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Mean Square Difference Cordial Labeling of Path Related Graphs

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ABSTRACT

Mean Square Difference Cordial Labeling of a graph G with vertex set V is a bijection from V(G) to {1, 2, ……,|V(G)| } such that an edge uv is assigned 0, if $\left\lfloor \frac{|[f(u)]^2-[f(v)]^2|}{2} \right\rfloor$ is even and an edge uv is assigned 1, if $\left\lfloor \frac{|[f(u)]^2-[f(v)]^2|}{2} \right\rfloor$ is odd then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a mean square difference cordial labeling is called a Mean Square Difference Cordial graph.

This paper elucidates Mean Square Difference Cordial Labeling of Centipede, Total graph of path, Quadrilatornal Snake, Hurdle, ladder and $P_n \odot A(k_1)$ graphs.

Keywords: Centipede, Total graph of path, Quadrilatornal Snake, Hurdle, ladder,$P_n \odot A(k_{1,n})$, Mean Square Difference Cordial Labeling, Mean Square Difference Cordial graph.

1. INTRODUCTION

Allocation of labels to vertices or edges or both under some constraints is known as graph labeling. If labels are given to vertices then the resultant labeling is known as vertex labeling and if labels are given to edges then the resultant labeling is known as edge labeling. Here we considered only simple, finite, connected and undirected graphs. Labeling techniques and

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extensive survey we refer Gallian[1]. For various definitions and review relevant to present study we refer F. Harary[2].

2. PRELIMINARIES

Definition 2.1

Mean Square Difference Cordial Labeling of a graph G with vertex set V is a bijection from V(G) to $\{1,2,\ldots\ldots,|V(G)|\}$ such that an edge uv is assigned 0, if $\left\lfloor\frac{|[f(u)]^2-[f(v)]^2|}{2}\right\rfloor$ is even and an edge uv is assigned 1, if $\left\lfloor\frac{|[f(u)]^2-[f(v)]^2|}{2}\right\rfloor$ is odd then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a mean square difference cordial labeling is called a Mean Square Difference Cordial graph.

Definition 2.2

A graph $G(V, E)$ obtained by a path by attaching exactly two pendent edges to each vertices of the path is called a centipede graph.

Definition 2.3

The total graph T(G) of G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G. When $G = P_n$, the total graph of path is $T(P_n)$.

Definition 2.4

Quadrilatornal Snake Q_n is obtained from a path $u_1, u_{2,} \ldots \ldots u_n$ by joining u_i to u_{i+1} to new vertices v_i and w_i repectively, and joining v_i and w_i. That is every edge of a path is replaced by a cycle c_4.

Definition 2.5

A graph obtained from a path P_n by attaching a pendent edges to every internal vertices of the path. It is called Hurdle graph with n-2 hurdles and is denoted by Hd_n.

Definition 2.6

A ladder graph L_n is defined by $L_n = P_n \times K_2$ where P_n is a path of n vertices and x denotes the Cartesian product and K_2 is a complete graph with two-vertices.

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Definition 2.7

Let G be the graph obtained by joining pendant edges alternately to the vertices of path $P_n (n > 3)$, where the pendant edges starts from the first vertex. G is denoted by the symbol $P_n \odot A(k_1)$.

3. MAIN RESULTS

Theorem 3.1

Centipede is a Mean Square Difference Cordial Graph.

Proof

Let $G = P_n \odot 2k_1$ be a centipede graph

Let V (G) = $\{v_i;\ 1 \leq i \leq 3n\}$ and

E (G) = $\{v_{3i-2}v_{3i-1}\ ; 1 \leq i \leq n\} \cup \{v_{3i-1}v_{3i}\ ; 1 \leq i \leq n\} \cup \{v_{3i-1}v_{3i+2}\ ; 1 \leq i \leq n-1\}$

Then G is of order 3n and size 3n-1.

The function f: V (G) →$\{1, 2\ldots\ 3n\}$is defined by,

$f(v_i) = i\ ; 1 \leq i \leq 3n$

The induced edge labels are defined by

$$f^*(uv) = \begin{cases} 0\ ; & \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is even} \\ 1\ ; & \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is odd} \end{cases}$$

Case (i) If n is odd

$$f^*(v_{3i-2}v_{3i-1}) = \begin{cases} 0 & ; \quad i = 2, 4, 6 \ldots . n-1 \\ 1 & ; \quad i = 1, 3, 5 \ldots . n \end{cases}$$

$$f^*(v_{3i-1}v_{3i}) = \begin{cases} 0 & ; \quad i = 1, 3, 5 \ldots . n \\ 1 & ; \quad i = 2, 4, 6 \ldots . n-1 \end{cases}$$

$$f^*(v_{3i-1}v_{3i+2}) = \begin{cases} 0 & ; \quad i = 1, 3, 5 \ldots . n-2 \\ 1 & ; \quad i = 2, 4, 6 \ldots . n-1 \end{cases}$$

Hence we observe that

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$e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor; \; e_f(1) = \left\lfloor \frac{3n}{2} \right\rfloor$

Case (ii) If n is even

$$f^*(v_{3i-2}v_{3i-1}) = \begin{cases} 0 \; ; \; i = 2,4,6 \ldots.n \\ 1 \; ; \; i = 1,3,5 \ldots.n-1 \end{cases}$$

$$f^*(v_{3i-1}v_{3i}) = \begin{cases} 0 \; ; \; i = 1,3,5 \ldots.n-1 \\ 1 \; ; \; i = 2,4,6 \ldots.n \end{cases}$$

$$f^*(v_{3i-1}v_{3i+2}) = \begin{cases} 0 \; ; \; i = 1,3,5 \ldots.n-1 \\ 1 \; ; \; i = 2,4,6 \ldots.n-2 \end{cases}$$

Hence we observe that

$e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor, \; e_f(1) = \left\lfloor \frac{3n-1}{2} \right\rfloor$

In both the cases we notice that $|e_f(0) - e_f(1)| \leq 1$

Hence centipede graph admits Mean Square Difference Cordial labeling.

Centipede graph is a Mean Square Difference Cordial graph.

Example 3. 2

$\mathbf{P_7 \odot 2k_1}$

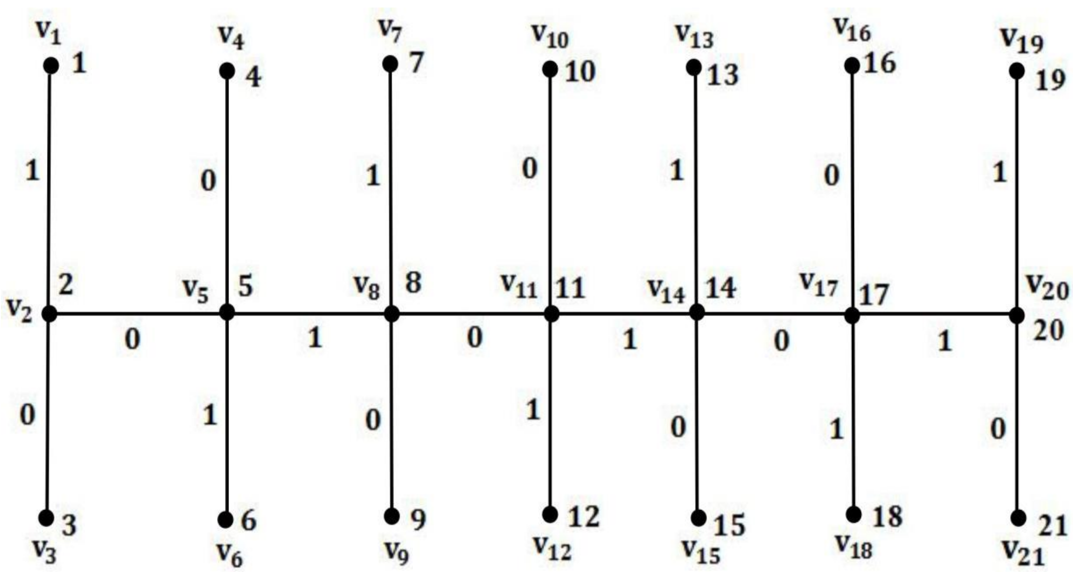

Figure 3.2.1

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$P_6 \odot 2k_1$

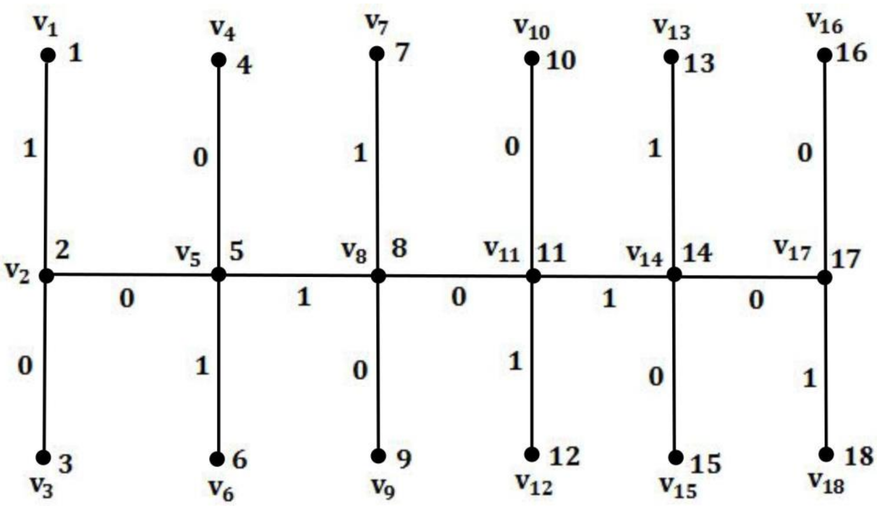

Figure 3.2.2

Theorem 3.3

The total graph of path is a Mean Square Difference Cordial Graph.

Proof

Let G = T (P_n) be a total graph of path P_n.

Let V (G)) = $\{ v_i ; 1 \leq i \leq n\} \cup \{u_i ; 1 \leq i \leq n-1\}$ and

E(G)) = $\{a_i, b_i, c_i ; 1 \leq i \leq n-1\} \cup \{d_i ; 1 \leq i \leq n-2\}$

Where $a_i = v_i v_{i+1}$, $b_i = v_i u_i$, $c_i = v_{i+1} u_i$ for $1 \leq i \leq n-1$ and

$d_i = u_i u_{i+1}$ for $1 \leq i \leq n-2$

Then G is of size order 2n-1 and 4n-5.

The function f: V(G) → {1, 2, …. 2n-1}is defined by

$f(v_1) = 2, f(v_n) = 2n-1$

$f(v_i) = 2i \;; 2 \leq i \leq n-1$

$f(u_i) = 2i-1 \;\; ; 1 \leq i \leq n-1$

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The induced edge labels are defined by

$$f^*(uv) = \begin{cases} 0 \; ; \; \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is even} \\ 1 \; ; \; \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is odd} \end{cases}$$

$f^*(a_i) = 0 \; ; \; 1 \le i \le n-1$

$f^*(b_i) = 1 \; ; \; 1 \le i \le n-1$

$f^*(c_i) = 1 \; ; \; 1 \le i \le n-2 \, , \; f^*(c_{n-1}) = 0$

$f^*(d_i) = 0 \; ; \; 1 \le i \le n-2$

Hence we observe that

$e_f(0) = 2n-2 \; ; e_f(1) = 2n-3$

Thus $|e_f(0) - e_f(1)| \le 1$

In the above two cases total graph of path $T(P_n)$ admits Mean Square Difference Cordial Labeling. Hence the total graph of path is Mean Square Difference Cordial graph.

Example 3.4

T (P_9)

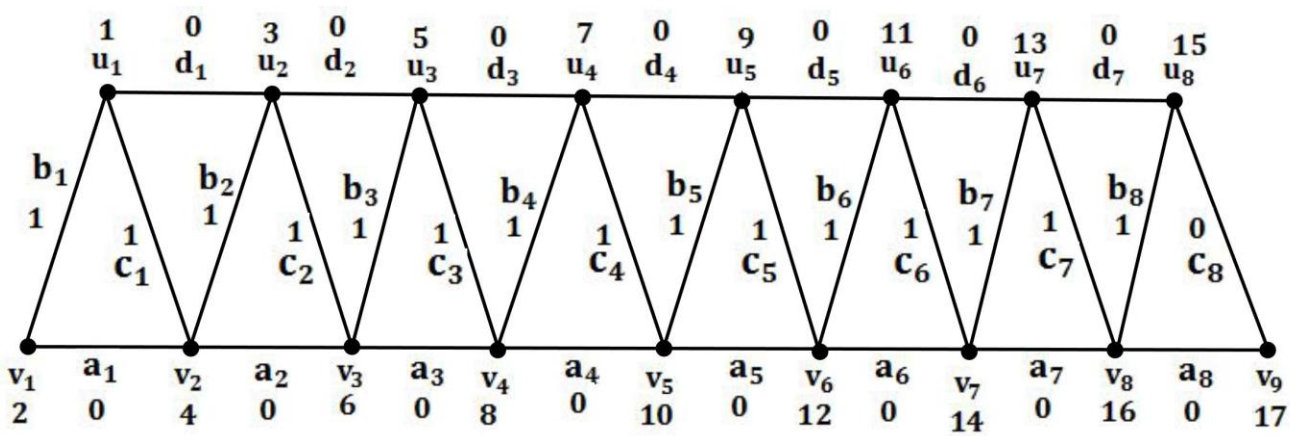

Figure 3.4.1

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Theorem 3.5

Quadrilateral Snake is a Mean Square Difference Cordial Graph.

Proof:

Let $G = QS_n$ be a Quadrilateral Snake graph.

Let $V(G) = \{u_i;\ 1 \le i \le n+1\} \cup \{v_i;\ 1 \le i \le n\} \cup \{w_i; 1 \le i \le n\}$ and

$E(G) = \{u_iu_{i+1};\ 1 \le i \le n\} \cup \{u_iv_i;\ 1 \le i \le n\} \cup \{u_{i+1}w_i;\ 1 \le i \le n\} \cup \{v_iw_i;\ 1 \le i \le n\}$

Then G of order 3n+1 and size 4n

Case(i) : If n is odd

The function f: V(G) → {1,2, ….3n+1}is defined by

$f(u_1) = 1$, $f(v_1) = 2$, $f(w_1) = 3$.

$f(v_i) = 3i-1$; $2 \le i \le n$

$f(w_i) = 3i$; $2 \le i \le n-1$

$f(w_n) = 3n+1$

$f(u_{i+1}) = 3i+1$; $1 \le i \le n-1$

$f(u_{n+1}) = 3n$

The induced edge labels are defined by

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$$f^*(uv) = \begin{cases} 0 \ ; \ \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is even} \\ 1 \ ; \ \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is odd} \end{cases}$$

$$f^*(u_i u_{i+1}) = \begin{cases} 0 \ ; \ i = 2, 4, 6, \ldots \ldots \ldots n-1 \\ 1 \ ; \ i = 1, 3, 5, \ldots \ldots n-2 \end{cases}$$

$$f(\, u_n u_{n+1}) = 0$$

$$f^*(v_i w_i) = \begin{cases} 0 \ ; \ i = 1, 3, 5, \ldots \ldots \ldots n \\ 1 \ ; \ i = 2, 4, 6, \ldots n-1 \end{cases}$$

$$f^*(u_i v_i) = \begin{cases} 0 \ ; \ i = 2, 4, 6, \ldots . n-1 \\ 1 \ ; \ i = 1, 3, 5, \ldots \ldots n \end{cases}$$

$$f^*(u_{i+1} w_i) = \begin{cases} 0 \ ; \ i = 2, 4, 6, \ldots . n-1 \\ 1 \ ; \ i = 1, 3, 5, \ldots \ldots n \end{cases}$$

Hence we observe that

$e_f(0) = 2n$; $e_f(1) = 2n$

Thus $|e_f(0) - e_f(1)| \leq 1$

Case(ii) : If n is even

The function f: V(G) → {1, 2, …..3n+1}is defined by

$f(u_1) = 1$, $f(v_1) = 2$, $f(w_1) = 3$.

$f(u_{i+1}) = 3i+1$; $1 \leq i \leq n$

$f(v_{i+1}) = 3i+2$; $1 \leq i \leq n-1$

$f(w_i) = 3i$; $2 \leq i \leq n$

The induced edge labels are defined by

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$$f^*(uv) = \begin{cases} 0 \ ; & \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is even} \\ 1 \ ; & \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is odd} \end{cases}$$

$$f^*(u_i u_{i+1}) = \begin{cases} 0 \ ; & i = 2, 4, 6, \dots \dots \dots n \\ 1 \ ; & i = 1, 3, 5, \dots \dots n - 1 \end{cases}$$

$$f^*(v_i w_i) = \begin{cases} 0 \ ; & i = 1, 3, 5, \dots \dots n - 1 \\ 1 \ ; & i = 2, 4, 6, \dots \dots \dots n \end{cases}$$

$$f^*(u_i v_i) = \begin{cases} 0 \ ; & i = 2, 4, 6, \dots \dots \dots n \\ 1 \ ; & i = 1, 3, 5, \dots \dots n - 1 \end{cases}$$

$$f^*(u_{i+1} w_i) = \begin{cases} 0 \ ; & i = 2, 4, 6, \dots \dots \dots n \\ 1 \ ; & i = 1, 3, 5, \dots \dots n - 1 \end{cases}$$

Hence the observe that

$e_f(0) = 2n$; $e_f(1) = 2n$

Thus $|e_f(0) - e_f(1)| \leq 1$

In both cases Quadrilateral Snake QS_n admits Mean Square Difference Cordial Labeling.

Hence Quadrilateral Snake is a Mean Square Difference Cordial graph.

Example 3.6

QS_3

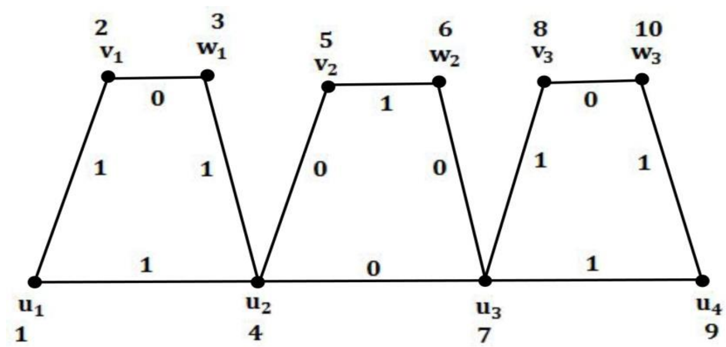

Figure 3.6.1

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$\mathbf{QS_4}$

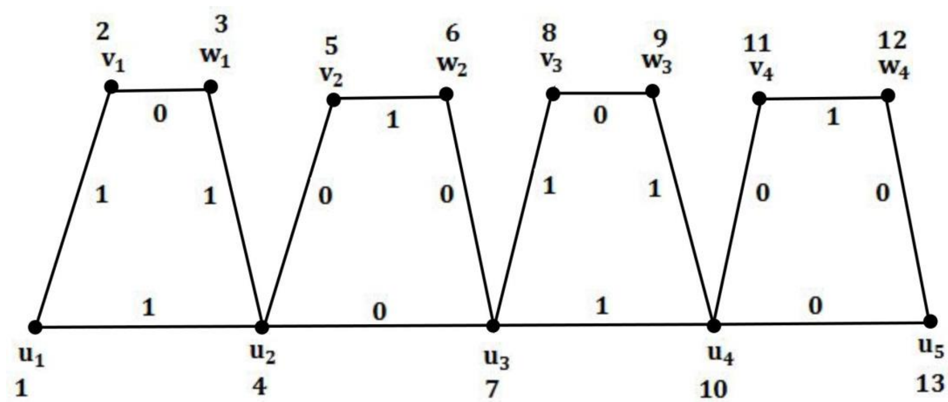

Figure 3.6.2

Theorem 3.7

Hurdle graph is a Mean Square Difference Cordial Graph.

Proof:

Let G = Hd_n be a Hurdle Graph,

Let $V(G) = \{u_i;\ 1 \le i \le n,\ v_i;\ 1 \le i \le n-2\}$ and

$E(G) = \{u_iu_{i+1};\ 1 \le i \le n-1\} \cup \{v_iu_{i+1};\ 1 \le i \le n-2\}$

Then G is of order 2n-2 and size 2n-3

Case (i) If n is odd

The function f: V(G) → {1, 2, ….. 2n-2} is defined by

$f(u_i) = 2i\ ;\ 1 \le i \le n-1$

$f(u_n) = 2n-3$

$f(v_i) = 2i-1;\ 1 \le i \le n-2$

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The induced edge labels are defined by

$$f^*(uv) = \begin{cases} 0 \; ; \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is even} \\ 1 \; ; \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is odd} \end{cases}$$

$f^*(u_i u_{i+1}) = 0 \; ; \; 1 \le i \le n-2$

$f^*(u_{n-1} u_n) = 1$

$f^*(u_{i+1} v_i) = 1 \; ; \; 1 \le i \le n-2$

We observe that

$e_f(0) = n-2 \; ; \; e_f(1) = n-1$

Thus $|e_f(0) - e_f(1)| \le 1$

Case (ii) If n is even

Define f: V(G) → {1, 2, ……, 2n-2} as follows

$f(u_i) = i; \; 1 \le i \le n$

$f(v_i) = n+i \quad 1 \le i \le n-2$

The induced edge labels are defined by

$$f^*(uv) = \begin{cases} 0 \; ; \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is even} \\ 1 \; ; \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is odd} \end{cases}$$

$$f^*(u_i u_{i+1}) = \begin{cases} 0 \; ; & i = 2, 4, 6, \dots .. n-2 \\ 1 \; ; & i = 1, 3, \dots .. n-1 \end{cases}$$

$$f^*(u_{i+1} v_i) = \begin{cases} 0 \; ; & i = 1, 3, 5, \dots .. n-3 \\ 1 \; ; & i = 2, 4, \; 6, \dots .. n-2 \end{cases}$$

We observe that

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$e_f(0) = n - 2$; $e_f(1) = n - 1$

Thus $|e_f(0) - e_f(1)| \leq 1$

In both the cases we observe that the hurdle graph Hd_n admits Mean Square Difference Cordial labeling.

Hence Hurdle graph Hd_n is a Mean square Difference graph.

Example 3.8

Hd_8

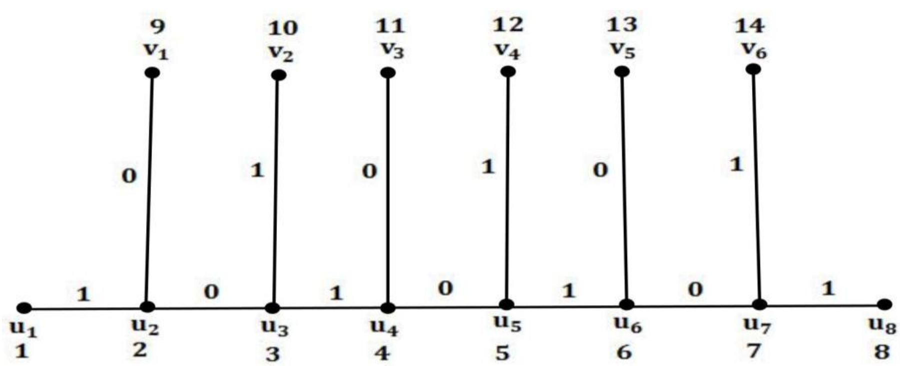

Figure 3.8.1

Hd_9

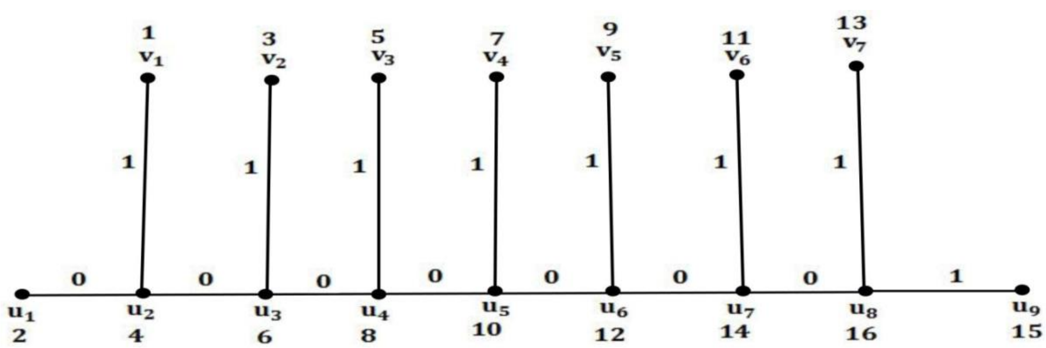

Figure 3.8.2

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Theorem 3.9

Ladder graph is a Mean Square Difference Cordial Graph.

Proof

Let $G = L_n$ be a ladder graph.

Let $V(G) = \{u_i, v_i \ ; 1 \leq i \leq n\}$ and

$E(G) = \{u_i u_{i+1} \ ; \ 1 \leq i \leq n-1\} \cup \{v_i v_{i+1} \ ; \ 1 \leq i \leq n-1\} \cup \{u_i v_i \ ; \ 1 \leq i \leq n\}$

Then G is of order 2n and size 3n-2

Case (i) If n is odd

The function $f : V(G) \rightarrow \{1, 2, \ldots..2n\}$ is defined by

$f(u_i) = i \ ; \ 1 \leq i \leq n$

$f(v_{i+1}) = 2n - i \ ; \ 0 \leq i \leq n-1$

The induced edge labels are defined by

$$f^*(uv) = \begin{cases} 0 \ ; & \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is even} \\ 1 \ ; & \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is odd} \end{cases}$$

$$f^*(u_i u_{i+1}) = \begin{cases} 0 \ ; & i = 2, 4, 6, \ldots \ldots n-1 \\ 1 \ ; & i = 1, 3, \ldots..n-2 \end{cases}$$

$$f^*(v_i v_{i+1}) = \begin{cases} 0 \ ; & i = 2, 4, 6, \ldots \ldots n-1 \\ 1 \ ; & i = 1, 3, \ldots..n-2 \end{cases}$$

$$f^*(u_i v_i) = \begin{cases} 0 \ ; & i = 2, 4, 6, \ldots \ldots n-1 \\ 1 \ ; & i = 1, 3, \ldots..n \end{cases}$$

Hence we observe that,

$$e_f(0) = \left\lfloor \frac{3n-2}{2} \right\rfloor ; \ e_f(1) = \left\lceil \frac{3n-1}{2} \right\rceil$$

Case (ii) If n is even

The function $f : V(G) \rightarrow \{1, 2, \ldots..2n\}$ is defined by

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$f(u_i) = i\ ;\ 1 \leq i \leq n$

$f(v_1) = 2n - 1$

$f(v_2) = 2n$

$f(v_{i+2}) = 2n - 1 - i\ \ ;\ \ 1 \leq i \leq n - 2$

The induced edge labels are,

$$f^*(u_i u_{i+1}) = \begin{cases} 0\ ;\ \ i = 2,4\,, \ldots..n-2 \\ 1\ ;\ \ i = 1,3,6, \ldots \ldots n-1 \end{cases}$$

$$f^*(v_i v_{i+1}) = \begin{cases} 0\ ;\ \ i = 2,4\,, \ldots..n-2 \\ 1\ ;\ \ i = 1,3, \ldots \ldots n-1 \end{cases}$$

$f^*(u_1 v_1) = 0$

$$f^*(u_i v_i) = \begin{cases} 0\ ;\ \ i = 2,4, \ldots..n \\ 1\ ;\ \ i = 3,5,, \ldots \ldots n-1 \end{cases}$$

Hence we observe that,

$e_f(0) = \frac{3n-2}{2}\ ;\ e_f(1) = \frac{3n-2}{2}$

In both cases, we notice that

$|e_f(0) - e_f(1)| \leq 1$

The ladder graph admits Mean square difference cordial labeling.

Hence ladder graph is a Mean square difference cordial graph.

Example 3.10

L_5

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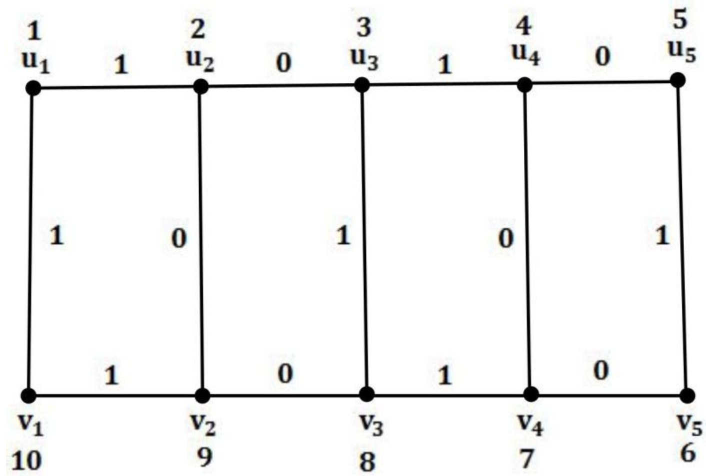

Figure 3.10.1

L_6

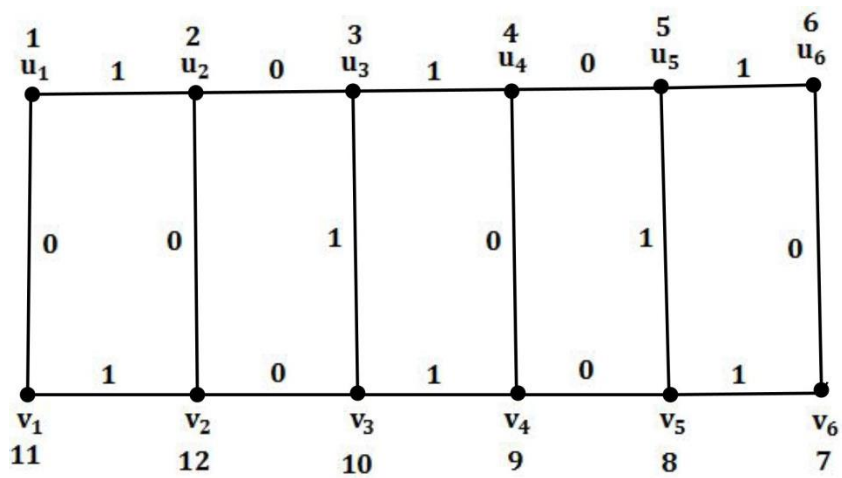

Figure 3.10.2

Theorem 3.11

The Graph $P_n \odot A(k_1)$ is a Mean Square Difference Cordial Graph.

Proof

Let $G = P_n \odot A(k_1)$ be a Graph.

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Case (i) If n is odd

Let V (G) = $\{v_i\,; 1 \le i \le n \text{ and } v_i'\,; 1 \le i \le \frac{n+1}{2}\}$ and

E (G) = $\{v_i v_{i+1}\,; 1 \le i \le n-1\} \cup \{v_1 v_1'\} \cup \{v_n v'_{\frac{n+1}{2}}\} \cup \{v_{2i+1} v'_{i+1}\,; 1 \le i \le \frac{n-3}{2}\}$

Then G is of order $\frac{3n+1}{2}$ and size $\frac{3n-1}{2}$

The function f: V (G) → $\{1, 2 \ldots \frac{3n+1}{2}\}$ is defined by,

$f(v_i) = i\,;\ 1 \le i \le n$

$f(v_i') = n + i\,;\ 1 \le i \le \frac{n+1}{2}$

The induced edge labels are defined by

$$f^*(uv) = \begin{cases} 0\ ; & \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is even} \\ 1\ ; & \left\lfloor \frac{|[f(u)]^2 - [f(v)]^2|}{2} \right\rfloor \text{ is odd} \end{cases}$$

$$f^*(v_i v_{i+1}) = \begin{cases} 0 & ;\ i = 2, 4, 6, \ldots . n-1 \\ 1 & ;\ i = 1, 3, 5, \ldots n-2 \end{cases}$$

$$f^*(v_{2i-1} v_i') = \begin{cases} 0\ ; & i = 2, 4, 6, \ldots . . \frac{n+1}{2} \\ 1\ ; & i = 1, 3, 5, \ldots . . \frac{n-1}{2} \end{cases}$$

n is classified into two cases

$$n = \begin{cases} 4k-1\ , & k = 1,2,3 \ldots . . \\ 4k+1\ , & k = 1,2,3 \ldots . \end{cases}$$

Sub case (i) when $n = 4k - 1$ where k= 1, 2, 3…..

$e_f(0) = 3k - 1\,;\ \ e_f(1) = 3k - 1$

Thus $|e_f(0) - e_f(1) =| \le 1$

Sub case (ii) when $n = 4k + 1$ where k=1, 2, 3…

$e_f(0) = 3k\,;\ e_f(1) = 3k + 1$

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Thus $|e_f(0) - e_f(1) =| \leq 1$

Case (ii) If n is even

Let V (G) = $\{v_i\,; 1 \leq i \leq n \text{ and } v_i'\,; 1 \leq i \leq \frac{n}{2}\}$ and

E (G) = $\{v_i v_{i+1}\ ; 1 \leq i \leq n-1\} \cup \{ v_1 v_1'\} \cup \{v_{2i+1} v_{i+1}'\,;\ 1 \leq i \leq \frac{n-2}{2}\}$

Then G is of order $\frac{3n}{2}$ and size $\frac{3n-2}{2}$

Define $f: V(G) \to \left\{1, 2 \ldots \frac{3n}{2}\right\}$ as follows

$f(v_i) = i\,;\ \ 1 \leq i \leq n$

$f(v_i') = n + i\,;\ \ 1 \leq i \leq \frac{n}{2}$

The induced edge labels are,

$$f^*(v_i v_{i+1}) = \begin{cases} 0\ ;\ \ i = 2, 4, 6 \ldots n-2 \\ 1\ ;\ \ i = 1, 3, 5 \ldots n-1 \end{cases}$$

$$f^*(v_{2i-1} v_i') = \begin{cases} 0\ ;\ \ i = 1, 3, 5 \ldots \frac{n-2}{2} \\ 1\ ;\ \ i = 2, 4, 6 \ldots \frac{n}{2} \end{cases}$$

n is classified into two cases

$$n = \begin{cases} 4k\,,\ \ k = 1,2,3 \ldots \\ 4k + 2\,,\ \ k = 1,2,3 \ldots \end{cases}$$

Sub case (i) when $n = 4k$ where k= 1, 2, 3…..

$e_f(0) = 3k - 1\,;\ e_f(1) = 3k$

Thus $|e_f(0) - e_f(1)| \leq 1$

Sub case (ii) when $n = 4k + 2$ where k=1, 2, 3…

$e_f(0) = 3k + 1\,;\ e_f(1) = 3k + 1$

Thus $|e_f(0) - e_f(1)| \leq 1$

In both the cases $P_n \odot A(k_1)$ admits Mean Square Difference Cordial Labeling.

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Hence the graph $P_n \odot A(k_1)$ is a Mean Square Difference Cordial Graph.

Example: 3.12

$\mathbf{P_7 \odot A(k_1)}$

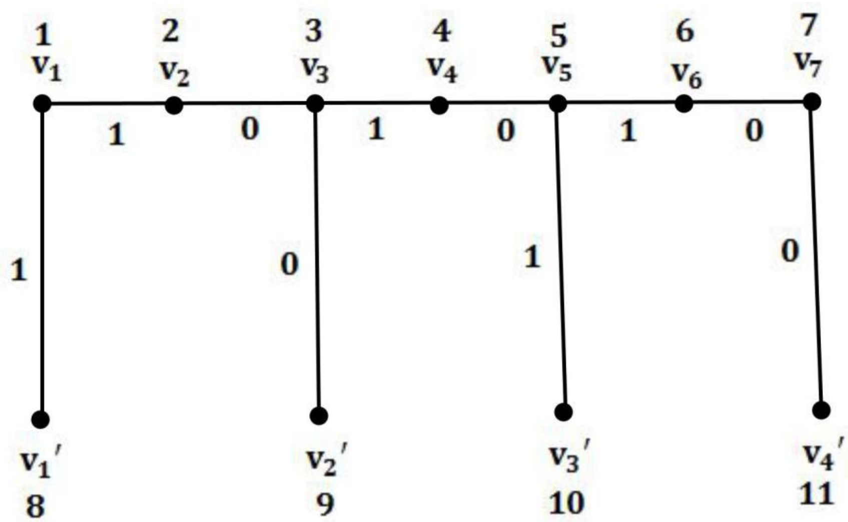

Figure 3.12.1

$\mathbf{P_8 \odot A(k_1)}$

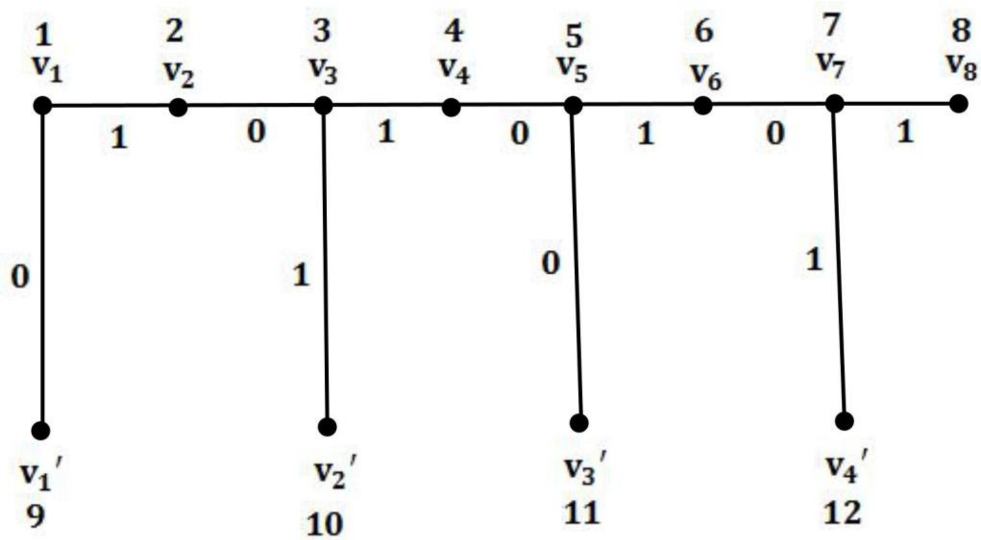

Figure 3.12.2

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4. CONCLUSION

In this paper, we have proved that the Centipede, Total graph of path, Quadrilatornal Snake, Hurdle, ladder, $P_n \odot A(k_1)$ graphs are Mean Square Difference Cordial Graphs.

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