Remark 12. Every BS -sgs-homeomorphism is BS -gs-homeomorphism. However the converse is not true as seen from the following example.

Example 9. Let X = Y = {α, β, γ}, τ_1 = {ϕ, X, {α, β}}, τ_2 = {ϕ, X}, σ_1 = {ϕ, Y, {β}} and σ_2 = {ϕ, Y, {α, β}}. We have BS -sgc(X) = BS -gsc(X) = {ϕ, X, {γ}, {α, γ}, {β, γ}}, BS -sgc(Y) = {ϕ, Y, {α}, {γ}, {α, γ}} and BS -gsc(Y) = {ϕ, Y, {α}, {γ}, {α, γ}, {β, γ}}. Then, the identity map I: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is BS -gs-homeomorphism but it is not BS -sgs-homeomorphism.

Example 10. The map I: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is given by Example 9 is BS -sg-homeomorphism but it is not BS -sgs-homeomorphism.

Remark 13. (1) From the Example 10, we can see that any BS -sg-homeomorphism is not BS -sgs-homeomorphism.

(2) Every BS -gsg-homeomorphism is BS -sgs-homeomorphism and the converse is not true as seen from the following example.

Example 11. Let X = Y = {α, β, γ}, τ_1 = {ϕ, X, {α}}, τ_2 = {ϕ, X, {α, β}}, σ_1 = {ϕ, Y, {α},{β}, {α, β}} and σ_2 = {ϕ, Y, {β, γ}}. Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) be defined by f(α) = β,f(β) = α and f(γ) = γ. Clearly f is BS -sgs-homeomorphism but it is not BS -gsg-homeomorphism.

Theorem 1. (1) Every BS -sgs-homeomorphism from BS -$T_{1/2}$ space onto itself is BS -gsg homeomorphism. This implies that BS -sgs-homeomorphism is both BS -sgc-homeomorphism and BS -gsc-homeomorphism.

(2) Every BS -sgs-homeomorphism from a BS -T_b space onto itself is BS -homeomorphism. This implies that BS -sgs-homeomorphism is BS -gs-homeomorphism, BS -sg-homeomorphism, BS -sgc-homeomorphism, BS -gsc-homeomorphism and BS -gsg-homeomorphism.

Proof. (i) In a BS -$T_{1/2}$ space, every BS -gs-closed set is BS -semi-closed. (ii) In a BS -T_b space, every BS -gs-closed set is BS -closed.

Example 11. Let X = Y = {α, β, γ}, τ_1 = {ϕ, X, {α}}, τ_2 = {ϕ, X, {α, β}}, σ_1 = {ϕ, Y, {α}, {β}, {α, β}} and σ_2 = {ϕ, Y, {β, γ}}. Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) be defined by f(α) = β, f(β) = α and f(γ) = γ. Clearly f is BS -sgs-homeomorphism but it is not BS -gsg-homeomorphism.

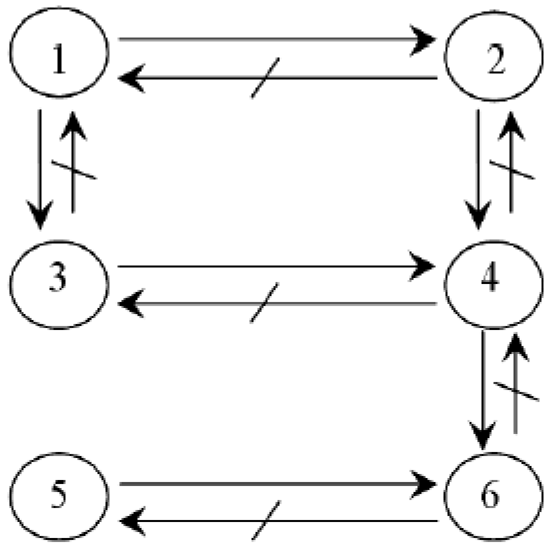
Theorem 1. (1) Every BS -sgs-homeomorphism from BS -$T_{1/2}$ space onto itself is BS -gsg homeomorphism.This implies that BS -sgs-homeomorphism is both BS -sgc-homeomorphism and BS -gsc-homeomorphism.
(2) Every BS -sgs-homeomorphism from a BS -T_b space onto itself is BS -homeomorphism. This implies that BS -sgs-homeomorphism is BS -gs-homeomorphism, BS -sg-homeomorphism, BS -sgc-homeomorphism, BS -gsc-homeomorphism and BS -gsg-homeomorphism.

Proof.

(i) In a BS -$T_{1/2}$ space, every BS -gs-closed set is BS -semi-closed. (ii) In a BS -T_b space, every BS -gs-closed set is BS -closed.

5. Conclusion

We obtain the following diagram from the above discussions.



Where

(1) BS -gsg-homeomorphism

(2) BS -sgc-homeomorphism

(3) BS -gsc-homeomorphism

(4) BS -sgs-homeomorphism

(5) BS -sg-homeomorphism

(6) BS -gs-homeomorphism

6. References

[1] Abd El-Monsef, M. E., El-Deeb, S. N. and Mahmoud. R. A. β-open sets and β-continuous mappings, Bull. Fac.Sci. Assiut Univ., 12(1983), 77-90.

[2] Athisaya Ponmani, S. and Lellis Thivagar, M. Remarks on ultra semi-T_Y S-spaces, Antartica Journal of Mathematics, XXXIIM(4)(2006), 1757-1766.

[3] Balachandran, K., Sundaram, P. and Maki, H. On generalized continuous maps in topological spaces, Mem.Fac. Sci. Kochi Univ. Math., 12(1991), 5-13.

[4] Bourbaki, N. General topology, Part I, Addison-Wesley, Reading, Mass., (1966).

[5] Dunham, W. T1/2-spaces, Kyungpook Math. J., 17(1977), 161-169.

[6] Kelly, J. C. Bitopological spaces, Proc. London Math. Soc., 13(1963), 71-89.

[7] Lellis Thivagar, M., Raja Rajeswari, R. and Athisaya Ponmani, S. Characterizations of ultra-separation axioms via (1,2)α-kernel, Lobachevski Journal of Mathematics, 25(2005), 50-55.

[8] Lellis Thivagar, M. and Raja Rajeswari, R. Bitopological ultra spaces, Southeast Asian Bulletin of Mathematics,31(2007), 993-1008.

[9] Levine, N. A decomposition of continuity in topological spaces, Amer. Math. Monthly, 68(1961), 44-46.

[10] Levine, N. Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19(2)(1970), 89-96.

[11] Levine, N. Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.

[12] Nj˚astad, O. On some classes of nearly open sets, Pacific Journal of Mathematics, 15(1965), 961-970.

[13] Raja Rajeswari, R., Lellis Thivagar, M., Athisaya Ponmani, S. and Saeid Jafari. Some weak separation axioms in bitopological spaces, Int. Journal of Math. Anal., 3(2)(2009), 87-93.

[14] Ravi, O. Studies on new class of functions in bitopological spaces, Ph. D., Thesis, Madurai Kamaraj University,Madurai (2006).

[15] Rose, D. On Levines decomposition of continuity, Canad. Math. Bull., 21(1978), 477-481.