e-ISSN 2320 –7876 www.ijfans.org
Research Paper
Vol.11, Iss.9, Dec 2022

A Study on Changing and Unchanging Independent Domination in Graphs

*1 Sophia Lavinia. B.
M.Phil. Scholar, Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai-72
*2 Dr D. Ahima Emilet,
Assistant Professor, Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai-73.
stanlysamjesus@gmail.com ahimaemilet.maths@bharathuniv.ac.in

Address for Correspondence

*1 Sophia Lavinia. B.
M.Phil. Scholar, Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai-72
*2 Dr D. Ahima Emilet,
Assistant Professor, Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai-73.
stanlysamjesus@gmail.com ahimaemilet.maths@bharathuniv.ac.in

Abstract

In this paper we study the effect of removal of vertices or edges and addition of edges on the independent domination number. Based on the changes, we define six classes of graphs. We investigate the changes and un-changes on certain special class of graphs. We also characterize some of the classes of graphs. Further we obtain some necessary conditions and some sufficient conditions for a graph to be a member of one of the six classes.

Keywords: Changing and unchanging graphs, independent domination in graphs, non-split domination of graphs, *CVR, CER, CEA, UVR,UER, and UEA*

Introduction:

The concept of domination in graphs was introduced by Ore [27]. In 1975, Cockayne and Hedetniemi unfolded its diverse aspects by surveying all the available results and citing its application potential in a variety of scientific areas. Prom then on, there has been rapid growth of research in this area.

Even though the concept of domination was introduced by Ore [27] in his book 'Theory of graphs 'in the year 1962, rapid growth has been made in the area only when Walikar et al. published their monograph [32] on domination. Since then, the vistas of domination was expanded by Graph theorists by defining hundreds of domination parameters. These parameters are defined by imposing additional conditions on a dominating set.

The vertex set of G is partitioned into three sets according to how the removal of vertices affect $i(G)$. Let $V = V^0 \cup V^+ \cup V^-$ where

$V^0 = \{v \in V: i(G - v) = i(G)\}$

$V^+ = \{v \in V: i(G - v) > i(G)\}$ and $V^- = \{v \in V: i(G - v) < i(G)\}$

Similarly, the edge set can be partitioned into

$E^0 = \{e \in E: i(G - e) = i(G)\}$and

$E^+ = \{e \in E: i(G - e) > i(G)\}$

For example. consider the graph given in Fig 2.1

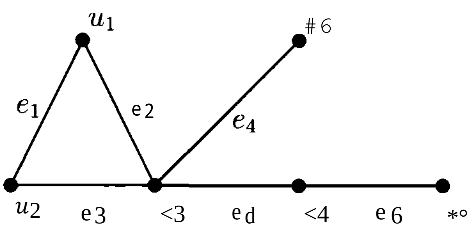

Fig.2.1

Here $V^0 = \{u_1, u_2, u_4, u_6\}, V^+ = \{u_3\}, V^- = \{u_5\}, E^0 = \{e_1, e_5, e_6\}$ and $E^+ = \{e_2, e_3, e_4\}$

1.1 Changing Vertex Removal (CVR)

Theorem 1.1.1. Let $G \cong K_{m,n} (m \leq n)$ with bipartition (V_1, V_2) where $|V_1| = m$ and $|V_2| =$

n. Then $V^- = V_1 \cup V_2$ if $m = n$ and $V^- = V_1, V^0 = V_2$ if $m < n$.

Proof. If $m = n$, since $i(G) = m$ we have $V^- = V_1 \cup V_2$. Suppose $m < n$. Then $i(G) = m$ and for all $v \in V_1, i(G - v) = m - 1$ and so $V_1 \subseteq V^-$. For all $v \in V_2$ $i(G - v) = m = i(G)$ and so $V_2 \subseteq V^0$. Hence $V^- = V_1$ and $V^0 = V_2$ if $m < n$.

Theorem 1.1.2. If $G \simeq C_p$, a cycle on p vertices, then $V = V^- \cup V^+$. Further more, $V = V^-$ if and only if $p = 3k + 1 (k \geq 1)$ and $V = V^0$ otherwise.

Proof. Let $C_p = (1,2, \dots, p, 1)$. Suppose $p = 3k (k \geq 1)$. Then $i(G) = \left\lceil \frac{p}{3} \right\rceil = k$. For any $j (1 \leq j \leq 3k), i(G - j) = \left\lceil \frac{3k-1}{3} \right\rceil = k - 1 + 1 = k$ and so $j \in V^0$ for every j. Suppose $p = 3k + 2 (k \geq 1)$. Now $i(G) = \left\lceil \frac{3k+2}{3} \right\rceil = k + 1$ and for any $j (1 \leq j \leq 3k + 2), i(G - j) = \left\lceil \frac{3k+1}{3} \right\rceil = k + 1 = i(G)$ so that $j \in V^0$ for every j. Suppose $p = 3k + 1 (k \geq 1)$. Now $i(G) = \left\lceil \frac{3k+1}{3} \right\rceil = k + 1$ and for any $j (1 \leq j \leq 3k + 1), i(G - j) = \left\lceil \frac{3k}{3} \right\rceil = k < i(G)$ so that $j \in V^-$ for every j. Thus $V = V^-$ if and only if $p = 3k + 1 (k \geq 1)$ and $V = V^0$ otherwise.

Theorem 1.1.3. (a) Let S be any i $(G) -$ set of a graph G. If $v \in V^+$, then $v \in S$ and $pn[v, S]$ contains at least two non-adjacent vertices.

(b) If $x \in V^+$ and $y \in V^-$, then x and y are not adjacent.

(c) $|V^0| \geq 2|V^+|$

(d) $i(G) \neq i(G - v)$ for all $v \in V$ if and only if $V = V^-$.

Hence

$< pn[v, S] >$ contains at least two non-adjacent vertices. (b) Suppose $x \in V^+, y \in V^-$ and $xy \in E(G)$. As $y \in V^-$, let S_y be an independent dominating set of $G - y$ such that $|S_y| = i(G) - 1$. If $x \in S_y$, S_y dominates G_1 a contradiction to minimality of $i(G)$. Suppose $x \notin S_y$; Now $S_y \cup \{y\}$ is an independent dominating set since otherwise S_y itself will dominate G. Thus $S_y \cup \{y\}$ is an $i(G)$ - set not containing x, a contradiction to (a) and so $xy \notin E(G)$ (c) For each $v \in V^+$, by (a), $pn[v, S]$ contains two non-adjacent vertices. These vertices are in $V - S$ and so again by (a), they are not in V^+. By (b), v cannot have neighbours in V^- and so these private neighbours must be in V^0. Hence $|V^0| \geq 2|V^+|$

(d) Suppose $i(G) \neq i(G - v)$ for all $v \in V$. Then $V^0 = \phi$ and so $v \in V^+$ or $v \in V^-$. If $v \in V^+$, then by (c), V^0 is non- empty, which is a contradiction. Hence $v \in V^-$. Converse is obvious.

Theorem 1.1.5. A vertex $v \in V^+$ if and only if (a) v is not an isolate and lies in every $i(G) -$ set of G and

(b) No subset $S \subseteq V - N(v)$ with cardinality $i(G)$ dominates $G - v$.

Proof. Suppose $v \in V^+$. Since every isolated vertex lies in V^-, v is not an isolate. By theorem 2.2.3 (a), v lies in every $i(G)$ - set of G. If there exists $S \subseteq V - N(v)$ with $|S| = i(G)$ such that S dominates $G - v$, then we get a contradiction as $v \in V^+$ Conversely assume that v satisfies (a) and (b). By (b) we observe that $v \notin V^0$. By (a) we have pn $[v, S] - \{v\} \neq \phi$ and so using theorem 2.2.4, we conclude that $v \notin V^-$. Hence $v \in V^+$

Theorem 1.1.6. If a graph $G \in CVR$ has order $n = (\Delta(G) + 1)(i(G) - 1) + 1$ then G is regular.

Proof. Suppose $G \in CVR$ with $n = (\Delta(G) + 1)(i(G) - 1) + 1$. By theorem 2.2.3 (d), since $G \in CVR$ we have $V = V^-$. Let S_t denote an $i(G)$. set of $G - u$, so that $|S_u| = i(G) - 1$. In order to dominate the $(\Delta(G) + 1)(i(G) - 1)$ vertices of $G - u$, each element of S_u must dominate exactly $\Delta(G) + 1$ vertices and so has degree $\Delta(G)$. Thus no two vertices in S_u have a common neighbor. To prove G is regular, it is enough to prove that for any arbitrary vertex $x, x \in S_u$ for some u. Let $r \in S_x$. We prove that $x \in S_r$. Suppose $x \notin S_r$. Since $G \in CVR$ $S_r \cap N[r] = \phi$. Each vertex in $S_x - \{r\}$ dominates a unique vertex of S_r. So the remaining vertex in S_r which is not x, must be dominated by S_z and so must be dominated by r, which is a contradiction as $S_r \cap N[r] = \phi$. Hence $x \in S_r$ and so G is regular.

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

The following are immediate.

Theorem 1.1.7. Let G be a graph with $\Delta(G) = p - 2$ and v be a verter of degree $p - 2$ such that $V - N[v] = \{u\}$. Then $u \in V^-$.

Corollary 1.1.8. If G is $(p - 2)$ -regular such that $p \neq 2$ then $V = V^-$.

1.2 Changing Edge Removal $(\boldsymbol{CER})$

Theorem 1.2.1. A graph $G \in CER$ if and only if G is a galaxy, i.e. a forest in which each component is a star.

Proof. If G is a galaxy, clearly $G \in CER$. Suppose $G \in CER$ and let S be an $i(G)$ - set of G. There can be no edge between two vertices in $V - S$ since otherwise S will still dominate $G - S$ if such an edge is removed. Also every vertex of degree at least two must lie in S. Thus G is a union of stars and so a galaxy. □

Theorem 1.2.2. If P_p is a path on p vertices where $p = 3k(k \geq 1)$ then $E = E^0 \cup E^+$ where $E^0 = \{e_j \ni j \equiv 0(\text{mod}3)\}$ and $E^+ = \{e_j \ni j \equiv 1{,}2(\text{mod}3)\}$.

Proof. Let $P_p = (1,2, \dots, p)$ and let $e_1, e_2, \dots, e_{3k-1}$ be the edges of P_p. Then $i(P_p) = k$. Consider $P_p - e_j (1 \leq j \leq 3k - 1)$. Let $P_p - e_j = P_1 \cup P_2$ where

$P_1 = (1,2, \dots, j)$ is a path on j vertices and $P_2 = (j + 1, j + 2, \dots, 3k)$ is a path on $3k - j$ vertices.

Case (i): $j \equiv 0(\text{mod}3)$

Now $3k - j \equiv 0(\text{mod}3)$ and so $i(P_p - e_j) = i(P_1) + i(P_2) = \left\lceil\frac{i}{3}\right\rceil + \left\lceil\frac{3k-j}{3}\right\rceil = \frac{j}{3} + \frac{3k-1}{3} = k = i(P_p)$

Case(ii): $j \equiv 1(\text{mod}3)$

Now $3k - j \equiv 2(\text{mod}3)$ and so

$$i(P_p - e_j) = \left\lceil\frac{j}{3}\right\rceil + \left\lceil\frac{3k - j}{3}\right\rceil = \frac{j - 1}{3} + 1 + \frac{3k - j - 2}{3} + 1 = \frac{ak + 3}{3} = k + 1 = i(P_p) + 1$$

e-ISSN 2320 –7876 www.ijfans.org
Research Paper
Vol.11, Iss.9, Dec 2022

Case(iii): $j \equiv 2(\mathrm{mod}3)$

Now $3k - j \equiv 1(\mathrm{mod}3)$ and so $i(P_p - e_j) = \left\lceil\frac{i}{3}\right\rceil + \left\lceil\frac{3k-j}{3}\right\rceil = \frac{i-2}{3} + 1 + \frac{3k-j-1}{3} + 1 = k + 1 = i(P_{\bar{D}}) + 1$. Hence $E^0 = \{e_j \ni j \equiv 0(\mathrm{mod}3)\}$ and $E^+ = \{e_j \ni j \equiv 1$ or $2(\mathrm{mod}3)\}$. □

Theorem 1.2.3.. If P_p is a path on p vertices where $p = 3k + 2(k \geq 0)$ then $E = E^0 \cup E^+$ where $E^0 = \{e_j \ni j \equiv 0$ or $2(\mathrm{mod}3)\}$ and $E^+ = \{e_j \ni j \equiv 1(\mathrm{mod}3)\}$.

Proof. Similar to Theorem 2.3.2.

1.3 Changing Edge Addition (CEA)

Example 1.3.1. If $G \cong \overline{K_p}$ then $G \in (CVR) \cup (CEA)$.

Theorem 1.3.1. No path is in the class CEA.

Proof. Let $P_p = (1,2, \dots, p)$ and $e_1, e_2, \dots, e_{p-1}$ be the edges of P_p. If $p \equiv 0$ or $2(\mathrm{mod}3)$, then let $e = (1,3)$. In the first case, $i(P_p) = i(P_p + e) = \frac{p}{3}$ and in the second case $i(P_p) = i(P_p + e) = \left\lceil\frac{[}{3}\right\rceil$. If $p \equiv 1(\mathrm{mod}3)$, let $e = (1, p)$. Now $i(P_p) = i(P_p + e) = \left\lceil\frac{p}{3}\right\rceil$ and $s0$ no path is in the class CEA. □

Remark 1.3.1. If G is a graph with $\Delta(G) = p - 1$ such that $G \not\cong K_p$, then G is not in the class CEA.

Theorem 1.3.2. If $G \cong K_{m,n}(m \leq n)$ then $G \in CEA$ if and only if $m = n$.

Proof. Let $G = (X, Y)$ where $X = \{x_1, x_2, \dots, x_m\}$ and $Y = \{y_1, y_2 \dots, y_n\}$. Suppose $G \in CEA$ and $m \neq n$. If $e = (y_1, y_2)$, then $i(G + e) = m = i(G)$ which is a contradiction. Conversely, suppose $m = n. i(K_{m,m}) = m$ and for all edges $e = (x_i, x_j)$ or $(y_i, y_j)(1 \leq i, j \leq m, i \neq j)$, we have $i(K_{m,m} + e) = m - 1 \neq i(K_{m,m})$ and so $K_{m,m} \in CEA$. □

Theorem 1.3.3. Let G be a graph with $i(G) = 2$. If $G \in CEA$ then $\bar{G}$ is a galaxy and if $\bar{G}$ is a galaxy with at lenst two components then $G \in CEA$.

e-ISSN 2320 –7876 www.ijfans.org
Research Paper
Vol.11, Iss.9, Dec 2022

Proof. Since $i(G) = 2$ and $G \in CEA$, we have $i(G + e) = 1\ \forall e \in E(\bar{G})$. Hence for $e = (u, v) \in E(\bar{G})$ at least one of u or v is of degree $p - 1$ in $G + e$ and so every edge in $E(\bar{G})$ is a pendent edge. Thus $\bar{G}$ is a galaxy. Conversely, by adding any edge from $E(\bar{G})$ in G, the respective pendent vertex in $\bar{G}$ becomes an universal vertex in $G + e$. So $i(G + e) = 1\ \forall e \in E(\bar{G})$ and hence $G \in CEA$.

1.4 Unchanging Vertex Removal (UVR)

Example 1.4. 1. If $G \cong K_p$ then $V = V^0$ and so $G \in UVR$.

2. If G is a unicyclic graph obtained by drawing an edge between a vertex of a cycle C_{3k} and a pendent vertex of a path P_{3l} then $G \in UVR$.

Lemma 1.4.1. Let G be any graph with $\delta(G) = 1$. If $V^0 = V$, then every support is adjacent to exactly one pendent vertex.

Proof. Suppose u is a support which is adjacent to two or more pendent vertices. If there exists an $i(G)$ -set containing u, removal of u increases $i(G)$. If not, removal of any pendent vertex adjacent to u decreases $i(G)$. These contradictions prove that every support is adjacent to exactly one pendent vertex. □

Remark 1.4.1. Converse of lemma 2.5.2 is not true. If $G \cong P_4, V^0(G) \neq V(G)$.

Definition 1.4.4. Two supports u and v are said to be consecutive if the unique $u - v$ path contains no other support.

Theorem 1.4.2. Let T be a tree. $V(T) = V_0$ if and only if every support is adjacent to exactly one pendent vertex and $d(u, v) \equiv 2(\mathrm{mod}3)$ for any two consecutives supports u and v.

Proof. Suppose that every support is adjacent to exactly one pendent vertex and $d(u, v) \equiv 2(\mathrm{mod}3)$ for any two consecutive supports u and v. Let u and v be two consecutive supports. Let $u = u_1, u_2, \dots, u_n = v$ be the $u - v$ path where$n \equiv 0(\mathrm{mod}3)$. Let S be any $i(T)$ - set. Without loss of generality we can assume that $\{u = u_1, u_4, u_7, \dots, v\} \subseteq S$. For $u_i \in S$ with $1 <$

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

$i < n, S - \{u_i\}$ does not dominate u_{i-1}. For $u_i, i = 1$ to $n, S - \{u_i\}$ does not dominate v_i, the pendent adjacent to u_i. Removal of any other vertex in T does not change $i(T)$. Hence $V^0 = V$. Conversely, assume $V^0 = V$. By lemma 2.5.2, every support is adjacent to exactly one pendent vertex. Suppose there exists two consecutive supports u and v such that $d(u, v) \not\equiv 2(\text{mod}3)$.

Case(i), $d(u, v) \equiv 0(\text{mod}3)$

Let $u = u_1, u_2, \dots, u_n = v$ be the $u - v$ path where $n \equiv 1(\text{mod}3)$. Without loss of generality we can assume that if S is any $i(T)$ -set then $\{u_1, u_4, \dots, u_n\} \subseteq S$. Now $S - \{u_4\}$ increases $i(G)$ since now u_3 is not dominated by this set. Hence $u_4 \notin V^0$ which is a contradiction.

Case(ii), $d(u, v) \equiv 1(\text{mod}3)$

Let u_1, v_1 be the pendent vertioes adjacent to u and v. Let S be any $i(T) -$ set. Since u, v are adjacent, without loss of generality, we can assume that $\{u_1, v_1\} \subseteq S$. Now $S - \{v_1\}$ decreases $i(T)$ and so $v_1 \notin V^0$ which is a contradiction. Thus $d(u, v) \equiv 2(\text{mod}3)$ for every two consecutive supports u and v. □

Corollary 1.4.6. If $G \cong P_p$ then $V^0 = V$ if and only if $p = 3k + 2(k \geq 0)$.

Theorem 1.4.7. For any tree T with at least two vertices, $V^0 \neq \phi$.

Proof. Let u be a pendent vertex and v be the support adjacent to u. Every $i(T)$ - set S of T contains either u or v. If $u \in S, v \in V^0$ and if $v \in S, u \in V^0$. Thus $V^0 \neq \phi$.

1.5 Unchanging Edge Removal $(\boldsymbol{UER})$

Example 1.5.1. (i) If $G \cong C_p$ then $E = E^0$ and so $G \in UER$.

(ii) If $G \cong K_p$ then $G \in UER$.

(iii) If $G \cong K_{m,n}(m, n \geq 2)$ then $G \cong UER$.

Theorem 1.5.1. If P_p is a path on p vertices where $p = 3k + 1(k \geq 0)$ then $E^0 = E$

Proof. Let $P_p = (1,2, \dots, p)$ and let $e_1, e_2, \dots, e_{3k}$ be the edges of P_p. Then $i(P_p) = k + 1$.

Consider $P_p - e_j (1 \leq j \leq 3k)$. Let $P_p - e_j = P_1 \cup P_2$ where $P_1 = (1,2, \ldots, j)$ is a path on j vertices and $P_2 = (j+1, j+2, \ldots, 3k+1)$ is a path on $3k+1-j$ vertices. Case(i). $j \equiv 0(\text{mod}3)$

Now $3k+1-j \equiv 1(\text{mod}3)$ and so

$$i(P_p - e_j) = i(P_1) + i(P_2) = \left\lceil\frac{i}{3}\right\rceil + \left\lceil\frac{3k+1-j}{3}\right\rceil = \frac{j}{3} + \frac{3k+1-j-1}{3} + 1 = \frac{3k+3}{3}$$

$$= k + 1 = i(P_p)$$

Case(ii). $j \equiv 1(\text{mod}3)$

Now $3k+1-j \equiv 0(\text{mod}3)$ and so

$$i(P_p - e_j) = i(P_1) + i(P_2) = \left\lceil\frac{j}{3}\right\rceil + \left\lceil\frac{ak+1-j}{3}\right\rceil = \frac{j-1}{3} + 1 + \frac{3k+1-i}{3} = k + 1$$

$$= i(P_p)$$

Case(iii). $j \equiv 2(\text{mod}3)$

Now $3k+1-j \equiv 2(\text{mod}3)$ and so

$$i(P_p - e_j) = i(P_1) + i(P_2) = \frac{j-2}{3} + 1 + \frac{3k+1-j-2}{3} + 1 = k + 1 = i(P_p)$$

Hence $E^0 = E$.

Proposition 1.5.3. For any connected graph $G, G \circ K_1 \in UER$.

Proof. Let $V(G) = \{v_1, v_2, \ldots, v_p\}$ and $E(G) = \{e_1, e_2, \ldots, e_q\}$. If $S = \{u_i, 1 \leq i \leq p\}, \{d_i, 1 \leq i \leq p\}$ are the sets of pendent vertios and pendent edges of $G \circ K_1$ respectively, then S is a minimum independent dominating set of $G \circ K_1$ and so $i(G \circ K_1) = p$. Also $i((G \circ K_1) - e_i) = p$ for every $1 \leq i \leq q$. Consider $(G \circ K_1) - d_j$ where $d_j = v_j u_j$, Since G is connected, there exists a vertex $v_1 \in V(G)$ such that v_1 is adjacent to v_j. Now $S - \{u_i\} \cup \{v_l\}$ is an $i(G)$ -set of $G \circ K_1$ and so $i\left((G \circ K_1) - d_j\right) = p$ for all $1 \leq j \leq p$. Thus $G \circ K_1 \in UER$. □

e-ISSN 2320 –7876 www.ijfans.org
Research Paper
Vol.11, Iss.9, Dec 2022

1.6 Unchanging Edge Addition (UEA)

Example 1.6.1. (i) If $G \cong mK_2$ then $G \in (CER) \cap (UEA)$.

(ii) If $G \cong K_{1,p-1}$ then $G \in (CER) \cap (UEA)$.

(iii) It $G \cong K_p - e$ then $G \in UEA$.

Theorem 1.6.2. If P_p is a path on p vertices, then $P_p \in UEA$ if and only if $p = 3$ or $p = 3k + 2(k \geq 1)$.

Proof. Suppose $P_p \in UEA$. Let $p = 3k$. Let $P_p = (1,2, \dots ,3k)$ and suppose that $k > 1$. Consider the edge $e = (2,5)$. Then $i(P_p + e) = k + 1$ whereas $i(P_p) = k$ and so $P_p \notin UEA$ which is a contradiction. Hence $p \neq 3k(k > 2)$. Similarly if $p = 3k + 1$ with $e = (1,3), i(P_p + e) = k$ whereas $i(P_p) = k + 1$ and so $P_p \notin UEA$ which is a contradiction. Hence $n \neq 3k + 1$ and so either $p = 3$ or $p = 3k + 2$.

Proposition 1.6.3. A cycle $C_p \in UEA$ if and only if $p \neq 1(\text{mod}3)$.

Proof. Suppose $C_p \in UEA$ and $p \equiv 1(\text{mod}3)$. Let $p = 3k + 1$ and $C_p = \langle v_1, v_2, \dots , v_{3k+1} \rangle$. Then $i(C_p + e) = k$ where $e = (v_1, v_3)$ and $i(C_p) = k + 1$. This violation of the assumption helps to conclude that $p \not\equiv 1(\text{mod}3)$. Conversely, if $p \equiv 0(\text{mod}3)$ or $p \equiv 2(\text{mod}3)$, it is easy to observe that $C_p \in UEA$. □

Proposition 1.6.4. Let T be a caterpillar in which altermate vertices are supports and at most one support is adjacent to two or more pendent vertices. Then $T \in$ UEA.

Proof. By choice of T, the set of all supports S is a minimum independent dominating set. Suppose an edge is drawn between two supports u and v where u is one which is adjacent to exactly one pendent vertex. Then $S - \{u\} \cup \{w\}$ is a minimum independent dominating set where w is the pendent vertex adjacent to u. Addition of any other edge leaves S unaffected and so $T \in UEA$. □

Remark 1.6.5. If the caterpillar described in proposition 2.7.4 has two or more supports which are adjacent to two or more pendent vertices, addition of an edge between these supports increases $i(G)$ and so $T \notin UEA$.

The hypothesis of proposition 2.7.4 is not necessary. For example, $P_3 \in UEA$ but P_i does not satisfy the hypothesis of proposition 2.7.4.

Proposition 2.7.6. If V^- is empty then $G \in UEA$.

Proof. Suppose $G \in UEA$ and $v \in V^-$. Then $i(G-v) < i(G)$ and let S be an $i(G)$ - set of $G - v$. Clearly $N(v) \cap S = \phi$. Now adding an edge $e = (v, u)$ for any $u \in S$ we have $i(G+e) < i(G)$ which is a contradiction. Hence V^- is empty. □

Remark 1.6.7. Converse of proposition 2.7.6 is not true.

Conclusion:

we obtain several results on changing and unchanging independent domination number of a graph.

References

[1] J. R. Carrington, F. Harary and T. W. Haynes, *Changing and unchanging the domination number of a graph*, J. Combin. Math. Combin. Comput., 9(1991),57-63.

[2] G. Chartrand and L. Lesniak, *Graphs and Digraphs*, Chapman and Hall, Madras (1996).

[3] E.J. Cockayne, R.M. Dawes and S.T. Hedetniemi, *Total domination in graphs*, Networks 10(1980),211-219.

[4] E. J. Cockayne, L. Hartnell, S. T. Hedetnieni and R. Laskar, *Perfect domination in graphs*, JCISS 18(1993), 136-148.

[5] E. J. Cockayne, *Domination of undirected graphs - A survey*. In theory and Applications of Graphs. LNM 642, Springer - Verlag, (1978), 141-147. 109

[6] E. J. Cockayne and S. T. Hedetniemi, *Towards a theory of domination in graphs*,

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

Networks, *7*, (1977), 247-261.

[7] E. J. Cockayne, B. L. Hartnell, S. T. Hedetniemi and R. Laskar. *Efficient domination in graphs,* Clemson University Dept. of Mathematical Sciences, Technical report 558 (1998).

[8] F. Harary, *Graph theory,* Addison Wesley, Reading Mass (1969).

[9] F. Harary, *Changing and unchanging invariants for graphs.* Bull. Malaysian Math. Soc, 83 5(1982),73-78.

[10] F. Harary and T.W. Haynes, *Nordhaus-Gaddum inequalities for domination in graphs,* Discrete Math 155(1996),99-105.