IJFANS International Journal of Food and Nutritional Sciences

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 10, Spl Iss 1, 2021

Method for Series Decomposition of Asymmetric Non-Linear Oscillations

B. NAGESWARA RAO, Department of Physics, Koneru Lakshmaiah Education Foundation, Green Fields, Guntur District, Vaddeswaram, Andhra Pradesh 522502, India

Abstract STSDM is recognized as an effective technique for addressing a broad spectrum of challenges encountered in Engineering and Sciences. It yields a series solution that diverges, akin to the approach employed in DTM. In the pursuit of obtaining periodic solutions, the strategy of modified DTM is harnessed, incorporating LT for achieving this goal. By integrating Pade's Approximant and ILT, researchers introduce a means to derive periodic solutions. Through an examination of a Standard Duffing Equation of Motion marked by symmetric oscillations, numerous scholars have delved into the efficacy of STSDM. This present research endeavor probes the utility of STSDM in deciphering the asymmetric oscillations inherent to a nonlinear Simple Helmholtz Equation of Motion.

1. INTRODUCTION

A ODE of the Duffing oscillator which is nonlinear and 2nd order is

$$\ddot{x} + \epsilon x + g(x) = G_0(t)$$

$$\ddot{x} = \frac{d^2x}{dt^2}.$$

$$\epsilon = dampingFactor.$$

$$G_0(t) = ForcingFunctiondependingont.$$

Restoring force function is g(x) and is given by

$$g(x) = ax + bx^2 + cx^3 + d \tag{1.1}$$

When either "n" is non-zero or "d" is non-zero, asymmetric oscillations are indicated by equation (1.1)[1]. If both "b" and "d" are zero, equation (1.1) describes symmetric oscillations. Positive values of "c" signify a hardening type system [2], while negative values of "c" represent a softening type system[3].

STSDM is recognized as a proficient technique for solving numerous complex physical problems in the fields of Engineering and Sciences [4]. STSDM simplifies the integration of highly nonlinear integral functions [5]. The series expansion of the solution yields consistently high rates of convergence. Nonlinear terms within the differential equation (DEq) are decomposed using the AP expression [6]. Akinola et al. applied the STSDM method, combining ST, Series Expansion, and AP Expressions, to investigate oscillatory behavior [7]. This study delves into the [8]adaptability of STSDM in obtaining solutions for asymmetric oscillations within a Simple Nonlinear Helmholtz Equation of Motion [9]. It ultimately concludes that STSDM falls short in distinguishing the non-periodic behavior from the nature of asymmetric oscillations [10].

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 10, Spl Iss 1, 2021

For additional references, please refer to sources [11].

2. ANALYSIS

Fig.1 depicts the comparison between (2.7) and (2.8) . (2.7) is diverging, while, (2.7) depicts behaviour of oscillation. In the same way, the STSDM solution for

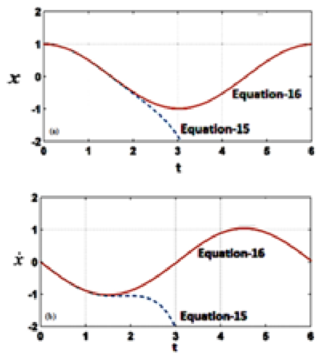

FIGURE 1. Comparison between (2.7) and (2.8)

B=5, and a=1,b=0.1 as

(2.9)

$$x(t) = (COSINE(1.186306t))(4.943389) + (COSINE(3.097204t))(0.056611)$$

The solution for , and is got as

(2.10) $x(t) = (COSINE(1.2205t))(5.9250) + (COSINE(3.2109t))(0.07496)$

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 10, Spl Iss 1, 2021

$$Sx(t) = B - v^2 S[ax(t) + bx^2(t)]$$

Applying the IST to (2.3) , one can see

(2.4) $$x(t) = B - S^{-1}[v^2(ax(t) + bx^2(t))]$$

Considering the series solution $x(t) = \sum_{m=0}^{\inf} x_m(t)$ and putting in (2.4) as

(2.5) $$\sum_{m=0}^{\inf} x_m(t) = B - S^{-1}[v^2 S(\sum_{m=0}^{inf} B_n)].$$

The AP functions in (2.5) are

(2.6) $$A_n = \frac{1}{m!}\frac{d^m}{d\theta^m}[\sum_{k=0}^{\inf}\theta^k a x_k + b(\sum_{k=0}^{\inf}\theta^k x_k)^2]|_\theta = 0 = a x_m + b\sum_{k=0}^{\inf} x_k x_{m-k}.$$

Inserting (2.6) in (2.5) and making a comparison imply

$$\begin{aligned}
x_0(t) &= B \\
x_{m+1}(t) &= -S^{-1}[v^2 S(A_m)]m \geq 0 \\
x_1(t) &= -(aB + bB^2)\frac{t^2}{2} \\
x_2(t) &= (a + 2bB)(aB + bB^2)\frac{t^4}{24} \\
x_3(t) &= -[(a + 2bB)^2 + 6b(aB + bB^2)](aB + bB^2)\frac{t^6}{720}.
\end{aligned}$$

For B=1, and the series solution of equations (2.1) and (2.2) are obtained is

$$x(t) = x_0(t) + x_1(t) + x_2(t) + x_3(t) + ...$$

(2.7) $$x(t) = 1 - 1.1\frac{t^2}{2} + 1.32\frac{t^4}{24} - 2.31\frac{t^6}{720}.$$

The series solution (2.7) is incapable to present the periodicity. Inserting LT, PA and the ILT as in the MDTM [19], the solution of the problem is got as
(2.8)

The solution described by equations (3) [12]is considered periodic when the amplitude range falls within the interval (-10, 5). If "B" does not fall within the range of (-10, 5), the solution becomes non-periodic.[13] For the case of B=1, the positive and negative amplitudes of oscillations from equation (5) are 1 and -1.0717, respectively[14]. This results in an asymmetry of the PHASE DIAGRAM with respect to the axis, while it remains symmetric with respect to the x-axis.[15] The PHASE DIAGRAM shown in Figure 2 illustrates the behavior based on the values of amplitude (B), specifically for B=1, 5, and 6. The PHASE DIAGRAM serves as a closed boundary for a range of values.[16] Notably, the PHASE DIAGRAM acts as a separatrix when representing . However, the PHASE DIAGRAMS do not manifest closed boundaries for [17]. In contrast, equations (2.8) to (2.10)[18] exhibit closed boundaries with a nearly symmetric oscillation pattern, as depicted in Figure 3[19].

3. CONCLUSION

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 10, Spl Iss 1, 2021

Asymmetric oscillations of the Helmholtz Equation of Motion were initially investigated through phase diagrams. While STSDM is purported to be an efficient method

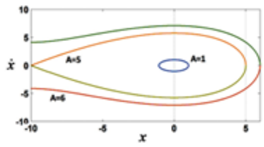

FIGURE 2. Phase diagrams from(2.3) for the amplitudes(B)

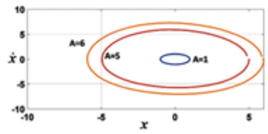

FIGURE 3. Phase diagrams from (2.8) to(2.10) for the amplitudes(B)

Although STSDM (Semi-Analytical Technique based on Simplification and Dominance Method) holds potential for addressing non-linear oscillatory systems, it falls short in effectively recognizing two essential aspects: asymmetric oscillations and the non-periodic nature inherent in a Simple Helmholtz Equation of Motion. Despite its seemingly straightforward approach, the practical application of STSDM entails a cumbersome computational process. This is primarily attributed to the intricate task of modifying initial conditions in every iteration of the procedure.

A notable challenge arises from the observed significant discrepancies, particularly evident in the negative amplitudes of asymmetrical oscillations. These discrepancies undermine the accuracy and reliability of the results generated by STSDM, particularly in cases involving oscillations of an asymmetric nature. Consequently, the outcomes provided by STSDM may not faithfully represent the true behavior of the oscillatory system under consideration.

In light of these limitations, the Simple Helmholtz Equation of Motion emerges as a valuable benchmark against which to gauge the effectiveness and accuracy of newly developed mathematical and numerical techniques.

REFERENCES

[1] M. S. SARMA, A. P. BEENA, B. NAGESWARA RAO: Applicability of perturbation tech nique to the periodic solution of x3 + x2 + x3 = 0, J Sound and Vibration, 183 (1995), 177– 184.

[2] R. H. ENNS, G. C. MCGUIRE: Nonlinear physics with mathematica for scientists and Engineers, Bikhauser, Boston, 2001.

[3] L. CVETICANIN: Ninety years of Duffing’s equation, Theoretical and Applied Mechanics, 40(1) (2013), 49– 63.

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 10, Spl Iss 1, 2021

[4] J. PETER PRAVEEN, B. NAGESWARA RAO: On the periodic solutions of the Helmholtz Equation using the modified differential transform method, MAYFEB Journal of Mathemat ics, 2 (2016), 7– 16

[5] A. O. EL-NANDY, M. M. A. LASHIM: Approximate Solution of Nonlinear Duffing Oscil lator Using Taylor Expansion, Journal of Mechanical Engineering and Automation , 6(5) (2016), 110– 116.

[6] R. NAJAFI, B. N. SARAY: Numericalsolution of the forced Duffing equations using Legendre multi-wavelets, Computational Methods for Differential Equations, 5(1) (2017), 43– 55.

[7] J. SUNDAY: The Duffing Oscillator: Applications and Computational Simulations ,Asian Research Journal of Mathematics, 2(3) (2017), 1– 13.

[8] J. PETER PRAVEEN, B. NAGESWARA RAO: Applicability of the MDTM to obtain the pe riodic solution of the Duffing type equations of motion, Asian Journal of Mathematics and Computer Research, 18(1) (2017), 19– 27.

[9] J. PETER PRAVEEN, B. NAGESWARA RAO: UncertaintiesinthePeriodic Solution of a Truly Non-linear Oscillator Differential Equation Using MDTM, International Journal of Applied and Computaional Mathematics , 5(4) (2019), 1– 12.

[10] S. MOMANI, V. S. ERTUK: Solutions of non-linear oscillators by the modified differential transform method, Computers and Mathematics with Applications, 55(4) (2008), 833 842.

[11] Z. H. KHAN, R. GUL, W. A. KHAN: Application of Adomian decomposition method for sumudu transform, NUST Journal of Engineering Sciences, 1(1) (2008), 40– 44.

[12] F. B. M. BELGACEM: Sumudu transform applications to Bessel functions and equations, Applied Mathematical Sciences, 4(74) (2010), 3665– 3686.

[13] F. B. M. BELGACEM, R. SILAMBARASAN : Laplace transform analytical restructure, Ap plied Mathematics, 4 (2014), 919– 932.

[14] M .A. RAMADAN, M. S. AL-LUHAIBI: Application of Sumudu Decomposition Method for Solving Linear and Nonlinear Klein-Gordon Equations, Int J Soft Computing and Engineer ing, 3(6) (2014), 138– 141.

[15] E. I. AKINOLA, I. A. OLOPADE, F. AKINPELU, A. O. AREO, A. A.OYEWUMI: Sumudu Transform Series Decomposition Method for Solving Nonlinear Volterra Integro-Differential Equations, IntJ Innovation and Applied Studies, 18(1) (2016), 138– 141.

[16] E. I. AKINOLA, J. OLOPADE, F. AKINPELU, J. OWOLABI: On the Application of Sumudu transform series decomposition method and oscillation Equations, Asian J Mathematics, 2(4) (2017), 1– 10.

[17] R. KHANDELWAL, P. KUMAWAT, Y. KHANDELWAL: Study of natural transform based on decomposition method for solving nonlinear ordinary differential equation, IntJ Statistics and Applied Mathematics, 3(2) (2018), 664– 669.

[18] S. M. KANG, Z. IQBAL, M. HABIB, W. NAZEER: Sumudu Decomposition Method for Solving Fuzzy Integro-Differential Equations, Axioms,8(74) (2019), 1– 18.

[19] J. PETER PRAVEEN, R. DONTHI, S. V. PRASAD, B. VENKATESWARLU: On stochastic linear regression model selection, AIP Conference Proceedings, 2177 (2019), ID020068-1 020068-5