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Research Paper

A Some Contribution to Graph Labelling and Graph Theory

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Abstract

Graph theory is the one of the most important concept which takes the great roll in the electronic devices IC's. These components are known as chips and include complicated, layered microcircuits that can be described by lines or arcs as sets of points. By utilizing graph theory, mathematicians create integrated chips with maximum part density and minimum total interconnecting conductor length.

Key Words: Prime Cordial Labelling , Graceful Graphs

Introduction:

Assume we wish to decompose a full graph G into trees that are all isomorphic to each other. In other words, we want to partition G's edges so that each set of the partition's edges induces a subgraph that is isomorphic to a given tree T. Ringel [20] proposed that the complete network K2n1 may be decomposed into 2n 1 trees isomorphic to T for every tree T with n vertices..

In 1966, Rosa [21] pioneered elegant labelling, which he dubbed _- labelling at the time. In 1972, Golomb [12] popularised the phrase "graceful." Rosa demonstrated that Ringel's

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hypothesis holds if every tree is elegant. Researchers have been attempting to verify Ringel's conjecture using the Graceful Tree Conjecture, which states that every tree is graceful, since then.

Over time, though, elegant graphs have garnered their own worth of study. The choice issue of graceful labelling is featured as the "Open Problem of the Month" by David S. Johnson in his NP-completeness column from 1983 [16]. There's also the International Workshop on Graph Labeling, where elegant labelling is one of the key issues, and Gallian's [11] comprehensive study of the field, which is continually updated.

The definitions used throughout the book are listed in Section 1.1. We offer a formal definition of graceful graph labelling, as well as the gracefulness of particular graph classes and some general findings on gracious graph labelling. We focus on the Graceful Tree Conjecture's outcomes, giving many techniques to solving the conjecture. Finally, we turn our attention to generalised cone graphs, which are a graph class created by the joining of two graphs. We examine existing theoretical results and provide new computational results that demonstrate the gracefulness of generalized cone graph families and make a hypothesis about the non-graceful ones..

1.1 Definitions

In this section, we give most of the definitions and notation of graph theory used in this text. For any missing definition, see Bondy and Murty [7].

An ordered pair (V, E) is a graph G, where V is a collection of components called vertices and E is a set of unordered pairs of different vertices from V called edges. We say an edge e connects two vertices u and v, denoting as $c = uv$, and we say u and v are adjucent if they are connected by an edge. The set of adjacent vertices of a vertex u is denoted as $N(u)$, and it is also called the set of neighbors of u. The degree of a vertex u is $d(u) = |N(u)|$, the number of neighbors of u.

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For a given graph G, when the vertex set and the edge set are not given explicitly, we refer to them as $V(G)$ and $E(G)$, and we use the letters n and m as the number of vertices and edges, respectively.

A walk in a graph is a finite sequence of vertices $W = (v_0, v_1, \dots, v_k)$ such that $v_1 v_{1+1}$ is an edge of the graph. If the walk W does not go through an edge twice, we say W is a trail, and if it does not go through a vertex twice, we say W is a path. A path starting in u and ending in v is called a uv-path.

The number of edges on a path determines its length, and the distance between two vertices u and v determines the length of the shortest path between them, which is indicated as dist (u, v). If there is no path between u and v, then dist $(u, v) = \infty$.

A walk is said to be closed if the first and the list vertices are the same. A cycle is a closed trail in which all vertices, but the last, are distinct.

A graph's Eulerian trail (or Eulerian route) is a path that passes over each edge of the graph precisely once. In the same way, an Eulerian tow (or Eulerian cycle) travels each edge precisely once. If a graph permits an Eulerian cycle, it is said to be Eulerian.

we are assigning labels (colors) to vertices and/or edges of the graph. Throughout this text, we have the codomsins as a finite subset of NN, and we denote $[a, b] = \{a, a + 1, \dots, b\}$.

Many problems of graph theory consist in finding a vertex or an edge labeling for a graph satisfying certain properties. For example, a proper vertex coloring is a vertex coloring such that adjacent vertices have different colors, and a very well known problem is to find for a given graph G the minimum k such that there exists a proper vertex coloring f of G with $|\text{Im}\ (f)| = k$. In our case, we are interested in graceful labeling.

In fact, Sylvester introduced the word "graph" in a paper published in Nature in 1878, where he drew a comparison between algebra and molecular diagrams of "quantic invariants" and "co-variants.

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Extremal Graph Theory

It is a subset of the philosophy of graphs. The theory of severe graphs studies maximal or minimal graphs that obey a certain property. We may take extremality in terms of numerous invariants of graphs, such as order, size or girth. It studies a graph 's global properties that affect the graph 's local substructures.

For eg, "graphs on n vertices have the highest number of edges" is a basic extreme graph theory. The extreme graphs are trees on n vertices with n-1 edges. We want to find the minimum value of m because of graph P, an invariant u, and a set of graphs H, such that any graph in H that has u greater than m has the property P. In the example above, H was the n-vertex graph collection, P was the cyclic property, and u was the number of edges in the graph. There must be a loop for each graph on n vertices with more than n−1 edges.

The above-mentioned type is concerned with many basic outcomes in severe graph theory. For eg, Turán 's theorem asks the question of how many edges an n-vertex graph may have until it must have a clique of size k as a sub graph. Instead of cliques, if total multi-party graphs are asked the same question, the Erdős-Stone theorem offers the answer..

With the theory of Ringel (1964) and a paper by Rosa (1967), much curiosity in graph marking started in the middle of 1960. The famous conjecture of Ringel-Kotzig (1964, 1965) that all trees are graceful remains unsettled. Rosa (1967) implemented β- valuation, alph-valuation and other marking in his classic paper as a method for decomposing complete graphs. The β- valuation was later called Golomb's graceful labelling (1972) and now this is the most commonly used term. In conjunction with their studies on the issue of additive bases resulting from error-correcting protocols, Graham and Sloane (1980) adopted harmonious marking.

The two basic labels that have been thoroughly researched are elegant labelling and harmonious labelling. In the field of graph labelling, varieties of graceful and harmonious

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labelling, including alpha-valuation, elegant labelling and cordial labelling, were implemented with various motives and domain restriction programming models. Therefore, separate labelling of graphs such as graceful labelling, prime labelling, cordial labelling, absolute cordial labelling, k-graceful labelling and unusual graceful labelling, etc., have been used in the subsequent years.

So far, multiple marking systems have been developed and many scholars are still exploring them. Within mathematics and other fields of information science and networking networks, graph marking has immense applications. In the work of Yegnanaryanan and Vaidhyanathan, different applications of graph labelling are mentioned.

More than six hundred articles have published on this issue over the course of four decades. This illustrates the field's accelerated development. The fundamental understanding, however, that the characterization of graceful and other labelled graphs appears to be one of graph theory's most difficult and difficult problems. Recognizing the simplest labelled graph, namely the cordial graph, is currently an NP-complete problem, see Kirchherr (1993). Many mathematicians have shown interest in having required criteria and different appropriate criteria on labelled graphs owing to these inherent difficulties of such marking, aiming to increase the comprehension of the characteristic existence of the labelled graphs. While the area of graph labelling deals essentially with theoretical analysis, the topic of graph labelling in the applied fields has also been the topic of research for a long time. The labelled graphs serve as useful templates for a large variety of applications such as coding theory, X-ray crystallography, radar, physics, circuit architecture and address of the communication network (refer to Golomb (1972), Bloom and Golomb (1997) and Bermond (1979)). It is important to notice that The labelling (or measurement) of graph G is the assigning of labels f from a collection of non-negative integers to a set of vertices of graph G, which induces a label identified by the labels f(u) and f(v) for each edge uv. The two basic marks that have

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been researched in the field of graph labelling are graceful and harmonious labelling.

BASIC DEFINITIONS:-

Graph Labeling

The marking of the graph is an attribution to the vertices or edges of the values or both using such criteria.

Vertex Labeling (Edge Labeling):-

If the set of vertices is the mapping(edges) area, so the labelling is called the labelling of the vertex.

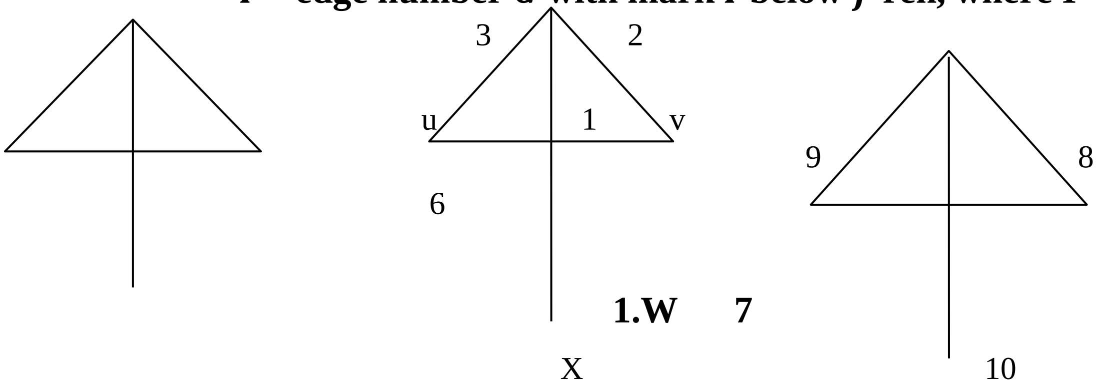
A label: (G) as $\{0\ ,1\}$ is classified as a binary vertex label of G and (v) is classified as G under f vertex of V.

Rating 1.If an edge is $e = u\ V$, edge marking is caused.

f: (G) to the point of $\{0\ ,1\}$ f to the point of) $-\ f(V)$.

$V(i)$ = number of vertices of G with the mark I below f

I = edge number G with mark I below f ren, where I = 0 or



Unlabelled graph edge-labeled graph vertex-labeled graph

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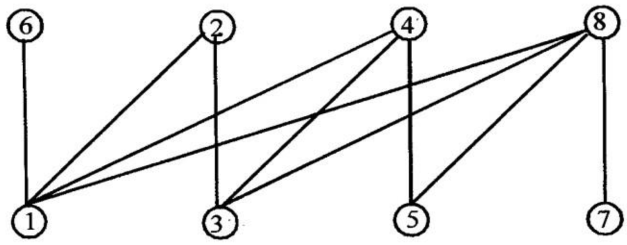
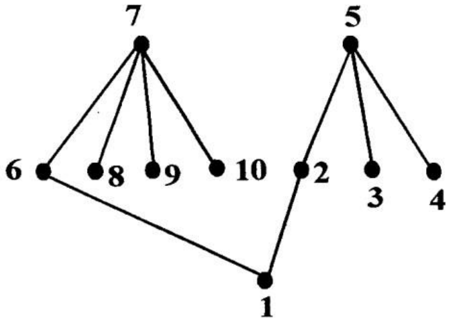
Prime Labeling





An injective function is the primary marking of a graph $\boldsymbol{G}$: $(\boldsymbol{G})$ da $\{\mathbf{1}, \mathbf{2}, . |\boldsymbol{V}(\boldsymbol{G})|\}$, for all adjacent pairs $\boldsymbol{u}$ and $\boldsymbol{v}$, $\boldsymbol{gcd}$ of $(\boldsymbol{f}(\boldsymbol{u}), \boldsymbol{f}(\boldsymbol{v})) = \mathbf{1}$. And the prime number graph is known as the main labelling graph.

Prime Cordial Labeling

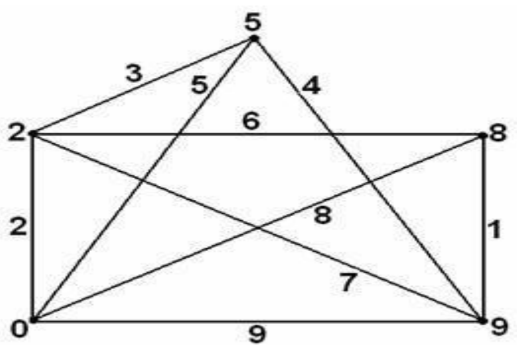
A prime cordial labeling of a graph G where vertex set (G) is a bijection$f$': $(G) \rightarrow$ {1, 2, 3, 4,. . . , $|V(G)|$} and if the induced function f: $E(G) \rightarrow \{0, 1\}$ is defined by

f'($e = u$V) = 1, if $gcd(f\ (u)\ , f$ (V)) = 1 = 0, otherwise, the number of edges labeled with 0 and edges labeled with 1 differ by at most 1. A graph which labeled with prime cordial labeling is called a prime cordial graph.

The values f (u), f' (u, v) are called graceful labels of the vertex U and the edge (U, V) respectively.

A cyclic decomposition of K_{2q+1} is obtained as follows

1) Choose any graph G with graceful labeling.
2) Identify the edges of G with q suitable edges of K_{2q+1} , whereq=|E(G)|
3) Each vertex and each edge of G is rotated 2q times from the original position.



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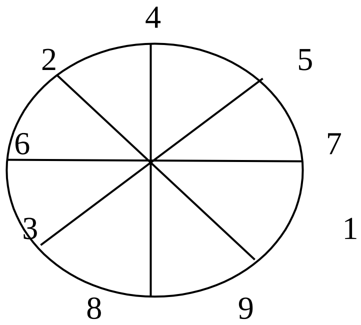
Wheel Labeling



Graceful graph

- ❖ May the graph G=(V, E) be a q edge graph. Graceful G labelling is an f-injective function: v by the way,{0,1,2,⋯,q} so that the induced labelling of the edge, f(uv)= is a bijection of E into the set{1,2,⋯,q}.
- ❖ The conjecture given is the problem of definition of all graceful graphs and the graceful tree. The explanation why the diagrams are named differently. Few variants of successful labelling have also recently been added, for example, edge graceful labelling, graceful labelling by Fibonacci unusual graceful labelling.
- ❖ **Steps for Graceful Labeling of a graph**

Use distinct nonnegative integers like {0,1,2,3,….,9} to label vertices no need to use all

Each edge value is the absolute difference of the values on adjacent vertices The adjacent 3 vertices should not be labeled with 0,1,2.

Values of edge must be distinct integers {1,2,3,…,9}.

The labeling of the vertices should be in the manner, where we get unique values to each edge.

Example: Graph with 5 vertices and 8 edges.

8 0

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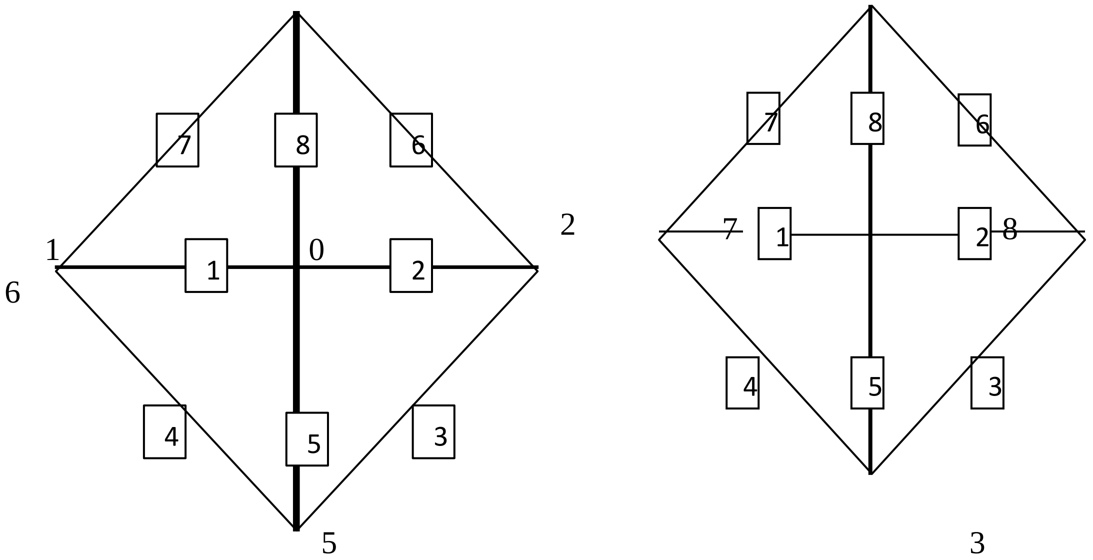
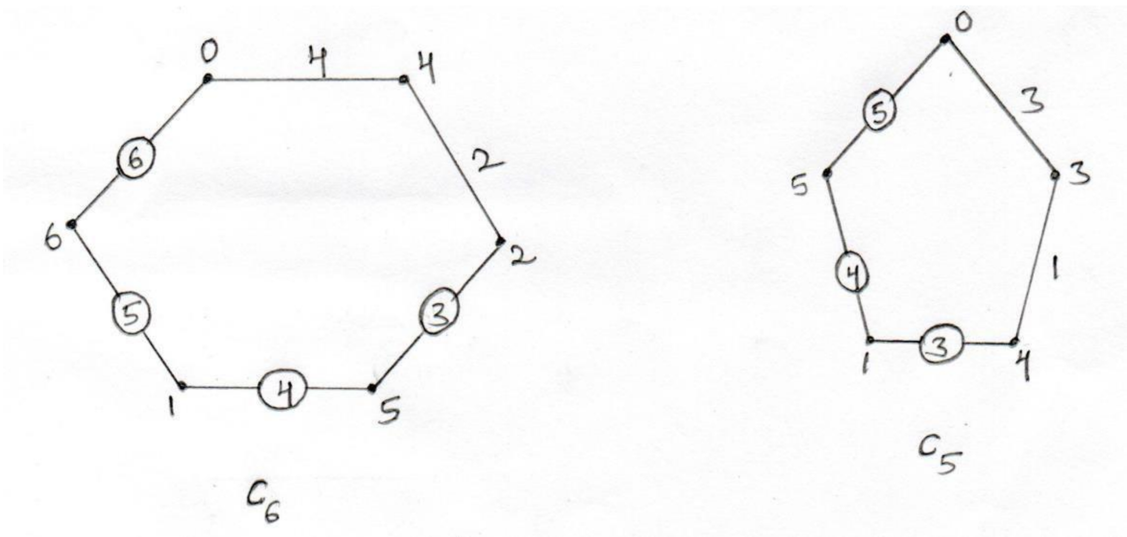


Graceful Labeling of cyclic graph C_n

According to S. P. Rao Hebbare, Graceful Cycles, Utilitas Mathematica, (1976) states that steps for graceful labeling of Cycle graph c_n is Graceful if n=4k or n=4k+3 for some integer k≥0, And non-graceful if n=4k+1 or n=4k+2. Non-graceful graphs: - If K=1 n=4K+1 then n=5 Or n=4K+2 = 6. Therefore C_6 and C_5 are non graceful graph.



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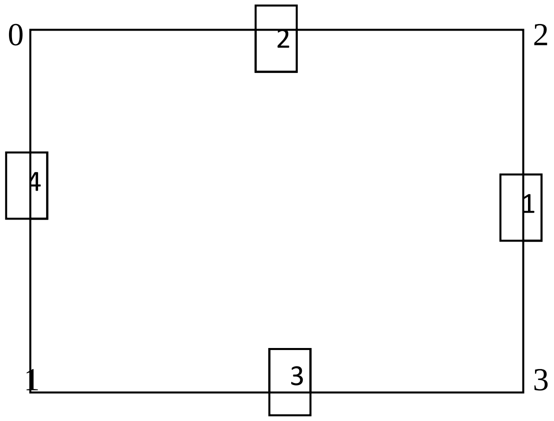
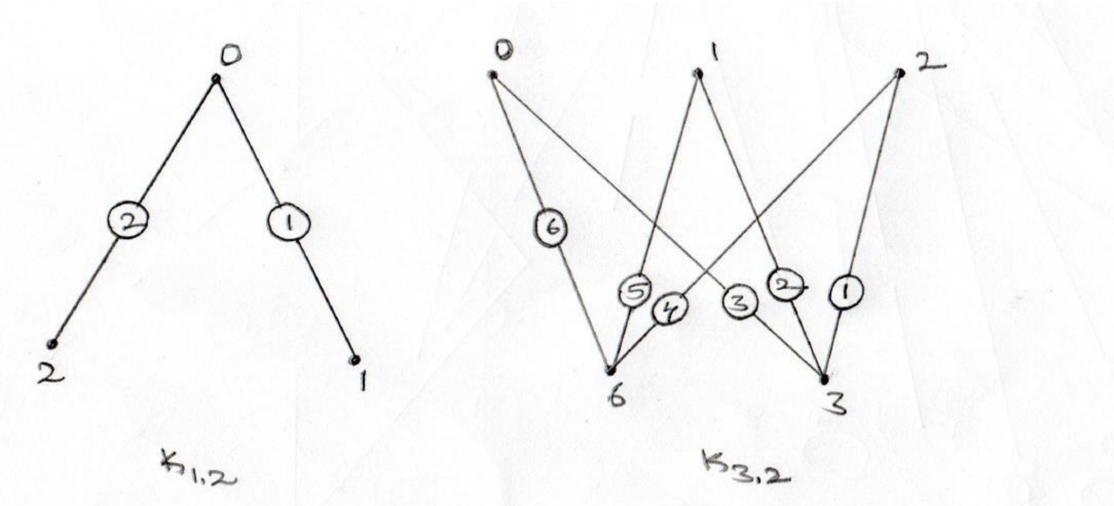
Graceful graphs: - C4



Graceful graph of $K_{m,n}$



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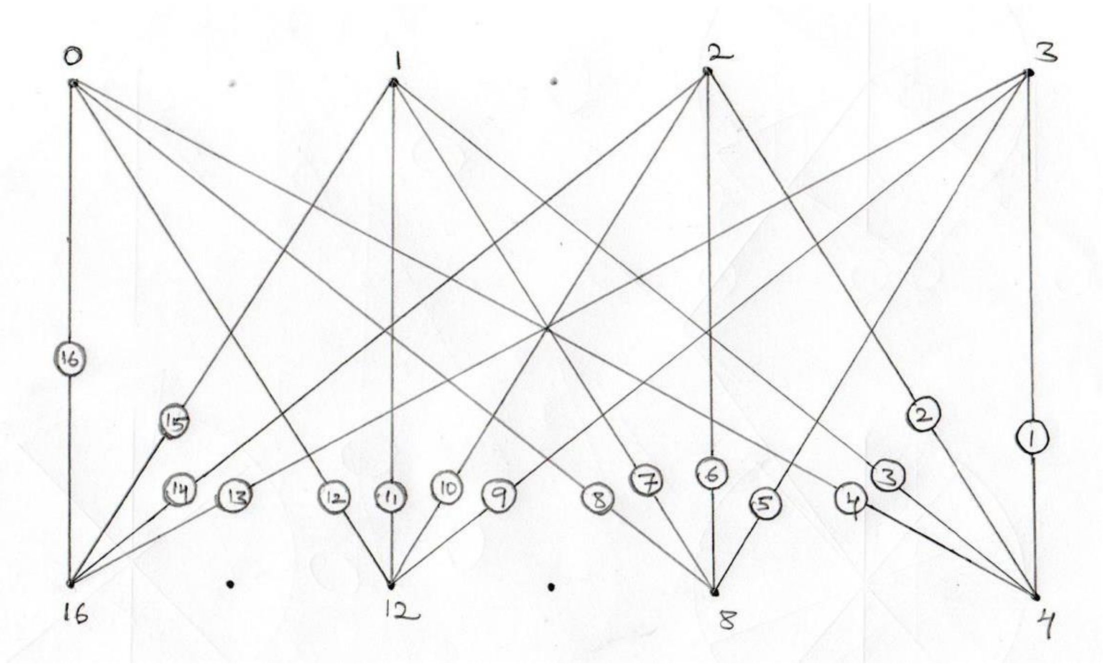

Prime Labeling Of $K_{4,4}$

Prime Labelling of Tree

Entringer conjectured around 1980 that all trees have a prime marking. Paths, stars, caterpillars, complete binary trees, spiders and all trees with fewer than 16 orders are among the groups with trees considered to have prime marking. For a tree to have a prime marking that would specify many families of prime trees, we give a necessary requirement. We are still studying the prime marking of banana trees in special groups. All cycles Cn and the disjoint union of C2k contain other graphs with prime naming. The full graph Kndoes does not have a primary labelling for n4 and the wheel Wn with n spokes is primary if and only if we review the primary labelling of such join graphs and product graphs, namely, n is also studied.C_{n+mk1}, P_{n+mk1}, $P_n \times P_2$, $P_n \times P_3$ *and* $K_{1n} \times P_2$.

Results Proved on Labeling Of Tree

Every tree is prime, Entringer conjectured. We give a necessary requirement in the theorems below for a tree to have prime marking. To the list of prime named plants, we even introduce a few unique groups of banana trees.

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Like above applications some other applications listed below

- Graph Labelling in Communication Relevant to Adhoc Networks
- Secure Communication in Graph
- By Using Key Graphs
- Identification of Routing Algorithm with Short Label Names
- Automatic Routing with labeling
- Security with reducing the packet size using labeling schema
- Fast Communication in sensor networks Using Radio Labelling.

Conclusion

Some findings corresponding to prime labelling and some results corresponding to elegant Fibonacci labelling are investigated and some results corresponding to tree graph prime labelling are investigated. For other graph families and in the sense of deferred graph labelling problems with the specified condition, related findings may be derived. We applied the conditions given in the theorem of graph prime labelling and tree prime labelling to the separate graphs here, and we succeeded in drawing the graph with the conditions given. Graph Labelling is a valuable tool that, as described above, allows it simple in different areas of networking.

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