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A Study on odd graceful graph labelling theory

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Abstract

Graph labeling one mainly rising areas mathematics which project integers vertices edges both subject definite setting. provide useful mock-up broad assortment application such as data safety, communications net-works, X-ray, radar, circuit design data base organization.

Keywords : Labeling, communication, graph, edges, Magic Labeling etc.,

Introduction:

The concept of odd graceful graphs was introduced by Gnanajothi [11]. In this chapter we prove that several classes of graphs such as $rC_{4k}(k > 1), C_n \cup K_{1,m}$ where $n = 4k, k > 1$ and m is any positive integer, and $C_n \cup P_m$ where $n = 2k$ are odd graceful.

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We also prove that the graph obtained hy identifying a vertex of an even cycle with the centre or with a pendant vertex of a star is odd graceful and the graph obtained by joining a vertex of an even cycle to a pendant vertex of a star is odd graceful. We also construct several families of odd graceful graphs

Theorem 1.1 Let G be the graph obtained by identifying a pendant vertex of P_m with a vertex of C_n where $n = 4k$. Then G is odd graceful.

Proof. Let $V(G) = \{u_1, u_2, \dots, u_{4k}, v_1, v_2, \dots, v_m\}$ $C_{4k} = (u_1, u_2, \dots, u_{4k}, u_1)$ and $P_m = (u_{4k}, v_1, v_2, \dots, v_m)$

Define $f: V(G) \to \{0,1,2, \dots, 2q-1\}$ as follows.

$$f(u_i) = \begin{cases} i-1 & \text{if } i \text{ is odd and } i \le 2k-1 \\ i+1 & \text{if } i \text{ is odd and } 2k+1 \le i \le 4k-1 \\ 2q-i+1 & \text{if } i \text{ is even} \end{cases}$$

The induced edge labeling f^* is given by

$$f^*(u_i u_{i+1}) = \begin{cases} 2q-(2i-1) & \text{if } 1 \le i \le 2k-1 \\ 2q-(2i+1) & \text{if } 2k \le i \le 4k-1 \end{cases}$$

$f^*(u_n u_1) = 2q - (n-1)$,

$f^*(v, v_{i+1}) = 2q - (2n + 2i + 1)$ if $1 \le i \le n-1$

and $f^*(u_n v_1) = 2q - (2n+1)$.

The set of edge labels of the cycle C_n

$$= \{2q-1, 2q-3, \dots 2q-(n-3), 2q-(n+1), \dots, 2q-(2n-1), 2q-(n-1)\}.$$

The set of edge labels of the path P_m

$= \{2q-(2n+1), 2q-(2n+3), \dots, 5, 3, 1\}$

Hence G is odd graceful.

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Example 1.2 The odd graceful labeling of the graph G obtained by attaching a pendant vertex of P_{19} to a vertex of C_{12} is given in Figure 4.1

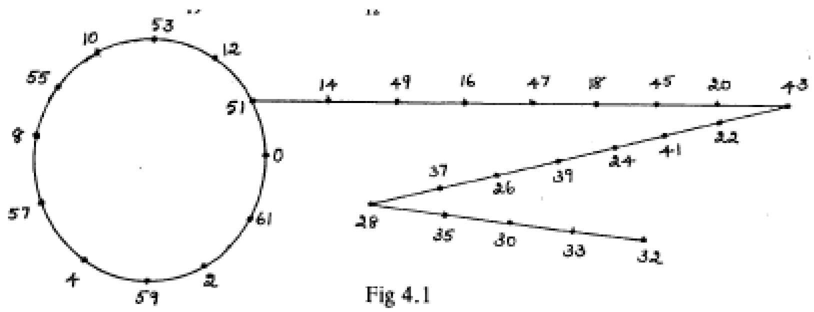

Fig 4.1

Theorem 1. 3 The graph G obtained by attaching an even cycle C to each vertex of K_2 is odd graceful.

Proof. Let $V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$

and $E(G) = \{u_1u_2, \dots, u_nu_1, u_1v_1, v_1v_2, \dots, v_nv_1\}$

Let $n = 2k$.

Define $f: V \to \{0,1,2, \dots, 2q-1\}$ as follows.

and

$$\begin{cases} 0 & \text{if } i = 1 \\ 8,20 & \text{if } i = 3,5 \\ 4i+4 & \text{if } i \text{ is odd and } 7 \leq i \leq k \\ 2q-4i+12 & \text{if } i \text{ is odd and } k+2 \leq i \leq n-3 \\ 14 & \text{if } i = n-1 \\ 2q-1 & \text{if } i = 2 \\ 2q-5 & \text{if } i = 4 \\ 2q-4i+15 & \text{if } i \text{ is even and } 6 \leq i \leq k+1 \\ 4i+7 & \text{if } i \text{ is even and } k+3 \leq i \leq n-4 \\ 2q-7 & \text{if } i = n-2 \\ 2q-3 & \text{if } i = n \end{cases}$$

If $n = 16,20,24, \dots$

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$$f(v_t) = \begin{cases} 1 & \text{if } i = 1 \\ 4i+1 & \text{if } i \text{ is odd and } 3 \le i \le k-1 \\ 4i-5 & \text{if } i = k+1 \\ 2q-4i+5 & \text{if } i \text{ is odd and } k+3 \le i \le n-3 \\ 7 & \text{if } i = n-1 \\ 4n-4 & \text{if } i = 2 \\ 2q-4i+6 & \text{if } i \text{ is even and } 4 \le i \le k-2 \\ 2n+6 & \text{if } i = k \\ 4i+2 & \text{if } i \text{ is even and } k+2 \le i \le n-2 \\ 4n-2 & \text{if } i = n \end{cases}$$

and,

$$f(u_i) = \begin{cases} 0 & \text{if } i = 1 \\ 8,20 & \text{if } i = 3,5 \\ 4i+4 & \text{if } i \text{ is odd and } 7 \le i \le k-1 \\ 2n+12 & \text{if } i = k+1 \\ 2q-4i+12 & \text{if } i \text{ is odd and } k+3 \le i \le n-3 \\ 14 & \text{if } i = n-1 \\ 2q-1 & \text{if } i = 2 \\ 2q-5 & \text{if } i = 4 \\ 2q-4i+15 & \text{if } i \text{ is even and } 6 \le i \le k \\ 4i+7 & \text{if } i \text{ is even and } k+2 \le i \le n-4 \\ 2q-7 & \text{if } i = n-2 \\ 2q-3 & \text{if } i = n \end{cases}$$

All the odd numbers between 1 and $2q-1$ appear as edge labels in the above labeling.

Hence G is odd graceful.

Example 1.4 An odd graceful labeling of the graph obtained by attaching a copy of C_{16} to each vertex of K_2 is given in Figure 4.2.

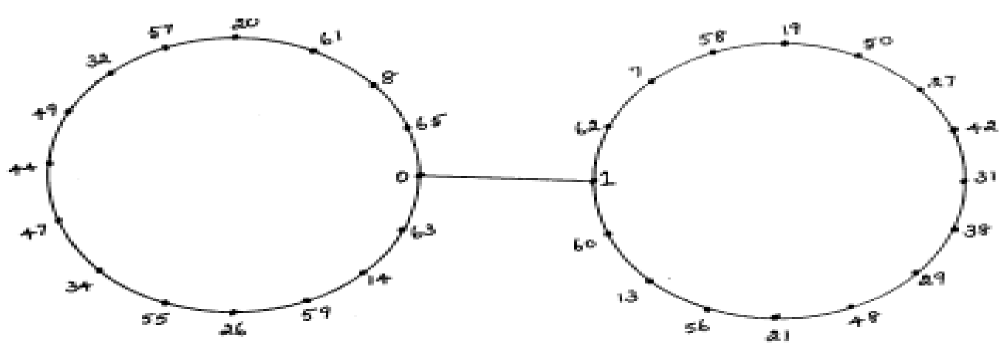

Fig 1.2

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Example 1.5 An odd graceful labeling of the graph obtained by attaching a copy of C_{18} to each vertex of K_2 is given in Figure 4.3.

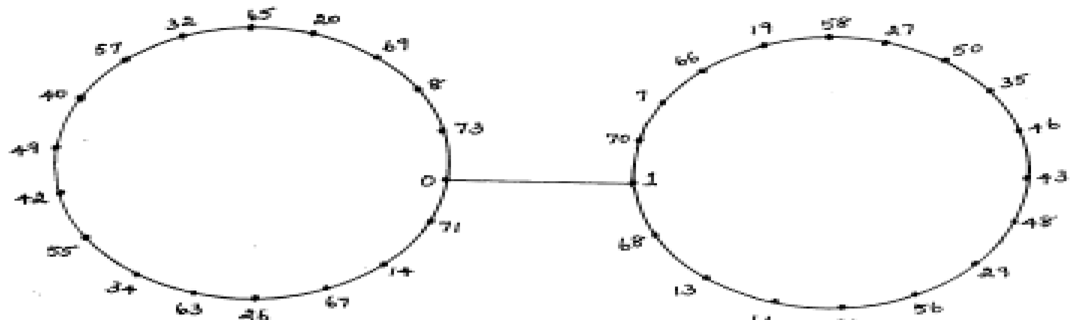

Fig 4.3

Theorem 1.6 Let $G = C_n \odot K_{1,n}$ be the graph obtained by identifying a vertex of $C_n = (u_1, u_2, \ldots, u_n, u_1)$ to the centre of the star $K_{1,\,m}$-Then G is odd graceful if $m \geq (n-6)/2$ and n is even.

Proof. Let $n = 2k$ and $m \geq k-3$.

Let $V(G) = \{u_1, u_2, \ldots, u_{2k}, v_1, v_2, \ldots, v_n\}$ and

$E(G) = \{u_1u_2, u_2u_3, \ldots, u_{tk-1}u_{2x}, u_{2k}u_1\} \cup \{u_1v_i / 1 \leq i \leq m\}$

Define $f: V \rightarrow \{0,1,2, \ldots, 2q-1\}$ as follows.

$$f(u_i) = \begin{cases} i-2 & \text{if } i \quad \text{is even and } 2 \leq i \leq 2k-2 \\ i+2k-4 & \text{if} \qquad i = 2k \\ 2q-i & \text{if} \qquad i \text{ is odd} \end{cases}$$

If $m = k-3, f(v_f) = 4k+2i-4,\ 1 \leq i \leq m.$

If $m > k-3$,

$$f(v_t) = \begin{cases} 4k+2i-4 & \text{if } 1 \leq i \leq k-2 \\ 4k+2i-2 & \text{if } i > k-2 \end{cases}$$

Let f " denote the induced edge labeling.

Then $f^*(u_iu_{i41}) = 2q-2i+1$ if $1 \leq i \leq n-2$,

$$f^*(u_{n-1}u_n) = 2m-2k+5 \text{ and}$$
$$f^*(u_nu_1) = 2q-4k+3$$

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If $m = k - 3$,

$$f^*(u_1 v_t) = 2q - 2i - 4k + 3, 1 \leq i \leq m$$

and if $m > k - 3$,

$$f^*(u_1 v_i) = \begin{cases} 2q - 2i - 4k + 3 & \text{if } 1 \leq i \leq k - 2 \\ 2q - 2i - 4k + 1 & \text{if } i > k - 2 \end{cases}$$

Thus $f^*(E(G)) = \{1,3, \ldots, 2q - 1\}$ and hence G is odd graceful.

Example 1.7 An odd graceful labeling of $C_{12} \odot K_{1,7}$ is given in Figure 4.4

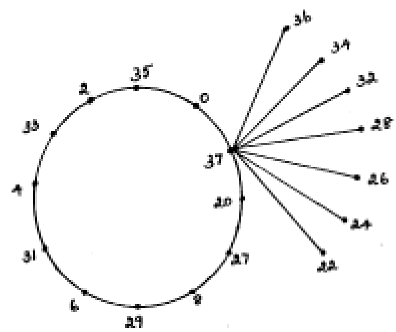

Fig:1;4

Theorem 1.9 $P_n X P_2$ is odd graceful where P is a path of length $n - 1$.

Proof. Let $G = P_n X P_2$, $V(G) = \{a_i, b / 1 \leq i \leq n\}$ and

$E(G) = \{aa_{i+1}, bb_{s+1} / 1 \leq i \leq n - 1\} \cup \{ab_i / 1 \leq i \leq n\}$

Define $f: V(G) \to \{0,1,2, \ldots, 2q - 1\} by$

$$f(a_i) = \begin{cases} 2q - 2i - 1 & \text{if } i \text{ is odd} \\ 4i - 4 & \text{if } i \text{ is even} \end{cases} \text{ and } f(b_i) = \begin{cases} 2q - 2i + 3 & \text{if } i \text{ is even} \\ 4i - 4 & \text{if } i \text{ is odd} \end{cases}$$

The induced edge labeling f^* is given by

$$f^*(ap_f) = \begin{cases} 2q - 6i + 3 & \text{if } i \text{ is odd and } 1 \leq i < n \\ 1 & \text{if } \quad i = n \\ 2q - 6i + 7 & \text{if } i \text{ is even} \end{cases}$$

$$f^*(aa_{i+1}) = \begin{cases} 2q - 6i - 1 & \text{if } i \text{ is odd} \\ 2q - 6i + 1 & \text{if } i \text{ is even} \end{cases}$$

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and $f^*(bb_{1+1}) = \begin{cases} 2q - 6i + 5 & \text{if } i \text{ is odd} \\ 2q - 6i + 3 & \text{if } i \text{ is even.} \end{cases}$

Clearly $f^*(E) = \{1,3,5, \ldots ,2q - 1\}$ so that G is odd graceful.■

Example 1. 5 Odd graceful labeling of P_7XP_2 is given in Figure 4.22.

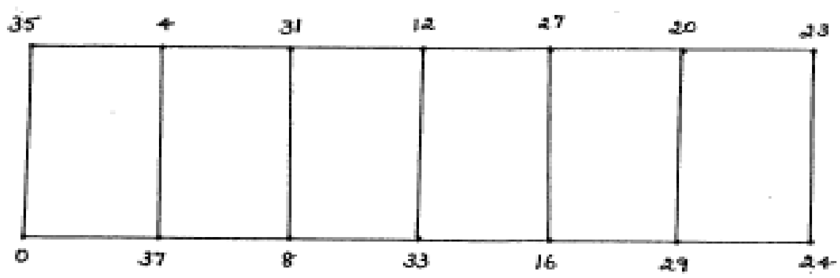

Fig 1. 5

Conclusion:

Labelled graphs useful models variety range applications such as: coding theory crystallography, astronomy, contact system, data base managing limit programming set area. Now days, graph theory-based results using in field of engineering science. Graph labeling is vital part graph theory. Many practical problems based on graph labeling. Methods application problems in communication network, radio astronomy problems etc. Bloom Golomb present graph labelingprms in different applications. Graph labeling well subject mobile Ad hoc networks. The model as connectivity, modeling network and recreation consider. For analyz the issues in the Ad hoc network, it be modelled graph. The model use to analyze the issues as node dense, mobility between nodes link information connecting the nodes.

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