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ECCENTRIC DOM- CHROMATIC NUMBER OF GRAPHS

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ABSTRACT

For a given χ-colouring of a graph a Eccentric dominating set $S \subseteq V(G)$ is said to be Eccentric dom- Colouring set (γ_{edc})if it contains atleast one vertex of each colour class of G. In this paper we establish Eccentric Dom- Chromatic Number of Graphs. In a particular, we investigate Eccentric dom- Chromatic number for star, Path.

Keywords: Domination, Eccentric Domination, Eccentric Dom-Colouring.

1. INTRODUCTION

In the article, all the terminologies from the graph theory are used in the case of Frank Haray. A simple Undirected graph without loops or Multiple edges are Considered here. As usual p, q denote the number of vertices and edges of a graph G respectively. A path on p vertices is denoted by P_p.

DEFINITION 1.1

A closed walk in which no vertices, except the end vertices, are repeated is called the cycle and the number of edges in a cycle is called its length.

DEFINITION 1.2

A set $D \subseteq V(G)$ of vertices in a graph G is a dominating set if every vertex v in $V - D$ is adjacent to a vertex in D. The Minimium Cardinality of a dominating set of G is called the domination number of G and is denoted by $\gamma(G)$.

DEFINITION 1.3

A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists atleast one eccentric point of v in D.

If D is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. But $D'' \subseteq D$ is not necessarily an eccentric dominating set.

A eccentric dominating set D is a minimal eccentric dominating set if no proper subset $D'' \subseteq D$ is an eccentric dominating set.

DEFINITION 1.4

The eccentric domination number $\gamma_{ed}(G)$ of a graph G equals the minimum Cardinality of an eccentric dominating set, That is, $\gamma_{ed}(G) = \min |D|$ where, the minimum is taken over D in D, Where D is the set of all Minimal eccentric dominating sets of G.

DEFINITION 1.5

A coloring of a graph G is an assignment of colors to its vertices so that no two adjacent vertices have the same color. The set of all vertices with any one color is independent and is called a color class. A graph which uses k- colours is called k- colouring. The chromatic number (G) is defined as the minimum k for which G has a k-coloring.

DEFINITION 1.6

For a given χ-colouring of a graph G, a dominating set $S \subseteq V(G)$ is said to be dom-Colouring set if it contains atleast one vertex of each colour class of G.

Eccentric Domination Chromatic Number of Graphs

DEFINITION

For a given χ-colouring of a graph G, a Eccentric dominating set $S \subseteq V(G)$ is said to be Eccentric dom- Colouring set (γ_{edc}) if it contains atleast one vertex of each colour class of G.

Theorem 1.1

For all Star graphs G denoted by $K_{1.p}, p \geq 1$

i) $\gamma_{edc} = 2$

ii) $\gamma(G) \neq \gamma_{edc}(G)$

iii) $\gamma_{edc}(G) = \gamma_{edc}(G)$.

Proof:

Let G be any Star graph $K_{1.p}, p \geq 1$.Label the vertices of the partite sets v_1 to indicate that they are a part of the first partite set and $\{v_{2,}v_3 \dots . v_p\}$ to indicate that they are a part of the second partite set. The minimum dominating set D of G is $\{v_1\}$. That is $D = \{v_1\}$. Therefore $\gamma(K_{1.p}) = 1$. But D is not a eccentric dominating set. Therefore it is necessary to add another vertex from the other partite set. Hence $D = \{v_{1,}v_i\}, i \geq 2$ is the minimum eccentric dominating set and also contains atleast one vertex from each colour class. Thus $\gamma_{edc}(K_{1.p}) = 2$. Since $\gamma(K_{1.p}) = 1$ and $\gamma_{edc}(K_{1.p}) = 2$. Therefore $\gamma(G) \neq \gamma_{edc}(G)$. The minimum eccentric dominating set D of G is $\{v_1, v_2\}$. Thus $\gamma_{edc}(K_{1.p}) = 2$. We have $\gamma_{edc}(K_{1.p}) = \gamma_{edc}(K_{1.p}) = 2$.

Theorem 1.2

For any path P_p with $p \geq 3$ vertices

$$\gamma_{edc}(P_p) = \left\lceil \frac{p}{3} \right\rceil, \text{ if } p = 3k + 1$$

$$\gamma_{edc}(P_p) = \left\lceil \frac{p}{3} \right\rceil + 1, \text{ if } p = 3k \text{ or } p = 3k + 2$$

Proof:

Case i) $p = 3k$

Let $v_1, v_2, v_3, \dots, v_n$ represent the path P_p. $D = \{v_{1,}v_{2,}v_3, \dots, v_{3k-1}\}$ is the minimum domination set of P_p, but not an eccentric dominating set.

$D_1 = \{ v_1, v_4, v_7, \dots . v_{3k-2}, v_{3k}\}$ is the minimum eccentric dominating set and it contains at least one vertex from each class colour.

Hence $\gamma_{edc}(P_{3k}) = \gamma_{edc}(P_{3k})$

$$= \left\lceil \frac{p}{3} \right\rceil + 1$$

Case ii) $p = 3k + 1$

Let $D = \{v_{1,}v_{4,}v_7 \dots . v_{3k-2}, v_{3k+1}\}$ is the minimum domination set and minimum eccentric dominating set in P_p and also it contains atleast one vertex from each class colour.

Hence $\gamma_{edc}(P_p) = \gamma_{edc}(P_p) = \gamma(P_p) = \left\lceil\frac{p}{3}\right\rceil$

Case iii) $\boldsymbol{p = 3k + 2}$

Let $D = \{v_{1,} v_{5,} v_8, \ldots, v_{3k+2}\}$ is the minimum eccentric dominating set and also it contains at least one vertex from each class colour.

Therefore $\gamma_{edc}(P_p) = \gamma_{edc}(P_p)$

$$= \left\lceil\frac{p}{3}\right\rceil + 1$$

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