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CERTAIN PROPERTIES OF MITTAG LEFFLER FUNTION IN KOBER OPERATOR OF ITS FIRST KIND

[1]Kannan P, [2] Siva M

Department of Mathematics
Faculty of Arts and Science

Faculty of Arts and Science, Bharath Institute of Higher Education and Research

[1] jsbkannan@gmail.com, [2] sivamurthy04@gmail.com

Address for Correspondence

[1]Kannan P, [2] Siva M

Department of Mathematics
Faculty of Arts and Science

Faculty of Arts and Science, Bharath Institute of Higher Education and Research

[1] jsbkannan@gmail.com, [2] sivamurthy04@gmail.com

ABSTRACT:

There are many ways to apply the mittag leffler function in causing field. Author used two parameters of mittag leffler function in this article. Also author tries to investigate the properties of $E_{\delta}(Z), E^{\gamma}_{\rho,\mu}(Z)$ in kober fractional integral operator of its first kind. This artefact contributes to the investigation of geometric properties of normalized Mittag-Leffler (M-L) function with three parameters in complex plane. We discuss the appropriate conditions required for the stabilized three parameter M-L functions to be starlike and convex in the open unit disk. Some interesting relationship between the normalized M-L function with three limitations and hypergeometric functions are given. Also, assured special cases are categorized concluded mappings over open unit disk.

KEYWORDS: Mittag Leffler Function, Kober Operator, Beta Function, Fractional Intergal Operator

INTRODUCTION:

The special form of Mittag leffler function [9,22,23],

$$E_{\delta}(Z) = \sum_{k=0}^{\infty} \frac{Z^k}{\Gamma(\delta k + 1)}$$

General function,

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$$E_{\alpha,\beta}(Z) = \sum_{k=0}^{\infty} \frac{Z^k}{\Gamma(\alpha k + \beta)} \qquad (2)$$

Where, $\Gamma(x)$ is the Gamma function.

α, β are real numbers

General function with three parameters,

$$E_{\rho,\mu}^{\mu}(Z) = \sum_{k=0}^{\infty} \frac{(\gamma)_k}{\Gamma(\rho k+\mu)} \frac{z^k}{k!}$$

Where γ_k is a Pochhammar symbol.

Beta function, $\beta(m,n) = \int_0^1 S^{m-1}\,(1-S)^{n-1}\mathrm{ds}$

Fractional integral operator is related to kober operator. Applying the mittag leffler function and general function in kober operator, one can yield the following results.

Numerous researchers investigated geometric properties viz star likeness, convexity and univalence of the many families analytic functions Many results concerning the geometric properties of hypergeometric functions [5, 8, 10, 15, 16], Bessel functions [3,4,5,17], Fox-Wright function [2] and M-L functions [1] are existing in the literature [6]

PRELIMINARIES:

The fractional ordered integral is the generalization of fold integral and fractional derivative means the derivative of arbitrary order, where not necessarily an nonnegative integer. Earlier the concept of fractional integral and derivatives were purely theoretical and useful only for mathematicians.

In recent days fractional derivatives and fractional integrals are applied in most of the branches of mathematics such as calculus of variations [1, 2, 5,], differential equations and numerical analysis [3, 4, 11, 21] there are various researchers in the field pointed out that integrals and derivatives of non-integer order are very suitable for the description of various applications such as physical, biological, engineering and pure sciences. In fact, differential equations of arbitrary order have applications in various fields. For example, the study of viscoelastic materials which have an intermediate behaviour of viscosity and elasticity. This kind of materials are studied by fractional diffusion equation.

The exponential function plays a vital role in the theory of ordinary differential equations. Similarly, two parameter Mittag-Leffler function given by equation (2) is quite significant in the theory of fractional differential equations [7].There are more than one definitions of

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fractional derivatives available in the literature such as Riemann Liouville fractional derivative,Gruwald-Letnikov-Letnikov fractional derivative and Caputo fractional derivative [4].

The normalized M-L function with three parameters $E\rho$, given by equation has some motivating assets. The function $E\rho$, has a close affiliation with hypergeometric functions. For the proper restraint on the parameter γ, the geometric properties are also preserved.

Here, author deals with Riemann Liouville fractional derivative for obtaining the following result.

DEFINITION[1]:

Mittag leffler function in kober fractional integral operator of first kind is denoted by,

$$I_x^{\eta,\alpha} E_\delta(x) = K_{1,x,\eta}^{-\alpha} E_\delta(x) = \frac{x^{-\eta-\alpha}}{\Gamma\alpha} \int_0^x (x-t)^{\alpha-1}\, t^\eta E_\delta(t) dt$$

MAIN RESULTS:

THEOREM 1:

Kober operator and mittag leffler function,

$$I_x^{\eta,\alpha} E_\delta(x) = K_{1,x,\eta}^{-\alpha} E_\delta(x) = \frac{x^{-\eta-\alpha}}{\Gamma\alpha} \int_0^x (x-t)^{\alpha-1}\, t^\eta \sum_{k=0}^{\infty} \frac{1}{\Gamma(\alpha k+1)} t^k\, dt$$

The above equation is reduced to,

$$I_x^{\eta,\alpha} E_\delta(x) = K_{1,x,\eta}^{-\alpha} E_\delta(x) = \frac{x^k}{\Gamma\alpha} \sum_{k=0}^{\infty} \frac{1}{\Gamma(\delta k+1)} \int_0^x (S)^{\eta+k+1-1} (1-S)^{\alpha-1} ds$$

Beta function is applied to the above equation,

$$K_{1,x,\eta}^{-\alpha} E_\delta(x) = \frac{x^k}{\Gamma\alpha} \sum_{k=0}^{\infty} \frac{1}{\Gamma(\delta k+1)} \beta(\eta+k+1,\alpha)$$

$$K_{1,x,\eta}^{-\alpha} E_\delta(x) = x^k \sum_{k=0}^{\infty} \frac{1}{\Gamma(\delta k+1)} \frac{\Gamma(\eta+k+1)}{\Gamma(\eta+k+1+\alpha)}$$

COROLLARY 1:

Put $\alpha = 0$,We obtain

$$I_x^{\eta,0} E_\delta(x) = x^k E_\delta(1)$$

THEOREM 2:

Extended Mittag leffler function and kober operator

$$[I_x^{\eta,\alpha} E_{\rho,\mu}^{\gamma}(t)]x = \frac{x^{-\eta-\alpha}}{\Gamma\alpha}\int_0^x (x-t)^{\alpha-1}\, t^{\eta} E_{\rho,\mu}^{\gamma}(t)dt$$

$$=\frac{x^{-\eta-\alpha}}{\Gamma\alpha}\int_0^x (x-t)^{\alpha-1}\, t^{\eta}\sum_{k=0}^{\infty}\frac{\gamma_k}{\Gamma(\rho k+\mu)}\frac{t^k}{k!}dt$$

Simplify the above equation , we get

$$[\, I_x^{\eta,\alpha} E_{\rho,\mu}^{\gamma}(t)]x = \frac{x^k}{\Gamma\alpha}\sum_{k=0}^{\infty}\frac{\gamma_k}{\Gamma(\rho k+\mu)k!}\int_0^1 S^{\eta+k+1-1}(1-S)^{\alpha-1}\, ds$$

Applying the beta function in above equation,

$$[\, I_x^{\eta,\alpha} E_{\rho,\mu}^{\gamma}(t)]x = \frac{x^k}{\Gamma\alpha}\sum_{k=0}^{\infty}\frac{\gamma_k}{\Gamma(\rho k+\mu)k!}\beta(\eta+k+1,\alpha)$$

$$[\, I_x^{\eta,\alpha} E_{\rho,\mu}^{\gamma}(t)]x = x^k\sum_{k=0}^{\infty}\frac{\gamma_k}{\Gamma(\rho k+\mu)k!}\frac{\Gamma(\eta+k+1)}{\Gamma(\eta+k+1+\alpha)}$$

COROLLARY:2

Put $\alpha = 0, \mu = 1, \gamma = 1, We\ obtain$

$$[I_x^{\eta,\alpha} E_\rho(t)]x = x^k E_\rho(1)$$

CONCLUSION :

Proposed area of research on Geometric Function Theory which is one of the branches of Complex Analysis relating to certain studies on interesting results connected with Geometry and Analysis. Author proposes certain properties of Geometric Function Theory with special applications. Geometric Function Theory deals with several physical properties like defining the magnetic field, electromagnetic field, signal processing and wavelets based on the significant classes of functions with reference to analytic functions.

We presented some of the geometric properties of normalized M-L function $E\rho$, under suitable assumptions on its parameters. This investigation provides the sufficient condition for $E\rho$, to be starlike in the unit disk $\mathbb{U}$ under specific conditions on the parameters ρ, and γ. Convexity and close-to-convexity property of $E\rho$, is studied and the sufficient conditions are also provided. The transformation of the unit disk $\mathbb{U}$ by the function $E\rho$, preserve geometric properties and it was illustrated through figures for various values of γ.

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