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A Critical Evaluation on Conditional Semantic Fuzzy n-Inner Pair Surface Static Point Equations

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Abstract

In the research paper, we begin by demonstrating that any n-inner product space can be condensed down to an n-1-inner product space as a means of getting this investigation off to a strong beginning. As a result of this, we come up with the innovative concept of a fuzzy n-inner product space. Not only that, but we also show that every fuzzy n-inner product space can be shrunk down to a fuzzy n-1-inner product space. This result of this research paper gives a highly significant finding.

Introduction

In 1965, [9] proposed a formalisation of what would later become known as the fuzzy set theory. Attempts are being undertaken right now to implement this concept in areas that have not been explored before. In mathematics and, more specifically, mathematical analysis, some applications of the fuzzy concept include the fuzzy n-normed space and the fuzzy n-inner product space. These are only two examples of how the fuzzy concept can be applied. The utility of the fuzzy idea is demonstrated by these two application areas. The n-normed domain is where these two fuzzy generalisations of the domain get their start. An "n-inner product space" is described for the first time in [5,] and this term is utilised whenever n is a number greater than 2. There is some discussion of it in [2], and there is additional discussion of it in [6].

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Our first conception of fuzzy n-inner product [4] is distinct from the definition of fuzzy n-inner product in [8], which, in turn, is distinct from the definition put forward by other writers and is derived from the definition of n-inner product space [2]. This is because the space described by the n-inner product is implicit in the definition of the fuzzy n-inner product given in [8]. The reason for this is that the n-inner product defines the space. It is possible to define the fuzzy inner product [3] independently of the definitions given in [9], [3], and [10], and it is also possible to define the fuzzy 2-inner product [2] regardless of the definition that is provided in [7]. The definitions of both of these phrases are provided down below for our convenience. Since it was first presented, the idea of a fuzzy normed space has been developed further by a number of authors, one of which being [4].

In contrast to [7], [10] presents the idea of fuzzy norm space, which demonstrates that any fuzzy n-normed space may be condensed down to a fuzzy (n-1)-normed space. However, the arguments in [5] indicate that any k1,..., n1 may be used to obtain an inner product, or more specifically, a (nk)-inner product, explicitly from the n-inner product. This is demonstrated by the fact that any k1 can be used to obtain an inner product. This property will hold true for any positive integer n that is more than 2, regardless of the value.

On the other hand, the concept of the fuzzy n-inner product has not been presented as of yet. Given that every fuzzy n-inner product space can be reduced to a fuzzy (n-1), inner product space, we prove theorems in this work that demonstrate how to derive a fuzzy inner product from any fuzzy n-inner product space. By adhering to the procedures described in these theorems, it will be possible to condense all fuzzy n-inner product spaces down to a fuzzy inner product. These theorems provide an illustration of every fuzzy n-inner product space, demonstrating how to compute a fuzzy inner product from each of those spaces.

Prelude

At the first of this section let we review concept of fuzzy set that have been given by [9] and definition of n-inner product space by [2].

Definition 1. [5] Let X be any set is not empty. Then a fuzzy set A˜ in X is characterized by a membership function $\mu_{\tilde{A}} : X \to [0,1]$. Then A˜ can be written as

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X, 0 \leqslant \mu_{\tilde{A}}(x) \leqslant 1\}.$$

Definition 2. [9]. A Fuzzy point P_x in X is a fuzzy set whose membership function is

$$\mu_{P_x}(y) = \begin{cases} \alpha & \text{if} \quad y = x \\ 0 & \text{if} \quad y \neq x \end{cases}$$

for all y ∈ X, where 0 < α < 1.

We denote fuzzy points as x_α or (x, α).

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Definition 3. [9] Two fuzzy points x_α and y_β is said to be equal if x = y and α = β where α, β ∈ (0, 1].
Definition 4. [10] Let x_α be a fuzzy point and $\tilde{A}$ be a fuzzy set in X. Then x_α is said belong to $\tilde{A}$ or $x_\alpha \in \tilde{A}$ if α 6 $\mu\tilde{A}(x)$.
Definition 5. [1] Let X be a vector space over a field K and $\tilde{A}$ be a fuzzy set in X. Then $\tilde{A}$ is said to be a fuzzy subspace in X if for all x, y ∈ X and λ ∈ K satisfy

(i) $\mu\tilde{A}(x + y) \geq \min\{\mu\tilde{A}(x), \mu\tilde{A}(y)\}$
(ii) $\mu\tilde{A}(\lambda x) \geq \mu\tilde{A}(x)$.

Definition 6. [2] Let n ∈ N and , n ≥ 2, X be a vector space over R and dim(X) ≥ n. Function $\langle \cdot, \cdot | \cdot, \ldots, \cdot \rangle : \underbrace{X \times X \times \cdots \times X}_{n+1\ times} \to R$ is called n-Inner Product on X if for every $x_1, x_2, \cdots, x_n, y, z \in X$ it satisfies the conditions:

(nIP1) $\langle x_1, x_1 | x_2, ..., x_n \rangle \geqslant 0$ and $\langle x_1, x_1 | x_2, ..., x_n \rangle = 0$ if only if $x_1, x_2, ..., x_n$ are linearly dependent.
(nIP2) $\langle x_1, y | x_2, ..., x_n \rangle = \langle y, x_1 | x_2, ..., x_n \rangle$.
(nIP3) $\langle x_1, y | x_2, ..., x_n \rangle$ invariant under any permutation of $x_2, ..., x_n$.
(nIP4) $\langle x_1, x_1 | x_2, ..., x_n \rangle = \langle x_2, x_2 | x_1, x_3, ..., x_n \rangle$.
(nIP5) $\langle \alpha x_1, y | x_2, ..., x_n \rangle = \alpha \langle x_1, y | x_2, ..., x_n \rangle$ for all $\alpha \in R$.
(nIP6) $\langle y + z, x_1 | x_2, ..., x_n \rangle = \langle y, x_1 | x_2, ..., x_n \rangle + \langle z, x_1 | x_2, ..., x_n \rangle$.
A pair $(X, \langle \cdot, \cdot | \cdot, \ldots, \cdot \rangle)$ is called n-Inner Product Space.

Next, in order to avoid any misunderstandings, we will refer to an n-inner product as. We present a theorem that demonstrates how, given an n-inner product space with n more than two, one can obtain a (n k)-inner product by deriving it from the n-inner product for each. In particular, it is possible to derive an inner product from the n-inner product in every space that contains an n-inner product.

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Theorem 1. [5] Let $(X, \langle \cdot, \cdot | \cdot, \ldots, \cdot \rangle_n)$ be n-inner product space with n ≥ 2. Fix a linearly independent set $\{a_1, a_2, \ldots, a_n\}$ in X. With respect to $\{a_1, a_2, \ldots, a_n\}$, define for each $k \in \{1, 2, \ldots, n-1\}$ the $\langle \cdot, \cdot | \cdot, \ldots, \cdot \rangle_{n-k}$ on X^{n-k+1} by

$$\langle x, y | x_2, \ldots, x_{n-k} \rangle_{n-k} = \sum_{\{i_1, \ldots, i_k\} \subseteq \{1, \ldots, n\}} \langle x, y | x_2, \ldots, x_{n-k}, a_{i_1}, \ldots, a_{i_k} \rangle.$$

Then the function $\langle \cdot, \cdot | \cdot, \ldots, \cdot \rangle_{n-k}$ defines an (n − k)- inner product on X.

A number of different definitions are available for the fuzzy inner product space. The definition that is derived from an institutionalistic methodology is distinct from the definition that is derived from a fuzzy point methodology. We propose a definition of the fuzzy inner product space by making use of the fuzzy point approach, which can be found detailed in [3]. This concept is distinct from those given in [8] and [2], respectively.

Definition 7. [3].Let X be a vector space over K = C or K = R.If we define function $\langle \cdot, \cdot \rangle_f : \tilde{A} \times \tilde{A} \to K$

with $\left\langle x_\alpha, y_\beta \right\rangle_f = \langle x, y \rangle (\lambda)$ for every $x_\alpha, y_\beta \in \tilde{A}$, where $\lambda = \min\{\alpha, \beta\}$, and $\alpha, \beta \in (0, 1]$. For every $x_\alpha, y_\beta, z_\gamma \in \tilde{A}, \lambda = \min\{\alpha, \beta, \gamma\}$ and $\alpha, \beta, \gamma \in (0, 1]$, then function $\langle \cdot, \cdot \rangle_f$ is called fuzzy inner product if it satisfies the conditions:

(FIP1) $\langle x, x \rangle (\lambda) \geqslant 0$ and $\langle x, x \rangle (\lambda) = 0$ if only if $x = 0$.

(FIP2) $\langle x, y \rangle (\lambda) = \overline{\langle y, x \rangle}(\lambda)$.

(FIP3) $\langle rx, y \rangle (\lambda) = r \langle x, y \rangle (\lambda)$ for all $r \in K$.

(FIP4) $\langle x + y, z \rangle (\lambda) = \langle x, z \rangle (\lambda) + \langle y, z \rangle (\lambda)$.

(FIP5) If $0 < \sigma \leqslant \alpha < 1$ then $\langle x, x \rangle (\alpha) \leqslant \langle x, x \rangle (\sigma)$, and there exist $0 < \alpha_n \leqslant \alpha$ so that $\lim_{n \to \infty} \langle x, x \rangle (\alpha_n) = \langle x, x \rangle (\alpha)$.

A pair $(X, \langle \cdot, \cdot \rangle_f)$ is called fuzzy inner product space.

Definition 8. [13]. Let X be a vector space over $K = C$ or $K = R$. Define function

$$\langle \cdot, \cdot | \cdot \rangle_f : \tilde{A} \times \tilde{A} \times \tilde{A} \to K$$

with $\left\langle x_\alpha, y_\beta | z_\gamma \right\rangle_f = \langle x, y | z \rangle (\lambda)$ for every $x_\alpha, y_\beta, z_\gamma \in \tilde{A}$, where $\lambda = \min\{\alpha, \beta, \gamma\}$, and $\alpha, \beta, \gamma \in (0, 1]$. For every $x_\alpha, y_\beta, z_\gamma, w_\delta \in \tilde{A}, \lambda = \min\{\alpha, \beta, \gamma, \delta\}$ and $\alpha, \beta, \gamma, \delta \in (0, 1]$. Function $\langle \cdot, \cdot | \cdot \rangle_f$ is called fuzzy 2-inner product if it satisfies the following properties:

(F2P1) $\langle x, x | w \rangle (\lambda) \geqslant 0$ for each $x_\alpha, w_\delta \in P(X)$, $\langle x, x | w \rangle (\lambda) = 0$ if only if x_α, w_δ are linearly dependent.

(F2P2) $\langle x, x | w \rangle (\lambda) = \overline{\langle w, w | x \rangle}(\lambda)$.

(F2P3) $\langle x, y | w \rangle (\lambda) = \overline{\langle y, x | w \rangle}(\lambda)$.

(F2P4) $\langle rx, y | w \rangle (\lambda) = r \langle x, y | w \rangle (\lambda)$ for all $r \in K$.

(F2P5) $\langle x + y, z | w \rangle (\lambda) = \langle x, z | w \rangle (\lambda) + \langle y, z | w \rangle (\lambda)$.

(F2P6) If $0 < \sigma \leqslant \rho < 1$then $\langle x, x | w \rangle (\rho) \leqslant \langle x, x | w \rangle (\sigma)$, and there exist $0 < \rho_n \leqslant \rho$ so that $\lim_{n \to \infty} \langle x, x | w \rangle (\rho_n) = \langle x, x | w \rangle (\rho)$

A pair $(X, \langle \cdot, \cdot | \cdot \rangle_f)$ is called fuzzy 2-inner product space.

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The proof that each n-inner product space may be reduced to an n-inner product space with one less dimension can be found in [5, theorem 7]. At the moment, we are going to work on attempting to demonstrate a theorem that asserts every fuzzy n-inner product space may be reduced in size to a fuzzy n-inner product space minus one.

Definition 9. [14]. Let $n \in N$, $n \geqslant 2$ and X be a vector space over R with $\dim(X) \geqslant n$. Define a function

$$\langle \cdot, \cdot | \cdot, \ldots, \cdot \rangle_f : \underbrace{\tilde{A} \times \tilde{A} \times \cdots \times \tilde{A}}_{n+1 \ times} \to R$$

with

$$\left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, ..., x^{(n)}_{\alpha^{(n)}} \right\rangle_f = \left\langle x^{(1)}, y | x^{(2)}, ..., x^{(n)} \right\rangle (\lambda)$$

where $\lambda = \min_{i \in \{1,\ldots,n\}} \{\alpha^{(i)}, \beta\}, \alpha^{(i)}, \beta \in (0, 1]$ and $x^{(i)}_{\alpha^{(i)}} \in \tilde{A}$ for $i = 1, 2, ...n$. Function $\langle \cdot, \cdot | \cdot, \ldots, \cdot \rangle_f$ is called fuzzy n-inner productif it satisfies the conditions:

(FnIP1) $\left\langle x^{(1)}_{\alpha^{(1)}}, x^{(1)}_{\alpha^{(1)}} | x^{(2)}_{\alpha^{(2)}}, ..., x^{(n)}_{\alpha^{(n)}} \right\rangle_f \geqslant 0$ for $\forall x^{(1)}_{\alpha^{(1)}}, x^{(2)}_{\alpha^{(2)}}, ..., x^{(n)}_{\alpha^{(n)}} \in P(X)$ and $\left\langle x^{(1)}_{\alpha^{(1)}}, x^{(1)}_{\alpha^{(1)}} | x^{(2)}_{\alpha^{(2)}}, ..., x^{(n)}_{\alpha^{(n)}} \right\rangle_f = 0$ if only if $x^{(1)}_{\alpha^{(1)}}, x^{(2)}_{\alpha^{(2)}}, ..., x^{(n)}_{\alpha^{(n)}}$ are linearly dependent.

(FnIP2) $\left\langle x^{(1)}, y | x^{(2)}, ..., x^{(n)} \right\rangle (\lambda) = \left\langle y, x^{(1)} | x^{(2)}, ..., x^{(n)} \right\rangle (\lambda)$

(FnIP3)$\left\langle x^{(1)}, y | x^{(2)}, ..., x^{(n)} \right\rangle (\lambda)$ is invariant under any permutation$x^{(2)}, ..., x^{(n)}$.

(FnIP4) $\left\langle x^{(1)}, x^{(1)} | x^{(2)}, ..., x^{(n)} \right\rangle (\lambda) = \left\langle x^{(2)}, x^{(2)} | x^{(1)}, x^{(3)}, ..., x^{(n)} \right\rangle (\lambda)$.

(FnIP5) $\left\langle rx^{(1)}, y | x^{(2)}, ..., x^{(n)} \right\rangle (\lambda) = r \left\langle x^{(1)}, y | x^{(2)}, ..., x^{(n)} \right\rangle (\lambda)$ for all $r \in R$.

(FnIP6) $\left\langle x^{(1)} + z, y | x^{(2)}, ..., x^{(n)} \right\rangle (\lambda) = \left\langle x^{(1)}, y | x^{(2)}, ..., x^{(n)} \right\rangle (\lambda) + \left\langle z, y | x^{(2)}, ..., x^{(n)} \right\rangle (\lambda)$.

where $\lambda = \min_{i \in \{1,\ldots,n\}} \{\alpha^{(i)}, \beta, \gamma\}$.

(FnIP7) If $0 < \sigma \leqslant \rho < 1$ then

$$\left\langle x^{(1)}, x^{(1)} | x^{(2)}, ..., x^{(n)} \right\rangle (\rho) \leqslant \left\langle x^{(1)}, x^{(1)} | x^{(2)}, ..., x^{(n)} \right\rangle (\sigma)$$

and there exist $0 < \rho_m \leqslant \rho$ with sequence (ρ_m) converge to ρ so that

$$\lim_{m \to \infty} \left\langle x^{(1)}, x^{(1)} | x^{(2)}, ..., x^{(n)} \right\rangle (\rho_m) = \left\langle x^{(1)}, x^{(1)} | x^{(2)}, ..., x^{(n)} \right\rangle (\rho).$$

A pair $(X, \langle \cdot, \cdot | \cdot, \ldots, \cdot \rangle_f)$ is called fuzzy n-inner product space.

Main Results

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Corollary 1:

The proof that each n-inner product space may be reduced to an n-inner product space with one less dimension can be found in [5, theorem 2.7]. At the moment, we are going to work on attempting to demonstrate a theorem that asserts every fuzzy n-inner product space may be reduced in size to a fuzzy n-inner product space minus one.

Let $(X, \langle \cdot, \cdot | \cdot, \ldots, \cdot \rangle_f)$ be fuzzy n-inner product space for $n > 2$. If we define

$$\left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f = \left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f$$

where a_1 is linearly independent vector, then $(X, \langle \cdot, \cdot | \cdot, \ldots, \cdot \rangle^*_f)$ is fuzzy $(n-1)$-inner product space.

Proof.

(F(n-1)IP1) $\left\langle x^{(1)}_{\alpha^{(1)}}, x^{(1)}_{\alpha^{(1)}} | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f = \left\langle x^{(1)}_{\alpha^{(1)}}, x^{(1)}_{\alpha^{(1)}} | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f \geqslant 0$ for all $x^{(1)}_{\alpha^{(1)}}, x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \in P(X)$. If $x^{(1)}_{\alpha^{(1)}}, x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}$ are linearly dependent, then $x^{(1)}_{\alpha^{(1)}}, x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1$ are also linearly dependent, so that

$$\left\langle x^{(1)}_{\alpha^{(1)}}, x^{(1)}_{\alpha^{(1)}} | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f = \left\langle x^{(1)}_{\alpha^{(1)}}, x^{(1)}_{\alpha^{(1)}} | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f = 0.$$

Conversely, if $\left\langle x^{(1)}_{\alpha^{(1)}}, x^{(1)}_{\alpha^{(1)}} | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f = 0$, then $\left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f = 0$. Then $x^{(1)}_{\alpha^{(1)}}, x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1$ are linearly dependent. By elementary linear algebra, this can only happen if $x^{(1)}_{\alpha^{(1)}}, x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}$ linearly dependent.

(F$(n-1)$IP2)
$$\begin{aligned}\left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f &= \left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f \\ &= \left\langle y_\beta, x^{(1)}_{\alpha^{(1)}} | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f \\ &= \left\langle y_\beta, x^{(1)}_{\alpha^{(1)}} | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f\end{aligned}$$

(F$(n-1)$IP3) Since $\left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, x^{(n)}_{\alpha^{(n)}} \right\rangle_f$ is invariant under any permutation, then $\left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f = \left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f$ is also invariant under any permutation.

(F$(n-1)$IP4)
$$\begin{aligned}\left\langle x^{(1)}_{\alpha^{(1)}}, x^{(1)}_{\alpha^{(1)}} | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f &= \left\langle x^{(1)}_{\alpha^{(1)}}, x^{(1)}_{\alpha^{(1)}} | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f \\ &= \left\langle x^{(2)}_{\alpha^{(2)}}, x^{(2)}_{\alpha^{(2)}} | x^{(1)}_{\alpha^{(1)}}, x^{(3)}_{\alpha^{(3)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f \\ &= \left\langle x^{(2)}_{\alpha^{(2)}}, x^{(2)}_{\alpha^{(2)}} | x^{(1)}_{\alpha^{(1)}}, x^{(3)}_{\alpha^{(3)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f\end{aligned}$$

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$$(\mathrm{F}(n-1)\mathrm{IP5})\ \left\langle rx^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f = \left\langle rx^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f \text{ for all } r \in R$$
$$= r\left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f$$
$$= r\left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f$$

$$(\mathrm{F}(n-1)\mathrm{IP6})\ \left\langle x^{(1)}_{\alpha^{(1)}} + z_\gamma, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f = \left\langle x^{(1)}_{\alpha^{(1)}} + z_\gamma, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f$$
$$= \left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f + \left\langle z_\gamma, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}}, a_1 \right\rangle_f$$
$$= \left\langle x^{(1)}_{\alpha^{(1)}}, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f + \left\langle z_\gamma, y_\beta | x^{(2)}_{\alpha^{(2)}}, \ldots, x^{(n-1)}_{\alpha^{(n-1)}} \right\rangle^*_f$$

(F(n-1)IP7) If $0 < \sigma \leqslant \rho < 1$ then

$$\left\langle x^{(1)}, x^{(1)} | x^{(2)}, \ldots, x^{(n-1)} \right\rangle^*_f (\rho) = \left\langle x^{(1)}, x^{(1)} | x^{(2)}, \ldots, x^{(n-1)}, a \right\rangle (\rho)$$
$$\leqslant \left\langle x^{(1)}, x^{(1)} | x^{(2)}, \ldots, x^{(n-1)}, a \right\rangle (\sigma)$$
$$= \left\langle x^{(1)}, x^{(1)} | x^{(2)}, \ldots, x^{(n-1)} \right\rangle^*_f (\sigma)$$

and there exist $0 < \rho_m \leqslant \rho$ with (ρ_m) converge to ρ, so that

$$\lim_{m\to\infty} \left\langle x^{(1)}, x^{(1)} | x^{(2)}, \ldots, x^{(n-1)} \right\rangle^*_f (\rho_m) = \lim_{m\to\infty} \left\langle x^{(1)}, x^{(1)} | x^{(2)}, \ldots, x^{(n-1)}, a \right\rangle (\rho_m)$$
$$= \left\langle x^{(1)}, x^{(1)} | x^{(2)}, \ldots, x^{(n-1)}, a \right\rangle (\rho)$$
$$= \left\langle x^{(1)}, x^{(1)} | x^{(2)}, \ldots, x^{(n-1)} \right\rangle^*_f (\rho)$$

Corollary 2:

Any fuzzy (n)-inner product space can be used to produce a fuzzy (n r)-inner product space, where r can be any positive integer between 1 and n minus 2, and fuzzy (n r)-inner product spaces can be constructed from any fuzzy (n)-inner product space. To provide a further level of detail, it is possible to express any n-inner product space of fuzzy values in terms of a 2-inner product space of fuzzy values. As can be seen [3,] each fuzzy inner product space is capable of being turned into a fuzzy 2-inner product space. However, as of yet, has not been able to supply. Following is a presentation of a theorem that illustrates the equivalence of fuzzy inner product spaces and fuzzy 2-inner product spaces. This demonstration demonstrates that any fuzzy 2-inner product space is equivalent to a fuzzy inner product space.

Theorem 3. Let $(X, \langle \cdot, \cdot | \cdot \rangle_f)$be fuzzy 2-inner product space where X is a vector space over field K. If we define

$$\left\langle x_\alpha, y_\beta \right\rangle_f = \left\langle x_\alpha, y_\beta | a_1 \right\rangle_f = \langle x, y \rangle (\lambda)$$

for every $x_\alpha, y_\beta \in \widetilde{A}$ where $\lambda = \min\{\alpha, \beta\}$,$\alpha, \beta \in (0, 1]$ and a_1 is linearly independent vector then $(X, \langle \cdot, \cdot \rangle_f)$ is fuzzy inner product space.

Proof.

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(FIP1) $\langle x_\alpha, x_\alpha\rangle_f = \langle x_\alpha, x_\alpha|a_1\rangle_f \geqslant 0$ for each $x_\alpha \in P(X)$
If $x_\alpha = 0$, then x_α is linearly dependent so thatx_α, a_1 are also linearly dependent. We have $\langle x_\alpha, x_\alpha\rangle_f = \langle x_\alpha, x_\alpha|a_1\rangle_f =$ 0.Conversely, if $\langle x_\alpha, x_\alpha\rangle_f = \langle x_\alpha, x_\alpha|a_1\rangle_f = 0$, then x_α, a_1 are also linearly dependent. By elementary linear algebra, this can only happen if x_α linearly dependent, so that $x_\alpha = 0$.

(FIP2) $\left\langle x_\alpha, y_\beta\right\rangle_f = \left\langle x_\alpha, y_\beta|a_1\right\rangle_f = \overline{\left\langle y_\beta, x_\alpha|a_1\right\rangle}_f = \overline{\left\langle y_\beta, x_\alpha\right\rangle}_f$

(FIP3) $\left\langle rx_\alpha, y_\beta\right\rangle_f = \left\langle rx_\alpha, y_\beta|a_1\right\rangle_f = r\left\langle x_\alpha, y_\beta|a_1\right\rangle_f = r\left\langle x_\alpha, y_\beta\right\rangle_f$ for all $r \in K$.

(FIP4) $\left\langle x_\alpha + y_\beta, z_\gamma\right\rangle_f = \left\langle x_\alpha + y_\beta, z_\gamma|a_1\right\rangle_f = \left\langle x_\alpha, z_\gamma|a_1\right\rangle_f + \left\langle y_\beta, z_\gamma|a_1\right\rangle_f = \left\langle x_\alpha, z_\gamma\right\rangle_f + \left\langle y_\beta, z_\gamma\right\rangle_f$

(FIP5) If $0 < \sigma \leqslant \rho < 1$ then

$$\langle x_\alpha, x_\alpha\rangle_f(\rho) = \langle x_\alpha, x_\alpha|a_1\rangle(\rho) \leqslant \langle x_\alpha, x_\alpha|a_1\rangle(\sigma) = \langle x_\alpha, x_\alpha\rangle_f(\sigma)$$

and there exist $0 < \rho_n \leqslant \rho$ with (ρ_n) converge to ρ, so that

$$\lim_{n\to\infty} \langle x_\alpha, x_\alpha\rangle_f(\rho_n) = \lim_{n\to\infty} \langle x_\alpha, x_\alpha|a_1\rangle(\rho_n) = \langle x_\alpha, x_\alpha|a_1\rangle(\rho) = \langle x_\alpha, x_\alpha\rangle_f(\rho)$$

Corollary 3:

Each fuzzy n-inner product space can be thought of as a fuzzy inner product space.

Proof. Apply Theorem 2 and 3

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