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# Pair sum labeling of some standard graphs.

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**Abstract**

In the mathematical discipline of graph theory, a graph labelling is the assignment of labels, traditionally represented by integers, to edges and vertices of a graph. In this case, the graph is called an edge labeled graph.

A graph with a pair sum labeling is called a pair sum graph. In this paper we investigate the pair sum label-ing behavior of some trees which are derived from stars and bistars. Finally, we show that all trees of order nine are pair sum graphs.

Keywords:

Graph, vertices, edge, corona, cardinality, map, labeling, etc.,

Introduction

The graphs considered here will be finite, undirected and simple. The symbols V and E will denote the vertex set and edge set of a graph G. The cardinality of the vertex set of a graph G is denoted by p and the cardinality of its edge set is denoted by q. The corona G1G2 of two graphs G1 and G2 is defined as the graph obtained by taking one copy of $G_1$ (with $p_1$ vertices) and $p_1$ copies of G2 and then joining the ith

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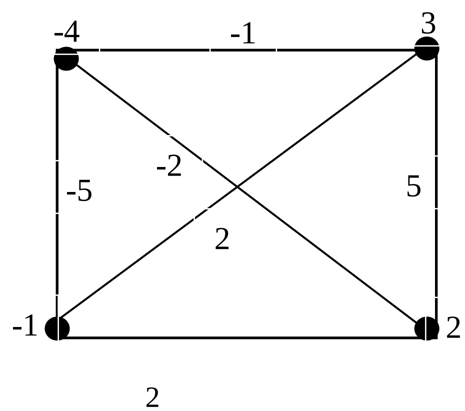
vertex of $G_1$ to all the vertices in the ith copy of $G_2$ . If e = xy is an edge of G and z is a vertex not in G then e is said to be sub divided when it is replaced by the edges xz and zy. The graph obtained by subdividing each edge of a graph G is called the subdivision graph of G Terms not defined here are used in the sense of Gary Chartrand [2] and Harary [3] . Here we introduce a new motion called pair sum labeling.

**Definition** :LetGbea(p, q)graph. Aone-onemap g:V(G) $\rightarrow\{\pm 1,\pm 2,...,\pm p\}$ issaid tobeapair sumlabelingiftheinduced edgefunction $g_e$:E(G) $\rightarrow Z-\{0\}$definedby $g_e(uv)= g(u)+ g(v)$ isone-one and $g_e(E(G))$iseither oftheform $\{\pm k_1,\pm k_2,...,\pm k_{q/2}\}$ or$\{\pm k_1,\pm k_2,...,\pm k_{(q-1)/2}\}\cup\{k_{(q+1)/2}\}$according asqisevenorodd. Agraphwithapair sum labeling defined on it is called pair sum graph

**Example** : A pair sum labeling of $k_4$ is




**Observation :**If g is a pair sum labeling defined on G then

$\sum_{x\in v} d(x) f(x) = 0$ iff G is of even size.

Proof : Follows since $\sum_{x\in v} d(x) f(x) = \sum_{xy\in E(G)} g_e(xy)$

**Observation :** If g is a pair sum labeling then y and –y are not labels of two adjacent vertices .

Proof: Otherwise,zero appears as a edge label.

Theorem : Any path is a pair sum graph

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Proof:

Let $P_x$ be the path $x_1x_2.....x_m$

Case (i) : m is odd

Define a map g: V ($P_x$) → {±1,±2,….±m} by

$g(x_{(m-1)/2+1}) = j$ ; $1 \leq j \leq (m+1)/2$

$g(x_{(m-1)/2-2j+2}) = -2-2j$ ; $1 \leq j \leq (m-1)/4$ if m=1 (mod 4)

$1 \leq j \leq (m+1)/4$ if m=3 (mod 4)

$g(x_{(m-3)/2 -2j +2}) = -2j +1$ ; $1 \leq j \leq (m-1)/4$ if m=1 (mod 4)

$1 \leq j \leq (m-3)/4$ if m=3 (mod 4)

Clearly g is a pair sum labeling

Case iii : m is even

Assign the label to the vertices of the path $P_{m-1}$ : $x_2$, $x_3$, ……$x_m$ as in case (i). Label the vertices $x_1$ by-m .Therefore in both care $P_m$ is a pair sum graph.

**Theorem** :- Cycle Cn is a pair sum graph

Poof: The vertices of the cycle Cn be $x_1x_2.....x_nx_1$

Case i : n = 4k +2

Define gV($C_n$)→ {±1,±2,...,±n} by

$g(x_j)= j$ j=1, 2…….2k+1

$g(x_{2k+1+j}) = -j$ j=1,2…..2k+1

Case ii : n=4k

Define $g(x_J) = j$ :j=1,2,3……$2_{k-1}$

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$g(x_{2k}) = 2k+1$

$g(x_{2k+1}) = -(2k+1)$

Case iii: n=2k+1

Define $g(x_j)=j;\ 1\leq j\leq k+1$

$g(x_{k-2j-2}) = -2-2j\ ;\ 1\leq j\leq k-2$

$g(x_{k-2j+1}) = 1-2j\ ;\ 1\leq j\leq k-2$

Obviously g is a pair sum labeling

Next we look into pair sum labeling of complete graph $K_m$.

**Theorem**: The complete graph $K_m$ is a pair sum graph if $m\leq 4$.

Proof:-

$K_1,K_2$ are pair sum graphs from theorem 4.1.$K_3$ is a pair sum graph by theorem 4.2 pair sum labeling of $K_4$ is given in the example 2.2 for the opposite direction let m>4 suppose g is a pair sum labeling by observation 3.2, either y or – y must be a vertex label for all $y\leq m$

**Case i:** 1 is a vertex label.

**Sub case (A)** : m is a vertex label.

**Claim 1:** m-1 is not a vertex label.

Suppose m-1 is a vertex label. Then 2m-1 is an edge label. But -2m+1 is not an edge label.

Hence m-1 is not a vertex label. By observation 3.2 –m+1 is a vertex label.

**Claim 2**: m-2 is not a vertex label.

Suppose m-2 is a vertex label. Then 2m-2 is an edge label. To get an edge label 2m+2 both –m and –m+2 are vertex label, This is not possible . Therefore –m+2 is a vertex label.

**Claim 3**: m-3 is not a vertex label.

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Suppose m-3 is a not a vertex label. If 2 is a vertex label then –m+3, occurs as an edge label in twice . If -2 are a vertex label then m-2 occurs as a edge label in twice. This is not possible.

**Claim 4** : -m +3 is not a vertex label.

In the case of –m+3 is a vertex label 3 or 1 or –m occurs as an edge label in at least twice. By claim 3 and 4 we get a contradiction .

**Sub case(B):** -m is a vertex label

similar arguments as in the sub case (A), -m+1, -m+2 are not vertex labels.

**Claim 5**: -m+3 is not a vertex label.

Suppose –m+3 is a vertex label then -2m+3 is an edge label. But 2m-3 is not an edge label. Therefore -m+3 is not a vertex label.

**Claim 6**: m-3 is not a vertex label.

Suppose m-3 is a vertex label. If 2 is a vertex then an edge label m occurs twice. If -2 is a vertex label the edge label-1 occurs twice. If -2 is a vertex table the edge label -1 occurs twice. By claim 5 and 6, we get a contradiction.

**Case ii**: -1 is a vertex label

**Sub case (C)** : m is a vertex label.

**Claim 7**: -m +1 is not a vertex label

Suppose –m+1 is a vertex label then -2m +1 is an edge label. But 2m-1 is not an edge label, therefore -m+1 is not a vertex label. This implies m-1 is vertex label.

**Claim 8: -** m+2 is a not a vertex label

Suppose –m+2 is a vertex label then -2m +2 is an edge label, But 2m-2 is not an edge label, therefore -m+2 is not a vertex label. This force m-2 is a vertex label.

**Claim 9:** n-3 is not a vertex label

If 2 and 3 are vertex label then an edge label m occurs twice as an vertex label

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If 2 and -3 is vertex then m-4 occurs twice as an edge label. If -2 is a vertex label

Then -3 occurs twice is an edge label, therefore m-3 is not an edge label.

**Claim 10**: -m+3 is not a vertex label

Suppose –m+3 is a vertex label. If 2 is a vertex label then 1 occurs twice as an edge level. If -2 is a vertex label than m-3 occurs twice as an edge label. By claim 9 and 10 we get contradiction

**Sub case(D): -** m is a vertex label

As in sub case (C) clearly both m-1 and m-2 are not vertex labels.

**Claim 11:** m-3 is not a vertex label.

Suppose m-1 is a vertex label. If 2 are a vertex label then 1 occurs twice as an edge label. If -2 is a vertex label then –m occurs as an edge label twice. Therefore m-3 is not an edge label.

**Claim 12:** -m+3 is not a vertex label .

Suppose –m+3 is not a vertex label. If 2 is a vertex label then 1 occurs twice as an edge label. If -2 is a vertex label then –m occurs twice as an edge label. By claim 11 and 12 we get a contradiction . By case (i) and (ii) neither 1 nor -1 is not a vertex label . Therefore $K_m$ ,m>4 is not a pair sum graph.

**Theorem :** The star $k_{1,m}$ is a pair sum graph

Proof:- Let $V(K_{1, m)} =\{x_i,x_j : 1\leq j\leq m\}$

and $E(K_{1, m})=\{xx_j: 1\leq j\leq m\}$

Define $g:V(K_{1,m})\rightarrow \{\pm 1, \pm 2 \ldots \pm(m+1)\}$

by

$g(x)=1$

$g(x_j) =j+1 :1\leq j\leq m/2$

$g(x_j) =-j-3 : 1\leq j\leq m/2$ if m is even

$1\leq j\leq (m+1)/2$ if m is odd

Then g is a pain sum labeling this implies $K_{1, m}$ is a pair sum graph .

**Theorem** : $K_{2,m}$ is a pair sum graph

Proof :

Let $V(K_{2,m}) = \{x, y, z_j : 1\leq j\leq m\}$

And $E(K_{2,m})=\{xz_j,yz_j: 1\leq j\leq m\}$

Define $g:V(K_{2,m})\rightarrow\{\pm1,\pm2,\ldots.\pm(m+2)\}$ by

$g(x) = -1$

$g(y)= -(m+2)$

$g(z_j)=-j+1 \quad : 1\leq j\leq m$

Thus $g_e(E(G)) =\{\pm1, \pm2, \ldots\ldots\pm m\}$ and hence

$K_{2,m}$is a pair sum graph.

**Theorem** Any Bistar $B_{n,m}$ $(n\geq m)$ is a pair sum graph.

**Proof**

Let $V(B_{n,m}) =\{ x,y,xi,yj,:1\leq i\leq m, 1\leq i\leq n\}$

$E(B_{n,m}) =\{xy,xxi,vvj: 1\leq i\leq m, 1\leq i\leq n\}$

**Case(i)** :n=m

Define $g:V(B_{m,m})\rightarrow\{ \pm1, \pm2, \ldots\ldots\pm(2m+2)\}$

By

$g(x)=-1$

$g(y)=2$

$g(x_i)=-2i \quad :i=1,2,3,\ldots m$

$g(y_j)=2i-1 \quad :i=1,2,3,\ldots m$

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Obviously g is a pair sum labeling

**Case ii** : n>m

**Sub case (A)** : n is odd and m is even (on) n is even and m is odd

$g(x) = -1$

$g(x_i) = -2i \quad : 1 \leq i \leq m$

$g(y) = 2$

$g(y_i) = 2i - 1 : 1 \leq i \leq m$

$g(y_{m+1}) = -3$

$g(y_{m+2i}) = -4i - 6 : 1 \leq i \leq (n-m-1)/2$

$g(y_{m+2i+1}) = 4i + 2 : 1 \leq i \leq (n-m-1)/2$

Then g is a pair sum labeling

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