Dynamics Of Viscoelastic Fluid Motion Over Expanding And Contracting Surfaces

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Abstract:

In the present article an analysis is carried out to study two dimensional flow of a viscoelastic fluid (Walter's liquid B) induced by a shrinking / stretching sheet. The governing partial differential equations are first reduced to ordinary nonlinear differential equations by using the appropriate similarity transformation. An exponential solution is assumed to solve the considered boundary value problem. The effects of governing pertinent physical parameters on the flow characteristics are presented graphically and discussed.

KeyWords: Shrinking / stretching sheet, mass suction, viscoelastic parameter, exact solution.

1. Introduction

Numerous applications of visco-elastic fluids in several industrial manufacturing processes have led to renewed interest among the researchers to investigate visco-elastic boundary layer flow over a stretching plastic sheet.

Some of the typical applications of such study are polymer sheet Extraction from a dye, glass fiber and paper production drawing of plastic fibers, etc crystal growing, and the boundary layer along a liquid film in condensation processes one examples of practical applications of continuous moving surface.

Glass blowing, continuous casting and spinning of fibers also involve the flow due to a stretching surface. It has several practical applications in the field of metallurgy & Chemical Engineering.

In 1961, Sakiadi's [1961-I, 1961-II] initiated the study of the boundary layer flow over a continuous solid surface moving with constant speed.

Erickson et al [1966] extended the work of sakiadi's to account for mass transfer at this stretched sheet surface.

As we know boundary layer problems due to a stretching sheet have relevance to extrusion in plastic and metal industries, so they have received considerable attention. However, works on the flow problems due to shrinking/stretching are scarce. Wang(1990,2008) was first to study viscous flow induced by a shrinking sheet and further he studied stagnation flow towards a shrinking sheet. The proof of existence and non uniqueness, the exact solutions, both numerical and closed for, are studied by Miklavcic and Wang (2006) for the steady hydrodynamics flow due to shrinking sheet for a particular value of the suction parameter, They also showed that the solution for shrinking sheet problems not be unique at certain suction rates for both two dimensional and axisymmetric flows. Fang (2008) studied the boundary layer flow over a shrinking sheet with power-law velocity with mass transfer, and also he showed that power-law shrinking sheet offers quite interesting nonlinear behaviors. Fang and Zhang et al.(2009) gave closed form exact solutions of an MHD viscous flow over a shrinking sheet and Fang et al. (2009) studied the viscous flow over an unsteady shrinking sheet with mass transfer. Further, Fang et al .(2008) gave a new solution branch for a Blasius equation induced by a shrinking sheet. Fang and Zhang (2009) studied the heat transfer characteristics for a viscous fluid due to shrinking sheet. Fang et al.(2009) investigated the viscous flow over a shrinking sheet with a second order slip flow model.

Hayat et al. (2007) gave an analytic solution of a magnet hydrodynamics flow of a second grade fluid over a shrinking sheet and Hayat et al. (2008) investigated the flow and flow and mass transfer characteristics of an MHD upper-converted Maxwell fluid past a porous shrinking sheet. Sajid and Hayat (2009) studied an MHD viscous flow due to a shrinking sheet with the application of homotopy analysis method. Noor et al. (2009) gave a simple non-perturbative solution for an MHD viscous flow induced by a shrinking sheet.

All above studies and investigation are restricted to flow of a viscous fluid except works of Hayat et al. (2007, 2008). But in reality most liquids are non-Newtonian in nature, and are abundantly used in many industrial applications, such as in the manufacture of plastic films and artificial fibers, aerodynamics extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, crystal growing, liquid film condensation process, continuous polymer sheet extrusion, heat treated materials traveling between a feed roll, wind up roll or on a conveyer belt, geothermal reservoirs and petroleum industries(see Altan et al., 1979; Fisher,(1976); Tadmor and Klein, 1970.

By analyzing the literature review, one can see that there is no analysis of flow of a non-Newtonian liquid induced by a shrinking /stretching sheet. Motivated by the above technical and industrial applications of non-Newtonian fluid flow characteristics, we intended to analyze such as flow of shrinking sheet concept to related to a more general situation of a viscoelastic fluid flow over a permeable sheet. . Effects of various governing parameters of the flow are shown graphically and discussed.

2. Mathematical formulation

For this flow configuration, the fluid is stretched towards a slot and the flow is quite different from the stretching sheet case and mass suction is required generally to maintain the flow over a shrinking sheet. This shrinking flow is essentially a backward flow as discussed by Goldstein (1965). For a backward flow configuration, the surface moving from $+\infty$ to the slot, the fluid losses any memory of the perturbation introduced by the leading edge (slot). Therefore the flow induced by the shrinking sheet shows quite distinct phenomena from the forward stretching flow.

Consider a steady two dimensional laminar flow over a continuously shrinking sheet. The shrinking sheet velocity is $u_w = -bx$, and wall mass suction velocity is $v_w = v_w(x)$, which will be determined later. The x-axis runs along the shrinking surface in the direction opposite to the sheet motion and the y-axis is perpendicular to it. Based on the boundary layer assumptions, the governing basic equations of this problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \vartheta\frac{\partial^2 u}{\partial y^2} - k_0\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right\} \qquad (2)$$

Where υ is the kinematics viscosity, k_0 is the elastic parameter.

The boundary conditions for the velocity fields are of the form

$$u_w = -bx \qquad v_w = v_w(x) \qquad at \quad y = 0$$
$$u \to 0 \qquad as \qquad y \to \infty \qquad (3)$$

Where *b* is the stretching rate.

We define the following new variables

$$u = bxf_\eta(\eta), \qquad v = -(b\vartheta)^{\frac{1}{2}} f(\eta) \qquad \eta = \left(\frac{b}{\vartheta}\right)^{\frac{1}{2}} y \qquad (4)$$

Using Eqn .(4), Eqn. (1) is trivially satisfied and Eqn. (2) and (3) take the form

$$f_\eta^2 - ff_{\eta\eta} = f_{\eta\eta\eta} - k_1\{2f_\eta f_{\eta\eta\eta} - f_{\eta\eta}^2\} \qquad (5)$$

$$f_\eta = -1 \quad and \quad f_\eta = 1 \quad f(\eta) = s \quad at \quad \eta = 0$$
$$f_\eta(\eta) \to 0 \qquad f_{\eta\eta}(\eta) \to 0 \qquad as \quad \eta \to \infty \qquad (6)$$

Here the subscript η denotes differentiation with respect to η, s is the wall mass transfer parameter showing the strength of the mass transfer at the sheet and $k_1\left(=\frac{k_0 b}{\vartheta}\right)$ is the viscoelastic parameter.

. There is an analytical solution for $k_1 = 0$ given by Miklavcic and Wang (2006) as

$$f(\eta) = C + \frac{1}{C}\exp[-C\eta] \qquad (7)$$

Where $C = f(\infty) = f(0)$, can be found by solving $s = C + \frac{1}{C}$. It is shown that there are two solutions for this equation for any s > 2 and there is one solution for s = 2. No solution exists for s<2. There is also an algebraically decaying solution for $k = 0$ as

$$f(n) = \frac{6}{n+6} \qquad for \qquad s = \sqrt{6} \qquad (8)$$

In this paper, we will show a closed form of exact solution of Eq.(5) subjected to boundary conditions (6) which can be found by assuming the solution of the form.

$$f(n) = s + \left(\frac{e^{-\beta n} - 1}{\beta}\right) \quad \text{and} \quad f(\eta) = s + \frac{1 - e^{-\beta\eta}}{\beta} \qquad (9)$$

Where β > 0 is constant satisfying the following cubic equations foe shrinking /stretching

$$\beta^3 - \left(\frac{(1| + k_1)}{sk_1}\right)\beta^2 + \left(\frac{1}{k_1}\right)\beta - \left(\frac{1}{k_1 s}\right) = 0 \qquad \beta^3 + \frac{(1 - k_1)}{sk_1}\beta^2 + \frac{\beta}{k_1} = 0 \qquad (10)$$

for arbitrary and positive values of s, $k1$.

Using the solution (9) in Eqn. (4) the velocity components are obtained in the form

$$u = -bxe^{-\beta n}$$

$$V = -\sqrt{b\vartheta}\left\{s + \left(\frac{e^{-\beta n} - 1}{\beta}\right)\right\} \qquad (11)$$

The wall shearing stress on the surface of the stretching sheet is given by

$$\tau_w = \left[\vartheta\left(\frac{\partial u}{\partial y}\right) - k_0\left(u\frac{\partial^2}{\partial x \partial y} - 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right)\right]_{y=0} \qquad (12)$$

The local skin-friction coefficient or the frictional drag is given by

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_w^2} = 2Re_x^{-1/2}(1 - 3k_1)\bar{f}(0) \qquad (13)$$

Where $Re_x = \frac{bx^2}{\vartheta}$ is the local Reynolds number based on the length of scale χ.

3. Results and discussion

An analysis has been carried out to study the flow characteristics of a viscoelastic fluid of Walter's liquid B type from a permeable shrinking sheet. The basic boundary layer non-linear partial differential equations have been converted into a set of non-linear ordinary differential equations by using suitable similarity transformations and their analytical solutions are obtained in an exponential form.

The effects of the viscoelastic parameter and mass suction parameter have been analyzed on different profiles $(f, f_\eta, f_{\eta\eta})$ and the presented graphically .

In Figure(1a) and (1b),the effect of increasing values of viscoelastic parameter k1, is to decrease velocity of the fluid significantly the boundary layer region, this is because of the fact that introduction of tensile stress due to viscoelasticity causes transverse contraction of the boundary layer and hence, velocity decreases..

The effect of increasing values of suction parameter (S>0) is to decrease the velocity where as is has opposite effect for S<0.

The same effect has been observed in Figs (4), for the plot of $f\eta$ with increasing parametric values of k_1 and s. We can conclude that the boundary layers over a shrinking sheet are greatly different from the boundary layers due to a stretching sheet and offer more nonlinear phenomena in the boundary layer theory.

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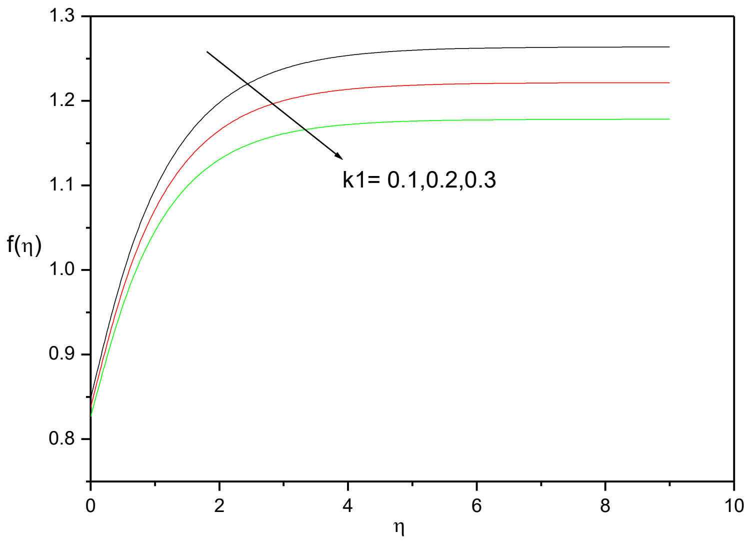
The effect of increasing values of suction parameter the imposition of the wall suction (S>0) gave the tendency to reduce the momentum boundary layer thickness this causes reduction in the velocity profiles. However opposite behavior is observed by imposition of the wall fluid blowing or injection (S<)0.

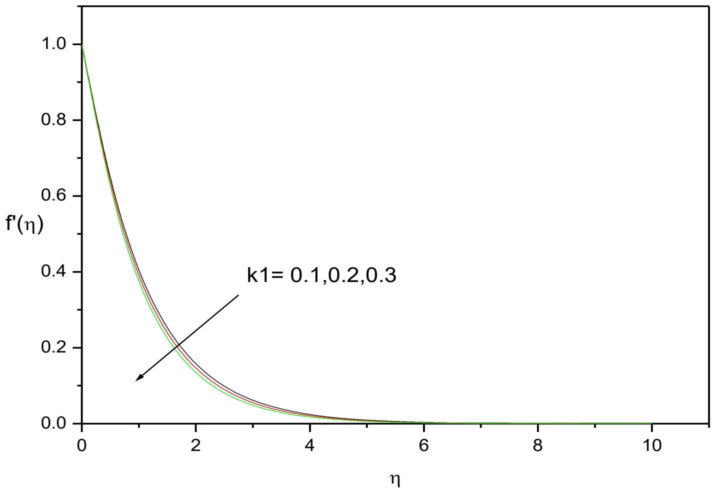
The order to have some insight of the flow characteristics results are plotted graphically for typical choice of physical parameters.

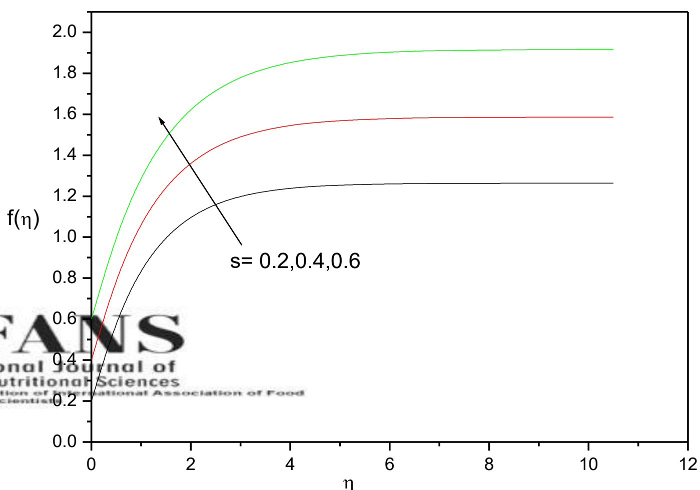
Fig (2a) & (2b) are graphical representation of horizontal velocity profile f(η) for different values of k1.Fig (4a) provides the information that the increase of viscoelastic parameter leads to the decrease of the horizontal velocity profile this is because of the fact that introduction of tensile stress due to visco-elasticity cause's transverse contraction of the boundary layer & hence velocity decreases.

Fig (2b) depicts the influence of suction blowing parameter on the velocity profiles in the boundary layer. It is known that imposition of the Wall suction (S>0) have the tendency to reduce the momentum of thickness. This causes reduction in the velocity profile. However, the opposite behavior is observed by imposition of the wall fluid or injection (S<0).

Fig(1a) and (1b) Plot of f(η) for different values of viscoelastic parameter k_1 and mass transfer parameter s in the case of stretching boundary condition







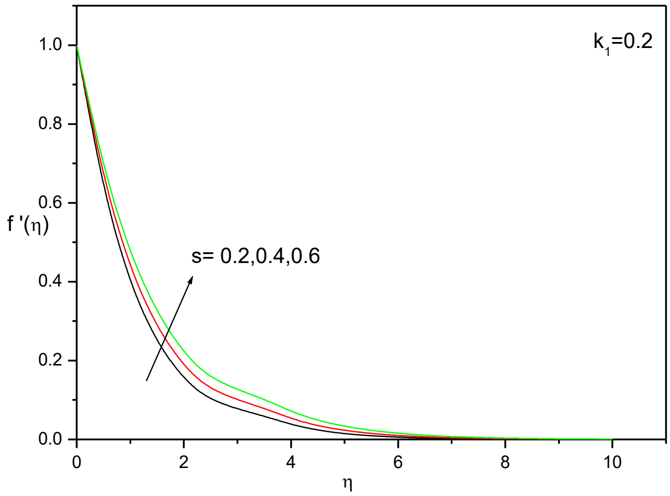


Fig (2a) and (2b) Plot of $f_\eta(\eta)$ for different values of viscoelastic parameter k_1 and mass transfer parameter s case for the stretching boundary condition.

Nomenclature

B	-	stretching rate
C	-	Constant
C	-	skin friction
f	-	dimensionless stream function
k	-	viscoelastic parameter
Re	-	Reynolds's number `
s	-	mass suction parameter
u	-	horizontal velocity components [ms]
x	-	horizontal coordinate [m]
y	-	vertical coordinate [m]
v	-	vertical velocity component [m s]
η	-	similarity variable
μ	-	dynamic viscosity [kg m s]
ρ	-	density [kg m]
t	-	Shear stress, [kgm s]
υ	-	Kinematic viscosity[m s]

Subscripts

η	-	first derivative w.r.t. η
t	-	second derivative w.r.t. η
υ	-	third derivative w.r.t. η

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